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HAMILTON'S STANDARD ARITHMETIC



BOOK THREE

WITH ANSWERS

AMERICAN BOOK COMPANY

HAMILTON'S STANDARD ARITHMETIC BOOK THREE

BY

SAMUEL HAMILTON, PH.D.

Superintendent of Schools, Allegheny County, Pennsylvania
Author of "The Recitation"



AMERICAN BOOK COMPANY

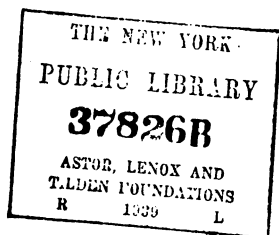
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HAM. STAND. AR. — BOOK THREE.

W. P. 4

PREFACE

THE aims of "Hamilton's Standard Arithmetic" are: first, to give the pupils such training as will lead to speed and accuracy in the use of numbers; second, to develop their power of mathematical reasoning; third, to appeal to their interests by presenting the problems in settings connected with their everyday experiences.

The series consists of three books. *Book Three* is intended to cover the work of the *seventh* and *eighth years*.

Attention is invited to the following features of this book:

1. The prominence given to drills intended to give skill.
2. The frequency of systematic reviews.
3. The easy steps in gradation.
4. The interesting character of the problems, drawn from the pupils' activities at home and at school, as well as from their relations to community life.
5. The close relation of business problems to real conditions.
6. The abundance of exercises for oral drills.
7. The utilization of the pupils' self-activity in constructive work.
8. The emphasis placed on correct interpretation of problems.
9. The encouragement of pupils to use the most economical methods for the solution of problems.

10. The training in estimating and in checking results.

11. The appeal made to observation as a stimulus to mathematical thought.

12. The omission of obsolete subjects and of problems unrelated to everyday experience.

The first two chapters contain the *seventh year's work*, and the third and fourth chapters, the *eighth year's work*. The first chapter is devoted to a review of processes through decimals and to a treatment of percentage. The second chapter places the chief emphasis on the applications of percentage, including discount, commission, taxes, simple interest, and savings bank accounts. Exercises in denominate numbers are also included. The third chapter deals chiefly with payments and collections by checks, drafts, and money orders; and with various problems on shop work, domestic science, and agriculture. The fourth chapter takes up mensuration, stocks and bonds, fire insurance, taxes and duties, longitude and time, as well as the equation and the use of letters as symbols of quantity.

SAMUEL HAMILTON.

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CHAPTER I

REVIEW OF FUNDAMENTAL OPERATIONS

NOTATION AND NUMERATION

A **unit** is a single thing ; as, one, one cent.

A **number** is a unit or a collection of units.

Numbers are used to tell *how many* ; they are expressed by figures or by letters.

The figures we now use are of Hindu origin, but the Arabs first introduced them into Europe. They are :

NAUGHT	ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE
0	1	2	3	4	5	6	7	8	9

These ten figures are, therefore, called **Arabic numerals**.

The figure 0 is called **naught**, **zero**, or **cipher**, and has no value.

Notation is a method of representing numbers by characters.

The **Arabic notation** is a method of expressing numbers by means of figures.

Numeration is a method of reading numbers expressed by figures or letters.

The method in common use in the United States is known as **French numeration**.

Arithmetic is the science of numbers and the art of computing by them.

ARABIC SYSTEM OF NOTATION AND NUMERATION

Any number containing but one figure simply stands for so many ones. Thus, 9 stands for 9 units, or ones.

In any number containing two figures, the *first place* at the right is called **ones**; the *second place* is called **tens**. Thus, in 25 there are 2 tens, or 20 ones, plus 5 ones, which equal 25 ones.

1. Name the *ones* and the *tens* in the following numbers and state how many ones each number equals :

15 25 30 75 82 60 72 45 70 99 20 10 39 47 90

In any number containing three figures, the *third place* from the right is called **hundreds**. Thus, in 325, the 3 stands for 300 ones. 325 is read "three hundred twenty-five."

2. Name the *ones*, *tens*, and *hundreds* in the following and state how many ones each number equals :

125 329 879 801 600 650 803 132 400 904
109 705 105 550 900 901 502 999 570 809

In any number containing four figures, the *fourth place* from the right is called **thousands**. 4635 is read "four thousand, six hundred, thirty-five."

3. Name the places in each number and then read :
2135 6005 6910 5604 5000 4025 2684 8709 8900

For convenience in reading large numbers, in the Arabic system, the figures are generally separated by commas into groups of three figures each, called **periods**.

The first period, counting from the right, is *units*; the second, *thousands*; the third, *millions*; the fourth, *billions*; the fifth, *trillions*; etc.

4. Write each number above from dictation.

The following table shows the arrangement of these periods, and the three orders of figures in each period:

TRILLIONS' PERIOD	BILLIONS' PERIOD	MILLIONS' PERIOD	THOUSANDS' PERIOD	UNITS' PERIOD
Hundred-trillions Ten-trillions Trillions	Hundred-billions Ten-billions Billions	Hundred-millions Ten-millions Millions	Hundred-thousands Ten-thousands Thousands	Hundreds Tens Ones
1 0 1	3 4 5	6 4 2	0 0 1	3 4 6

The number in the table is read, "101 trillion, 345 billion, 642 million, 1 thousand, 346."

In reading numbers the *name* of the units' period is omitted.

Since *ten ones* make 1 *ten*, and *ten tens* make 1 *hundred*, etc., our system of writing numbers is called a **decimal system**. The word decimal comes from the word *decem*, meaning *ten*.

Name the places and periods. Read the numbers and then write them from dictation:

- | | | |
|------------|------------------------|-------------------|
| 5. 129,475 | 9. 1,700,425 pounds | 13. 400,000,000 |
| 6. 407,575 | 10. 9,609,500 tons | 14. 709,050,050 |
| 7. 600,905 | 11. 5,505,608 yards | 15. 8,000,000,000 |
| 8. 790,505 | 12. 40,000,905 dollars | 16. 9,075,074,098 |

Copy, point off, and read :

- | | | |
|---------------|----------------|----------------|
| 17. 700102 | 24. 5050504 | 31. 60414 |
| 18. 6067004 | 25. 12126012 | 32. 102365 |
| 19. 8011100 | 26. 876491101 | 33. 14763487 |
| 20. 90314607 | 27. 104706950 | 34. 243540038 |
| 21. 910723586 | 28. 1202486414 | 35. 2452603467 |
| 22. 837421012 | 29. 3183456109 | 36. 8703005020 |
| 23. 987654321 | 30. 7891234560 | 37. 6010100100 |

ROMAN SYSTEM OF NOTATION AND NUMERATION

The seven letters used in Roman notation are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The other numbers are represented by combinations thus:

I. *When a letter is followed by the same letter or by one of less value, the values of the letters are to be added.* Thus, XX represents 20; XI represents 11.

II. *When a letter is followed by one of greater value, the value of the smaller is to be subtracted from that of the greater.* Thus, IV represents 4; IX represents 9.

III. *When a letter is placed between two letters of greater value, the value of the smaller is to be subtracted from the sum of the other two.* Thus, XIV represents 14; XIX represents 19.

A bar placed over a letter multiplies its value by 1000. Thus, \overline{V} represents 5000.

The following table further illustrates the system:

I, 1	VIII, 8	XVI, 16	LXXX, 80	DCC, 700
II, 2	IX, 9	XX, 20	XC, 90	DCCC, 800
III, 3	X, 10	XXX, 30	C, 100	CM, 900
IV, 4	XI, 11	XL, 40	CCC, 300	M, 1000
V, 5	XII, 12	L, 50	CD, 400	MCM, 1900
VI, 6	XIV, 14	LX, 60	D, 500	\overline{V} , 5000
VII, 7	XV, 15	LXX, 70	DC, 600	\overline{M} , 1,000,000

Read:

1. XLIII CDXLIX \overline{MDII} MCDXCII
2. XCIX MCMVIII MDLXXVI MDCCCLXI

3. Express in Roman notation: 41, 63, 84, 99, 107, 218, 572, 735, 996, 1907, 1564, 1616, 1000, 1600.

UNITED STATES MONEY



10 mills	= 1 cent
10 cents	= 1 dime
10 dimes	= 1 dollar
10 dollars	= 1 eagle



1. The dollar sign is \$; it is placed before the number of dollars. The sign for cent or cents is ¢; it is placed after the number of cents.

When dollars and parts of a dollar are written as one number, a period, called a **decimal point**, separates the dollars from the cents. 8 dollars and 15 cents is written \$8.15.

Parts of a dollar may be written in three ways. Thus, 15 cents may be written 15¢, \$0.15, or ¢.15.

The first two places to the right of the point are for cents.

2. Read in as many ways as you can :

15¢	\$8.32	5¢	105¢	\$8	\$.25	\$0.50
900¢	\$0.01	\$0.35	\$0.05	\$8.00	1101¢	\$9.90

The first place to the right of the point is for dimes, or **tenths** of a dollar; the second, for cents, or **hundredths** of a dollar; the third, for **mills**, or **thousandths** of a dollar. \$.025 is read, "two cents, 5 mills."

3. Read; then write from dictation, using the dollar sign :

\$0.02	\$255	\$9.055	\$.901	\$.801	\$.005
\$.022	\$8.06	\$1.005	\$.80	\$515	\$.50
\$2.50	275 dollars and 5 cents		100 cents, 8 mills		
\$0.75	10 dollars and 1 cent		525 cents, 5 mills		

In business, the seller usually regards any part of a cent as an additional cent. In computations in percentage and interest, however, less than $\frac{1}{2}$ cent in the final result is discarded and $\frac{1}{2}$ cent or a greater fraction of a cent is counted as an extra cent.

ADDITION

Addition is the process of uniting two or more numbers to form one number.

The sign $+$, called *plus*, indicates *addition*. The sign $=$, called *equal* or *equals*, indicates *equality*.

The **sum** or **amount** is the result of addition.

Like numbers are numbers that express the same units; as, 3 pounds, 5 pounds, or 3, 5. **Unlike numbers** are numbers that express different units; as, 7 days, \$3.

Only like numbers can be added.

TO THE TEACHER. — Give much oral drill on the "45 combinations," on addition by endings, on horizontal addition, etc. In column addition, encourage the pupils to look for combinations that make 5, 10, 15, 20, etc.

Written Work

1. Find the sum of 326, 736, 284.

$$\begin{array}{r} 326 \\ 736 \\ 284 \\ \hline \text{Sum, } 1346 \end{array}$$

Test by adding downwards.

In adding the first column, think 10 16 ones; then write the 6 ones and carry the 1 ten to the tens' column. 9, 12, 14 tens; then write the 4 tens and carry the 1 hundred to hundreds' column. 10, 13 hundreds. The sum is 1346.

Practice until you can add these problems rapidly:

2.	4.	6.	7.	8.	9.
154	561	\$ 729.15	206793	534891	726432
371	729	804.90	429874	567321	874921
<u>723</u>	<u>358</u>	728.16	358079	623413	785341
		574.34	427451	347621	634541
3.	5.	420.85	874297	983122	442211
534	147	345.78	298743	246893	532891
826	684	223.32	123456	972341	500030
<u>943</u>	<u>493</u>	<u>999.99</u>	<u>789034</u>	<u>826107</u>	<u>900765</u>

NOTE. — Give much drill with 3-figure numbers.

In adding two or more columns, business men frequently write the sum of each column separately on a slip of paper, and then add the separate sums as illustrated in example 10. In case of interruption or error it is then not necessary to go over all the work.

10.	11.	12.	13.	14.	15.
279	224	746	581	910	375
874	921	317	841	725	421
398	234	567	789	234	291
569	321	104	902	803	123
768	572	571	846	931	146
343	937	847	641	261	143
<u>41</u>	921	803	731	283	197
39	321	830	450	847	190
28	731	476	903	275	913
<u>3231</u>					
16.	17.	18.	19.	20.	21.
649	587	439	867	945	348
935	693	381	722	178	629
633	854	397	921	752	956
242	334	727	285	838	782
298	222	928	569	746	832
697	587	962	379	637	779
758	938	778	588	959	678
<u>758</u>	<u>938</u>	<u>778</u>	<u>588</u>	<u>959</u>	<u>678</u>

Add the following, checking your work as above :

22.	23.	24.	25.
456732	596345	734586	943867
867849	654980	496752	599689
549268	847529	876972	567458
375697	368536	534895	737869
296489	779658	789648	695745
<u>584856</u>	<u>486795</u>	<u>359485</u>	<u>845399</u>

Write from dictation and add each of the following problems in one minute :

26.	27.	28.	29.
596784	652498	734677	345967
854853	798574	872698	989689
698723	465273	984458	754865
456838	989963	769872	937358
789675	585778	898569	673947
<u>594789</u>	<u>825899</u>	<u>457394</u>	<u>495768</u>
30.	31.	32.	33.
643594	385962	498735	645774
897487	956483	876494	585396
685729	567845	539653	458742
436896	584427	765853	967638
794257	896515	428767	384380
234586	786923	295854	518987
<u>234586</u>	<u>786923</u>	<u>468769</u>	<u>625945</u>
34.	35.	36.	37.
548673	293476	493271	742506
285338	568728	867526	258959
847268	343516	978313	168723
695575	435685	439679	698465
459786	765455	746285	372925
583325	437867	483255	459685
<u>448639</u>	<u>757677</u>	<u>671639</u>	<u>776779</u>
38.	39.	40.	41.
329546	527641	486735	942178
952532	689334	357692	389767
434632	775135	343843	683245
586975	283867	718938	749687
177423	163792	532546	588459
789512	948531	863285	256524
<u>327819</u>	<u>976984</u>	<u>685397</u>	<u>351839</u>

42.	43.	44.	45.
472596	921563	764955	857694
523947	487379	485685	258776
115689	472258	321364	584259
876954	289675	378178	964524
752643	977345	523535	492537
382313	586187	619898	867954
429756	742914	564686	348693
<u>874795</u>	<u>685947</u>	<u>587464</u>	<u>985488</u>

SUBTRACTION

Subtraction is the process of finding the difference between two numbers, or of taking one number from another.

The **minuend** is the number from which we subtract.

The **subtrahend** is the number to be subtracted.

The **difference**, or **remainder**, is the result of subtraction.

The difference added to the subtrahend equals the minuend.

The sign $-$, called *minus*, indicates *subtraction*.

Only like numbers can be subtracted.

TO THE TEACHER. — Give much oral drill on the “45 combinations,” on subtraction by endings, on making change, etc.

Written Work

1. From 632 take 374.

Minuend, $632 = 500 + 120 + 12 = 5 \text{ hundreds, } 12 \text{ tens, } 12 \text{ ones}$

Subtrahend, $374 = 300 + 70 + 4 = 3 \text{ hundreds, } 7 \text{ tens, } 4 \text{ ones}$

Difference, $258 = 200 + 50 + 8 = 2 \text{ hundreds, } 5 \text{ tens, } 8 \text{ ones}$

Since 4 ones cannot be taken from 2 ones, take 1 ten = 10 ones, from 3 tens; 10 ones + 2 ones = 12 ones; 12 ones - 4 ones = 8 ones. Since 7 tens cannot be taken from the 2 tens remaining, take 1 hundred = 10 tens, from the 6 hundreds; 10 tens + 2 tens = 12 tens; 12 tens - 7 tens = 5 tens. 5 hundreds - 3 hundreds = 2 hundreds.

Test: $374 + 258 = 632$. By adding the difference to the subtrahend, you can see whether the answer is correct.

In the **Austrian method** of subtraction, we find what number added to the subtrahend will make the minuend.

Since 4 and 8 are 12, write 8 under the ones and take 1 ten from the 3 tens, leaving 2 tens. Since 7 and 5 are 12, write 5 under the tens and take 1 hundred from 6 hundreds, leaving 5 hundreds. Since 3 and 2 are 5, write 2 under the hundreds.

$$\begin{array}{r} 632 \\ 374 \\ \hline 258 \end{array}$$

TO THE TEACHER. — Teach only one method. — whichever is preferred.

Explain the steps in finding each remainder :

2. 800	3. 9004	4. 9080	5. 7001	6. 9040
594	7907	5987	4908	5879
<u>206</u>	<u>1097</u>	<u>3093</u>	<u>2093</u>	<u>3161</u>

Subtract:

7. 30984	8. 54009	9. 81704	10. 41711	11. 50000
<u>24987</u>	<u>31047</u>	<u>54270</u>	<u>39111</u>	<u>42001</u>

Write, subtract, and test four problems in 3 minutes :

12. 85980	18. 90012	24. 59001	30. 458995
<u>71409</u>	<u>79875</u>	<u>34678</u>	<u>233450</u>
13. 57004	19. \$670.01	25. 61070	31. 54100
<u>20098</u>	<u>340.97</u>	<u>46581</u>	<u>36798</u>
14. 39702	20. \$590.10	26. 590680	32. 70001
<u>21308</u>	<u>210.89</u>	<u>289796</u>	<u>58976</u>
15. 70001	21. \$953.01	27. 998076	33. 6459871
<u>39005</u>	<u>391.54</u>	<u>433015</u>	<u>2987598</u>
16. 98235	22. \$401.97	28. 598301	34. 5798371
<u>60104</u>	<u>207.58</u>	<u>303397</u>	<u>3099384</u>
17. 80021	23. \$701.49	29. 743019	35. 8342901
<u>51037</u>	<u>511.10</u>	<u>556601</u>	<u>5217809</u>

MULTIPLICATION

Multiplication is the process of taking one number as many times as there are units in another number.

The **multiplicand** is the number multiplied.

The **multiplier** is the number by which we multiply.

The **product** is the result of multiplication.

The sign \times indicates multiplication. It is read "times," when the multiplier *precedes* the sign, and "multiplied by," when the multiplier *follows* the sign.

A **concrete number** is a number used with reference to a particular object; as, 5 days, 10 pounds, 8 inches.

An **abstract number** is a number used without reference to a particular object; as, 5, 8, 20.

The multiplier is always regarded as an abstract number. The multiplicand may be either abstract or concrete.

TO THE TEACHER.—Drill on these tables until pupils thoroughly know them.

1	2's	3's	4's	5's	6's	7's	8's	9's	10's	11's	12's
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Written Work

1. Multiply \$1.25 by 43.

$$\begin{array}{r}
 \$1.25 \\
 \times 43 \\
 \hline
 375, \text{ 1st partial product} \\
 500, \text{ 2d partial product} \\
 \hline
 \$53.75, \text{ entire product}
 \end{array}$$

In practice the 0 in the right-hand place in the second partial product is omitted, as shown in the solution.

Place the decimal point in the product *directly under* the decimal point in the multiplicand.

2. \$2.53	5. \$8.09	8. \$3.27	11. \$2.41	14. \$3.1
<u>39</u>	<u>69</u>	<u>78</u>	<u>10</u>	<u>3</u>
3. \$6.08	6. \$2.25	9. \$1.04	12. \$3.74	15. \$8.9
<u>149</u>	<u>110</u>	<u>311</u>	<u>312</u>	<u>41</u>
4. \$9.09	7. \$1.29	10. \$3.05	13. \$5.05	16. \$9.0
<u>459</u>	<u>411</u>	<u>510</u>	<u>402</u>	<u>30</u>

DRILLS AND SHORT METHODS IN MULTIPLICATION

Oral and Written Work

To multiply by 11, multiply by 10 and add the multiplicand

Thus, $10 \times 253 = 2530$; $2530 + 253 = 2783$.

To multiply by 21, multiply by 20 and add the multiplicand

To multiply by 10, 100, 1000, etc., add as many ciphers to the right of the multiplicand as there are ciphers in the multiplier. Thus, $28 \times 1000 = 28,000$.

To multiply by 40, 400, 4000, etc., multiply by 4, and add as many ciphers to the product as there are ciphers in the multiplier.

To multiply by 99, first multiply by 100 and subtract the multiplicand.

Give a short method of multiplying by 9, 19, 24, 49, 99, 101, 102, etc.

Multiply the following first by 9; then by 10; then by 11; then by 100; by 19; by 40; by 99; by 12; by 24; by 101; by 400.

- | | | | | |
|--------|--------|---------|-----------|-----------|
| 1. 110 | 4. 198 | 7. 1234 | 10. 72810 | 13. 19345 |
| 2. 256 | 5. 345 | 8. 3069 | 11. 63454 | 14. 73216 |
| 3. 327 | 6. 254 | 9. 4189 | 12. 72018 | 15. 87209 |

The square of a number is the number multiplied by itself. Thus, the square of $8 = 64$; written, $8 \times 8 = 64$, or $8^2 = 64$.

16. Commit the following squares to memory :

$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$
$6^2 = 36$	$11^2 = 121$	$16^2 = 256$	$21^2 = 441$
$7^2 = 49$	$12^2 = 144$	$17^2 = 289$	$22^2 = 484$
$8^2 = 64$	$13^2 = 169$	$18^2 = 324$	$23^2 = 529$
$24^2 = 576$		$25^2 = 625$	

17. Build the multiplication tables through 15, and commit them to memory.

Multiply and test :

18. 6425	by	a. 245
19. 1024		b. 344
20. 8720		c. 564
21. 9652		d. 746
22. 8665		e. 804
23. 7894		f. 961
24. 8465		g. 869
25. 7695		h. 796
26. 8425		i. 968
27. 9476		j. 898

Form 100 problems by multiplying each multiplicand by each of the multipliers, thus:

$$18 \text{ a. } 245 \times 6425 = ?$$

$$18 \text{ b. } 344 \times 6425 = ?$$

$$25 \text{ i. } 968 \times 7695 = ?$$

Write, solve, and test each problem in 1 minute.

DIVISION

Division is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

The **dividend** is the number to be divided.

The **divisor** is the number by which we divide.

The **quotient** is the result of division.

The **remainder** is the part of the dividend remaining when the quotient is not exact.

The sign \div indicates division, and is read "divided by."

Division is indicated in *three* ways: $14 \div 2$; $2 \overline{)14}$; and $1\frac{1}{2}$.

If both the dividend and divisor are concrete, they must be like numbers.

In separating a number into equal parts, the divisor is always an abstract number and the quotient is like the dividend.

This kind of division is called **partition**.

TO THE TEACHER.—Give much oral drill on the inverses of the multiplication tables to $225 \div 15$.

LONG DIVISION

1. Divide 4310 by 21.

Steps

$$\begin{array}{r}
 205 \\
 21 \overline{)4310} \\
 \underline{42} \\
 110 \\
 \underline{105} \\
 5, \text{ remainder}
 \end{array}$$

Test: $21 \times 205 = 4305$
 $4305 + 5 = 4310$

1. Divide 43 by 21. Write the quotient figure 2 over the figure 3 of the dividend.
2. Multiply 21 by 2.
3. Subtract 42 from 43.
4. Bring down the next figure. Is 21 contained an integral number of times in 11? Write 0 in the quotient.
5. Bring down the next figure and proceed as before. Write 5 in the quotient.

Find quotients and test :

- | | | |
|------------------|-------------------|--------------------|
| 2. $252 \div 21$ | 6. $345 \div 31$ | 10. $1344 \div 42$ |
| 3. $525 \div 21$ | 7. $2214 \div 21$ | 11. $1024 \div 32$ |
| 4. $724 \div 22$ | 8. $4601 \div 22$ | 12. $1326 \div 51$ |
| 5. $642 \div 31$ | 9. $1271 \div 31$ | 13. $1922 \div 62$ |

Removing one naught from the right of a number divides the number by 10 ; removing two naughts divides it by 100 ; removing three naughts divides it by 1000 ; etc.

14. Divide 225 by 20, thus : $2 \overline{)022} \overline{)5}$ 2 tens is contained in 11 $\overline{)5}$ 22 tens 11 times, with 5 remaining ; $5 \div 20 = \frac{5}{20}$.

15. Divide 2375 by 20 ; by 200.

Divide each number by 20 ; by 50 ; by 80 ; by 500.

- | | | | |
|------------|------------|-------------|-------------|
| 16. 37,845 | 18. 90,200 | 20. 409,805 | 22. 390,075 |
| 17. 50,240 | 19. 74,079 | 21. 790,086 | 23. 985,000 |

Divide and test :

- | | | | |
|--------------|------|---------|---|
| 24. 6464341 | } by | a. 268 | Form 196 problems by dividing each dividend by each of the divisors, thus : |
| 25. 7846760 | | b. 354 | |
| 26. 5864548 | | c. 676 | |
| 27. 8645341 | | d. 758 | 24 a. $6464341 \div 268 = ?$ |
| 28. 9624872 | | e. 865 | 24 b. $6464341 \div 354 = ?$ |
| 29. 7784100 | | f. 984 | 29 c. $7784100 \div 676 = ?$ |
| 30. 6810404 | | g. 789 | Write, solve, and test each problem in $1\frac{1}{2}$ minutes. |
| 31. 7904025 | | h. 897 | |
| 32. 4867045 | | i. 509 | |
| 33. 3284567 | | j. 890 | |
| 34. 52745897 | | k. 8009 | |
| 35. 63743829 | | l. 7907 | |
| 36. 52374098 | | m. 6980 | |
| 37. 40987634 | | n. 7486 | |

PROBLEMS

1. If 48 chairs cost \$324, how much will 275 chairs cost?

\$324, cost of 48 chairs

$\$324 \div 48 = \6.75 , cost of 1 chair

$275 \times \$6.75 = \1856.25 , cost of 275 chairs

Study of Problem

1. What is given in this problem?

a. Number of chairs in each purchase.

b. Cost of 48 chairs.

2. What is required? a. Cost of 1 chair.

b. Cost of 275 chairs.

3. How do you find what is required? a. Divide cost of first purchase by the number of chairs. b. Multiply the cost of 1 chair by the number of chairs purchased.

NOTE.—The purpose of these studies is: (1) To train the pupil to understand the conditions of the problem. (2) To lead him to discover the logical steps in the solution of the problem. (3) To place emphasis upon the development of mathematical power.

2. If 2675 bushels of apples cost \$2728.50, how much are 196 bushels worth?

3. A water tank holds 8640 gallons. If it receives 728 gallons an hour by one pipe and discharges 512 gallons by another, in what time will it be filled?

4. Two steamers sail towards each other from opposite sides of the Pacific Ocean. If the distance across is 9872 miles, and one sails at the rate of 285 miles a day, and the other 332 miles a day, in how many days will they meet?

5. The daily pay of a car conductor is \$3.45. If he works 310 days in a year, and spends on an average \$65 a month, how much has he left at the end of the year?

6. The receipts of a street railway for 365 days were \$119,685.23. Find the average daily profits if the total expenses were \$96,478.02.

7. A man sold 128 acres of land at \$70 an acre, and 96 acres at \$90 an acre. He invested the money in town lots at \$550 each. How many did he buy?

Study of Problem

\$ 8960, value of 1st farm

\$ 8640, value of 2d farm

\$17600, value of both

1. What is given in the problem?

2. What is required?

3. What is the first step in the solution? the second? the third? the fourth?

$\$17600 \div \$550 = 32$, number of lots bought.

8. If it costs 40 cents to ship a 10-gallon can of milk from Hickory to Pittsburgh, how much does the railroad realize in 5 days from a shipment of 135 cans a day?

9. A shipper pays 20 cents a barrel, per month, cold storage charges on apples, and 15 cents a firkin on butter. Find the charges for three months on 45 firkins of butter and 328 barrels of apples.

10. A locomotive in making a certain trip uses 18 tons of coal. If a trip is made in 2 days, how much coal will the engine consume in 190 days?

11. A dealer buys three boxes of oranges for \$3.50, \$2.75, and \$2.50, respectively. If he sells 10 dozen at 50 cents a dozen, 9 dozen at 40 cents a dozen, 12 dozen at 35 cents a dozen, and the remaining 5 dozen at 25 cents a dozen, find his gain.

12. A sale of tickets is as follows: 450 @ \$1.50; 380 @ \$1.00; 520 @ \$.75; 310 @ \$.50; and 240 @ \$.25. Find the total sale of the tickets, and the average cost of each ticket.

13. A steamboat consumes 23 tons of coal a day. Find the cost of the coal, at \$5.85 a ton, for a trip of 39 days.

REVIEW OF FACTORS AND DIVISORS

An **integer** or an **integral number** is a whole number.

The **factors** of a number are the integers whose product is the number. Thus, 5 and 2 are factors of 10.

1. Name two factors that produce 24, 32, 40, 56, 49, 72, 96.

A factor of a number is an **exact divisor** of the number ; that is, it is contained in the number an *integral number* of times.

2. Name the exact divisors of 54, 81, 48, 36, 66, 64, 63.

3. Observe the two equal factors that produce 4; 9; 16.

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

4. Observe the three equal factors that produce 8; 27; 64.

$$2 \times 2 \times 2 = 8$$

$$3 \times 3 \times 3 = 27$$

$$4 \times 4 \times 4 = 64$$

A number is divisible by 2 if its ones' figure is 2, 4, 6, 8, or 0.

A number is divisible by 5 if its ones' figure is 5, or 0.

A number is divisible by 3 if the sum of its digits (that is, of the figures) is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9.

Tell by inspection which of the following numbers are divisible by 2; by 5; by 3; by 9.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
5.	36	25	162	369	639	3453
6.	69	40	125	279	819	2340
7.	86	96	123	918	515	3672
8.	94	72	321	819	450	1909

REVIEW OF COMMON FRACTIONS

FRACTIONAL UNITS

In 8, 6 ft., \$2, 6 rd., 7 mi., 5 in., what are the units of measure?

Observe that in each case the number and its unit of measure are of the same denomination. Thus, 1 ft. is the unit of measure in 6 ft.

A **unit**, therefore, is any single quantity with which another quantity of the same kind is measured or compared. Thus, 1 is the unit of 10; 1 ft. is the unit of 8 ft.; 1 yd., of 2 yd.; 1 mi., of 12 mi.; 1 acre, of 5 acres, etc.

A **fractional unit** is one of the equal parts into which an integral unit has been divided; as, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{15}$, etc.

A **fraction** is one or more fractional units; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{3}$, $\frac{5}{8}$, etc.

The **terms** of a fraction are the numerator and the denominator.

The **denominator** indicates the *size* of the fractional unit. It is written below the line, and shows into how many parts the integral unit has been divided. Thus, in the fraction $\frac{4}{5}$, 5 is the denominator, and shows that some unit has been divided into 5 parts.

The **numerator** indicates the *number* of fractional units. It is written above the line, and shows how many parts are taken. Thus, in the fraction $\frac{4}{5}$, 4 is the numerator, and shows that 4 parts have been taken.

Read the following fractional units in order of their size, beginning with the largest: $\frac{1}{16}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{21}$.

The use of the numerator and the denominator in $\frac{9}{12}$ yd. may be explained thus: $\frac{9}{12}$ yd. = $9 \times \frac{1}{12}$ yd.

As the *integral unit* is the basis by which we measure whole numbers, so the *fractional unit* is the basis by which we measure fractions of the same kind.

A **common fraction** is a fraction that has both terms expressed; as, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{4}$.

A **proper fraction** is a fraction whose value is less than 1; as, $\frac{1}{2}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{8}{9}$, $\frac{1}{7}$, $\frac{3}{4}$, etc.

An **improper fraction** is a fraction whose value is equal to or greater than 1; as, $\frac{8}{8}$, $\frac{9}{8}$, $\frac{4}{3}$, $\frac{5}{2}$, $\frac{6}{5}$, $\frac{10}{9}$, etc.

A **mixed number** is a number expressed by a whole number and a fraction; as, $3\frac{1}{2}$, $12\frac{3}{4}$.

REDUCTION

1. Divide both terms of the fraction $\frac{4}{8}$ by 2. How does $\frac{4}{8}$ compare in value with $\frac{1}{2}$?

2. Multiply both terms of the fraction $\frac{3}{4}$ by 2. How does $\frac{6}{8}$ compare in value with $\frac{3}{4}$? How may we obtain $\frac{6}{8}$ or $\frac{9}{12}$ from $\frac{3}{4}$?

Reduction is the process of changing the terms of a fraction without changing its value.

Multiplying or dividing both terms of a fraction by the same number does not change its value.

Changing a fraction to higher terms.

Oral Work

1. Explain why a fraction is expressed in *smaller* fractional units when it is changed to *higher* terms.

2. Explain why changing a fraction to *higher* terms does not change the *value* of the fraction.

Change :

3. $\frac{1}{2}$ to 12ths

6. $\frac{3}{10}$ to 40ths

9. $\frac{11}{12}$ to 72ds

4. $\frac{3}{4}$ to 24ths

7. $\frac{7}{8}$ to 56ths

10. $\frac{7}{9}$ to 63ds

5. $\frac{5}{8}$ to 18ths

8. $\frac{8}{9}$ to 81sts

11. $\frac{8}{9}$ to 96ths

Written Work

1. Change $\frac{5}{9}$ to 27ths.

$$27 \div 9 = 3$$

$$\frac{5}{9} = \frac{5 \times 3}{9 \times 3} = \frac{15}{27}$$

Since multiplying both terms of a fraction by the same number does not change its value, multiply both terms of the fraction by the quotient of $27 \div 9$, or 3.

Change:

2. $\frac{3}{4}$ to 20ths

9. $\frac{11}{25}$ to 100ths

3. $\frac{7}{8}$ to 56ths

10. $\frac{17}{25}$ to 200ths

4. $\frac{11}{12}$ to 96ths

11. $\frac{15}{16}$ to 64ths

5. $\frac{2}{13}$ to 78ths

12. $\frac{21}{32}$ to 96ths

6. $\frac{15}{12}$ to 66ths

13. $\frac{21}{50}$ to 500ths

7. $\frac{17}{18}$ to 72ds

14. $\frac{12}{20}$ to 600ths

8. $\frac{7}{15}$ to 60ths

15. $\frac{12}{35}$ to 360ths

Changing a fraction to lower terms or to lowest terms.

Oral Work

1. Explain why a fraction is expressed in *larger* fractional units when it is changed to *lower* terms. Explain also why changing a fraction to lower terms does not change its value.

A fraction is in its **lowest terms** when the numerator and denominator have no common divisor greater than 1.

Change to lowest terms:

2. $\frac{6}{12}$

4. $\frac{35}{40}$

6. $\frac{60}{72}$

8. $\frac{24}{54}$

10. $\frac{27}{63}$

3. $\frac{24}{36}$

5. $\frac{48}{80}$

7. $\frac{80}{96}$

9. $\frac{55}{80}$

11. $\frac{48}{108}$

Written Work

1. Change
- $1\frac{3}{4}$
- to lowest terms.

$$\frac{18+3}{24+3} = \frac{6}{8}; \quad \frac{6+2}{8+2} = \frac{3}{4}$$

Or

$$\frac{18+6}{24+6} = \frac{3}{4}$$

Since dividing both terms of a fraction by the same number does not change its value, divide both terms by 3 and both terms of the resulting fraction by 2, leaving the factors 3 and 4. Hence $1\frac{3}{4} = \frac{7}{4}$.

Or, divide both terms of the fraction directly by 6.

Change to lowest terms :

- | | | | | |
|-------------------|--------------------|--------------------|--------------------|--------------------|
| 2. $1\frac{1}{2}$ | 7. $4\frac{1}{2}$ | 12. $2\frac{1}{2}$ | 17. $3\frac{1}{4}$ | 22. $1\frac{1}{2}$ |
| 3. $2\frac{1}{6}$ | 8. $1\frac{1}{3}$ | 13. $5\frac{1}{4}$ | 18. $1\frac{2}{3}$ | 23. $3\frac{1}{6}$ |
| 4. $1\frac{1}{3}$ | 9. $1\frac{1}{3}$ | 14. $3\frac{1}{6}$ | 19. $1\frac{1}{4}$ | 24. $4\frac{1}{6}$ |
| 5. $1\frac{1}{4}$ | 10. $2\frac{1}{6}$ | 15. $2\frac{1}{2}$ | 20. $2\frac{1}{6}$ | 25. $1\frac{1}{6}$ |
| 6. $1\frac{1}{2}$ | 11. $2\frac{1}{4}$ | 16. $1\frac{1}{4}$ | 21. $2\frac{1}{2}$ | 26. $1\frac{1}{4}$ |

Changing a mixed number to an improper fraction, or an improper fraction to a mixed number.

Oral Work

Change to an improper fraction at sight :

- | | | | |
|-------------------|-------------------|--------------------|----------------|
| 1. $1\frac{1}{4}$ | 4. $3\frac{1}{4}$ | 7. $6\frac{2}{10}$ | 10. 12 to 3ds |
| 2. $1\frac{1}{8}$ | 5. $5\frac{1}{3}$ | 8. $3\frac{2}{16}$ | 11. 8 to 6ths |
| 3. $2\frac{1}{8}$ | 6. $2\frac{1}{6}$ | 9. 8 to 12ths | 12. 10 to 5ths |

Written Work

1. Change
- $111\frac{1}{3}$
- to an improper fraction.

$$1 = \frac{3}{3}$$

$$111 = 111 \times \frac{3}{3} = \frac{333}{3}$$

$$\frac{333}{3} + \frac{1}{3} = \frac{334}{3}$$

In 1 there are $\frac{3}{3}$; in 111 there are 111 times $\frac{3}{3}$, or $\frac{333}{3}$, which added to $\frac{1}{3}$ equal $\frac{334}{3}$. Hence $111\frac{1}{3}$ equals $\frac{334}{3}$.

In small numbers the work may be done *mentally*, only the result being written.

Change to improper fractions:

- | | | | |
|---------------------|----------------------|-------------------------|------------------------|
| 2. $12\frac{3}{4}$ | 6. $75\frac{15}{28}$ | 10. $268\frac{27}{120}$ | 14. $391\frac{45}{88}$ |
| 3. $15\frac{5}{8}$ | 7. $95\frac{31}{8}$ | 11. $324\frac{7}{8}$ | 15. $16\frac{1}{30}$ |
| 4. $22\frac{7}{8}$ | 8. $103\frac{22}{8}$ | 12. $502\frac{22}{105}$ | 16. $901\frac{5}{2}$ |
| 5. $30\frac{2}{10}$ | 9. $118\frac{21}{4}$ | 13. $109\frac{47}{44}$ | 17. $100\frac{7}{15}$ |

18. Change $12\frac{2}{7}$ to a mixed number.

In 1 there are $\frac{7}{7}$, and in $12\frac{2}{7}$ there are as many times 1, as 7 is contained in 128, or $18\frac{2}{7}$.
 $12\frac{2}{7} = 128 \div 7 = 18\frac{2}{7}$
 Hence $12\frac{2}{7}$ equals $18\frac{2}{7}$.

Every fraction is an indicated division.

Change to integers or mixed numbers:

- | | | | |
|----------------------|----------------------|----------------------|-----------------------|
| 19. $\frac{31}{16}$ | 23. $\frac{715}{82}$ | 27. $\frac{745}{84}$ | 31. $\frac{2362}{16}$ |
| 20. $\frac{527}{11}$ | 24. $\frac{770}{35}$ | 28. $\frac{876}{88}$ | 32. $\frac{3440}{75}$ |
| 21. $\frac{256}{7}$ | 25. $\frac{333}{18}$ | 29. $\frac{576}{24}$ | 33. $\frac{5286}{48}$ |
| 22. $\frac{157}{15}$ | 26. $\frac{525}{28}$ | 30. $\frac{862}{84}$ | 34. $\frac{4645}{50}$ |

Changing to least similar fractions.

A common denominator of two or more fractions is a number that contains all the denominators of the fractions an integral number of times. Thus, 24 is a common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

The least common denominator (l.c.d.) of two or more fractions is the *least* number that contains all the denominators of the fractions an integral number of times. Thus, 12 is the l.c.d. of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Similar fractions are fractions that express the same unit value. Similar fractions must have a common denominator.

Oral and Written Work

1. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{9}$ to similar fractions having the least common denominator.

It is evident by inspection that 18 is the least common denominator of the given fractions. Therefore change the given fractions to 18ths. $\frac{1}{2} = \frac{9}{18}$; $\frac{2}{3} = \frac{12}{18}$; and $\frac{5}{9} = \frac{10}{18}$. Hence the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{9}$ may be changed to the similar fractions $\frac{9}{18}$, $\frac{12}{18}$, and $\frac{10}{18}$.

NOTE. — When you cannot find the l. c. d. by inspection, try multiples of the largest denominator until you find one in which each of the other denominators is exactly contained. Thus, in $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{9}$, try 14, 28, 42.

Change to least similar fractions :

- | | | | |
|---|--|---|--|
| 2. $\frac{1}{3}, \frac{1}{4}, \frac{5}{12}$ | 5. $\frac{1}{5}, \frac{3}{10}, \frac{7}{40}$ | 8. $\frac{2}{3}, \frac{1}{6}, \frac{13}{36}$ | 11. $\frac{5}{6}, \frac{3}{7}, \frac{2}{14}$ |
| 3. $\frac{2}{3}, \frac{1}{5}, \frac{2}{15}$ | 6. $\frac{1}{3}, \frac{5}{12}, \frac{7}{24}$ | 9. $\frac{1}{3}, \frac{2}{5}, \frac{1}{6}$ | 12. $\frac{1}{3}, \frac{5}{18}, \frac{17}{24}$ |
| 4. $\frac{5}{6}, \frac{2}{3}, \frac{7}{18}$ | 7. $\frac{2}{5}, \frac{3}{8}, \frac{9}{40}$ | 10. $\frac{2}{7}, \frac{11}{14}, \frac{13}{28}$ | 13. $\frac{2}{5}, \frac{2}{3}, \frac{3}{20}$ |

ADDITION AND SUBTRACTION

Before fractions can be added or subtracted, they must be expressed in similar fractional units.

Oral Work

Add quickly :

- | | | | |
|--------------------------------|--------------------------------|-----------------------------------|---------------------------------|
| 1. $\frac{1}{2} + \frac{1}{4}$ | 5. $\frac{2}{5} + \frac{1}{2}$ | 9. $\frac{1}{12} + \frac{3}{4}$ | 13. $\frac{5}{6} + \frac{1}{3}$ |
| 2. $\frac{2}{3} + \frac{1}{6}$ | 6. $\frac{3}{4} + \frac{1}{2}$ | 10. $\frac{2}{3} + \frac{7}{9}$ | 14. $\frac{2}{3} + \frac{3}{6}$ |
| 3. $\frac{3}{4} + \frac{1}{8}$ | 7. $\frac{2}{3} + \frac{1}{9}$ | 11. $\frac{4}{5} + \frac{1}{2}$ | 15. $\frac{3}{4} + \frac{5}{8}$ |
| 4. $\frac{1}{4} + \frac{2}{3}$ | 8. $\frac{2}{4} + \frac{3}{2}$ | 12. $\frac{11}{12} + \frac{2}{3}$ | 16. $\frac{7}{8} + \frac{1}{3}$ |

Written Work

1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{3}{9}$.

36 = l. c. d.

$$\begin{array}{r|l} \frac{2}{3} & 24 \\ \frac{3}{4} & 27 \\ \frac{3}{9} & 32 \\ \hline & 83 \\ & 36 = 2\frac{11}{36} \end{array}$$

The least common denominator is 36. Change the given fractions to 36ths. $\frac{2}{3} = \frac{24}{36}$; $\frac{3}{4} = \frac{27}{36}$; $\frac{3}{9} = \frac{4}{12} = \frac{1}{3} = \frac{12}{36}$. The sum of these fractions = $\frac{83}{36}$, or $2\frac{11}{36}$. What is the *first* step in the work? the *second* step? the *third* step?

Add or subtract, as indicated :

- | | | | |
|---|--|--|------------------------------------|
| 2. $\frac{1}{2} + \frac{3}{4}$ | 12. $\frac{5}{12} - \frac{3}{14}$ | 22. $\frac{3}{4} + \frac{11}{12}$ | 32. $\frac{2}{3} + \frac{7}{12}$ |
| 3. $\frac{7}{12} - \frac{1}{3}$ | 13. $\frac{5}{6} - \frac{1}{7}$ | 23. $\frac{2}{3} - \frac{4}{9}$ | 33. $\frac{3}{8} - \frac{5}{16}$ |
| 4. $\frac{4}{15} + \frac{1}{5}$ | 14. $\frac{3}{4} + \frac{5}{8}$ | 24. $\frac{5}{8} - \frac{9}{16}$ | 34. $\frac{2}{3} - \frac{4}{9}$ |
| 5. $\frac{3}{4} - \frac{1}{12}$ | 15. $\frac{3}{10} + \frac{5}{6}$ | 25. $\frac{2}{3} - \frac{3}{10}$ | 35. $\frac{3}{5} - \frac{3}{10}$ |
| 6. $\frac{11}{12} - \frac{2}{3}$ | 16. $\frac{4}{5} - \frac{7}{15}$ | 26. $\frac{5}{8} + \frac{5}{16}$ | 36. $\frac{2}{3} + \frac{8}{9}$ |
| 7. $\frac{5}{9} + \frac{2}{3}$ | 17. $\frac{5}{6} - \frac{5}{12}$ | 27. $\frac{3}{10} - \frac{3}{20}$ | 37. $\frac{5}{12} + \frac{4}{6}$ |
| 8. $\frac{1}{3} + \frac{5}{6}$ | 18. $\frac{2}{3} + \frac{3}{4}$ | 28. $\frac{2}{7} - \frac{2}{14}$ | 38. $\frac{5}{16} + \frac{7}{12}$ |
| 9. $\frac{3}{5} - \frac{3}{8}$ | 19. $\frac{3}{8} + \frac{9}{16}$ | 29. $\frac{2}{9} - \frac{2}{18}$ | 39. $\frac{5}{9} - \frac{2}{30}$ |
| 10. $\frac{5}{14} + \frac{3}{7}$ | 20. $\frac{7}{9} + \frac{11}{18}$ | 30. $\frac{3}{4} + \frac{4}{5}$ | 40. $\frac{2}{3} + \frac{5}{6}$ |
| 11. $\frac{3}{16} + \frac{1}{2}$ | 21. $\frac{1}{2} + \frac{2}{9}$ | 31. $\frac{4}{5} - \frac{3}{10}$ | 41. $\frac{11}{16} - \frac{5}{12}$ |
| 42. $\frac{5}{8} + \frac{3}{4} - \frac{2}{3}$ | 43. $2\frac{1}{6} + 2\frac{1}{5} - 3\frac{1}{4}$ | 54. $3\frac{1}{3} - \frac{5}{6} + 3\frac{5}{8}$ | |
| 43. $\frac{3}{4} - \frac{5}{16} + \frac{5}{9}$ | 44. $1\frac{7}{10} + 51\frac{1}{3} + 1\frac{4}{5}$ | 55. $6\frac{1}{12} - 3\frac{1}{10} + 1\frac{2}{3}$ | |
| 44. $7\frac{1}{4} - 3\frac{2}{3} + \frac{3}{8}$ | 50. $\frac{9}{16} + \frac{3}{5} - \frac{9}{10}$ | 56. $2\frac{1}{4} - 1\frac{7}{8} + 5\frac{3}{10}$ | |
| 45. $\frac{2}{3} - \frac{1}{5} + 1\frac{3}{10}$ | 51. $2\frac{2}{16} + 5\frac{1}{4} - 3\frac{7}{8}$ | 57. $1\frac{1}{2} + 2\frac{1}{14} - 3\frac{5}{16}$ | |
| 46. $\frac{5}{16} - \frac{1}{4} + \frac{7}{8}$ | 52. $\frac{3}{4} - \frac{2}{3} + 5\frac{7}{16}$ | 58. $1\frac{7}{9} + 2\frac{3}{4} - 1\frac{7}{16}$ | |
| 47. $2\frac{5}{7} + \frac{1}{2} - 1\frac{1}{4}$ | 53. $3\frac{1}{5} + 41\frac{1}{2} - 7\frac{2}{3}$ | 59. $3\frac{1}{4} + 2\frac{1}{3} - 3\frac{11}{20}$ | |

First add ; then subtract :

- | | | | | |
|--|--|---|--|--|
| 60. $\begin{array}{r} 5\frac{3}{4} \\ 2\frac{1}{2} \\ \hline \end{array}$ | 65. $\begin{array}{r} 14\frac{11}{18} \\ 8\frac{2}{9} \\ \hline \end{array}$ | 70. $\begin{array}{r} 6\frac{7}{8} \\ 3\frac{2}{5} \\ \hline \end{array}$ | 75. $\begin{array}{r} 4\frac{2}{3} \\ 1\frac{3}{10} \\ \hline \end{array}$ | 80. $\begin{array}{r} 61\frac{1}{12} \\ 3\frac{4}{9} \\ \hline \end{array}$ |
| 61. $\begin{array}{r} 31\frac{5}{16} \\ 3\frac{5}{9} \\ \hline \end{array}$ | 66. $\begin{array}{r} 18\frac{2}{7} \\ 8\frac{2}{9} \\ \hline \end{array}$ | 71. $\begin{array}{r} 4\frac{9}{13} \\ 3\frac{2}{9} \\ \hline \end{array}$ | 76. $\begin{array}{r} 12\frac{7}{8} \\ 7\frac{4}{15} \\ \hline \end{array}$ | 81. $\begin{array}{r} 22\frac{7}{12} \\ 16\frac{5}{6} \\ \hline \end{array}$ |
| 62. $\begin{array}{r} 54\frac{11}{12} \\ 46\frac{5}{18} \\ \hline \end{array}$ | 67. $\begin{array}{r} 4\frac{7}{10} \\ 2\frac{5}{12} \\ \hline \end{array}$ | 72. $\begin{array}{r} 56\frac{7}{9} \\ 49\frac{3}{11} \\ \hline \end{array}$ | 77. $\begin{array}{r} 47\frac{2}{5} \\ 28\frac{1}{3} \\ \hline \end{array}$ | 82. $\begin{array}{r} 32\frac{1}{4} \\ 18\frac{1}{6} \\ \hline \end{array}$ |
| 63. $\begin{array}{r} 13\frac{3}{7} \\ 7\frac{2}{9} \\ \hline \end{array}$ | 68. $\begin{array}{r} 15\frac{5}{7} \\ 2\frac{1}{2} \\ \hline \end{array}$ | 73. $\begin{array}{r} 33\frac{4}{5} \\ 21\frac{3}{4} \\ \hline \end{array}$ | 78. $\begin{array}{r} 15\frac{7}{8} \\ 2\frac{6}{7} \\ \hline \end{array}$ | 83. $\begin{array}{r} 40\frac{2}{7} \\ 37\frac{5}{12} \\ \hline \end{array}$ |
| 64. $\begin{array}{r} 7\frac{7}{12} \\ 4\frac{2}{3} \\ \hline \end{array}$ | 69. $\begin{array}{r} 14\frac{8}{9} \\ 10\frac{3}{21} \\ \hline \end{array}$ | 74. $\begin{array}{r} 41\frac{11}{12} \\ 2\frac{3}{14} \\ \hline \end{array}$ | 79. $\begin{array}{r} 16\frac{2}{5} \\ 12\frac{3}{10} \\ \hline \end{array}$ | 84. $\begin{array}{r} 4\frac{3}{8} \\ 2\frac{1}{4} \\ \hline \end{array}$ |

MULTIPLICATION

Multiplying a fraction by an integer.

Oral Work

1. How many eighths are $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$, or 3 times $\frac{3}{8}$?

In multiplying $\frac{3}{8}$ by 3 we do not change the denominator eighths, but we *multiply* the number of eighths, or the *numerator*, by 3.

2. Multiply $\frac{3}{8}$ by 4 in the same manner : $\frac{3}{8} \times 4 = \frac{12}{8} = \frac{3}{2}$.

3. Multiply $\frac{3}{8}$ by 4 in this way : 4 times $\frac{1}{8} = \frac{4}{8}$, or $\frac{1}{2}$; 4 times $\frac{3}{8} = 3 \times \frac{1}{2}$, or $\frac{3}{2}$.

We may multiply $\frac{3}{8}$ by 4 by increasing the *size* of the parts without changing their number. This is done by dividing the denominator 8 by 4.

Multiplying the numerator or dividing the denominator of a fraction by a number multiplies the value of the fraction by that number.

The second method should be used when the denominator exactly contains or exactly divides the multiplier.

Give products : /

4. $8 \times \frac{2}{3}$

9. $15 \times \frac{2}{5}$

14. $25 \times \frac{3}{75}$

5. $9 \times \frac{7}{18}$

10. $11 \times \frac{4}{77}$

15. $15 \times \frac{4}{45}$

6. $11 \times \frac{8}{11}$

11. $9 \times \frac{5}{11}$

16. $18 \times \frac{5}{54}$

7. $6 \times \frac{5}{12}$

12. $16 \times \frac{7}{32}$

17. $12 \times \frac{11}{108}$

8. $10 \times \frac{7}{10}$

13. $7 \times \frac{14}{84}$

18. $36 \times \frac{1}{12}$

Finding a fractional part of an integer.

Oral Work

1. Find $\frac{3}{4}$ of 8; $\frac{1}{4}$ of 24. Compare the results. Notice that 3 times $\frac{1}{4}$ of 8 = $\frac{1}{4}$ of 3 times 8.

2. Notice also that 2 times $\frac{1}{2}$ of 10 = $\frac{1}{2}$ of 2 times 10. In what two ways, then, may we find a fractional part of a number?

3. Compare $\frac{3}{4}$ of 8 with $8 \times \frac{3}{4}$; $\frac{2}{5}$ of 10 with $10 \times \frac{2}{5}$.

Finding a fractional part of a number is the same as multiplying a number by a fraction.

In multiplying by a fraction, we multiply by the numerator and divide by the denominator.

NOTE.—Divide first, when the multiplicand exactly contains the denominator. In other cases, multiply first.

Find:

4. $\frac{7}{8}$ of 80

9. $\frac{5}{12}$ of 60

14. $\frac{11}{2} \times 120$

5. $\frac{5}{6} \times 42$

10. $\frac{7}{12}$ of 35

15. $\frac{3}{8} \times 176$

6. $\frac{7}{9} \times 63$

11. $\frac{4}{5}$ of 47

16. $\frac{2}{7} \times 450$

7. $\frac{3}{8} \times 48$

12. $\frac{5}{7} \times 100$

17. $\frac{3}{8} \times 720$

8. $\frac{7}{8} \times 15$

13. $\frac{8}{9} \times 95$

18. $\frac{1}{2} \times 108$

Cancellation is the process of shortening an operation by striking out equal factors from dividend and divisor.

Written Work

1. Multiply $\frac{7}{16}$ by 5.

$$5 \times \frac{7}{16} = \frac{35}{16} = 2\frac{3}{16}$$

Since multiplying the numerator multiplies the fraction, multiply the numerator 7 by 5.
 $5 \times \frac{7}{16} = \frac{35}{16}$.

2. Multiply $\frac{7}{16}$ by 8.

$$8 \times \frac{7}{16} = \frac{7}{16 \div 8} = \frac{7}{2}, \text{ or } 3\frac{1}{2}$$

Since dividing the denominator multiplies the fraction, divide the denominator 16 by 8. Whenever possible, use cancellation.

$$\text{Or, } 8 \times \frac{7}{16} = \frac{7}{2} = 3\frac{1}{2}$$

3. Multiply $14\frac{3}{8}$ by 9.

$$\begin{array}{r} 14\frac{3}{8} \\ 9 \\ \hline 3\frac{3}{8} = 9 \times \frac{3}{8} \\ 126 = 9 \times 14 \\ \hline 129\frac{3}{8} = 9 \times 14\frac{3}{8} \end{array}$$

$$9 \times \frac{3}{8} = \frac{27}{8} = 3\frac{3}{8}.$$

$$9 \times 14 = 126. \quad 3\frac{3}{8} + 126 = 129\frac{3}{8}.$$

Multiply:

- | | | |
|----------------------------|----------------------------|-----------------------------|
| 4. $\frac{8}{21}$ by 7 | 17. $\frac{7}{54}$ by 16 | 30. $64\frac{7}{8}$ by 56 |
| 5. $\frac{5}{8}$ by 10 | 18. $\frac{7}{50}$ by 100 | 31. $44\frac{11}{16}$ by 30 |
| 6. $\frac{29}{80}$ by 15 | 19. $\frac{5}{72}$ by 54 | 32. $78\frac{1}{4}$ by 63 |
| 7. $\frac{4}{9}$ by 8 | 20. $\frac{8}{136}$ by 28 | 33. $89\frac{4}{16}$ by 105 |
| 8. $\frac{17}{81}$ by 27 | 21. $\frac{3}{85}$ by 105 | 34. $715\frac{3}{8}$ by 45 |
| 9. $\frac{11}{16}$ by 35 | 22. $\frac{6}{64}$ by 96 | 35. $101\frac{1}{8}$ by 63 |
| 10. $\frac{17}{56}$ by 28 | 23. $\frac{10}{77}$ by 121 | 36. $205\frac{3}{8}$ by 75 |
| 11. $\frac{25}{88}$ by 72 | 24. $\frac{15}{28}$ by 48 | 37. $47\frac{3}{8}$ by 96 |
| 12. $\frac{17}{27}$ by 108 | 25. $12\frac{2}{7}$ by 28 | 38. $609\frac{3}{8}$ by 21 |
| 13. $\frac{3}{19}$ by 57 | 26. $11\frac{7}{8}$ by 45 | 39. $105\frac{1}{4}$ by 49 |
| 14. $\frac{19}{24}$ by 144 | 27. $14\frac{5}{12}$ by 60 | 40. $290\frac{3}{10}$ by 45 |
| 15. $\frac{8}{16}$ by 135 | 28. $25\frac{1}{8}$ by 75 | 41. $213\frac{1}{4}$ by 78 |
| 16. $\frac{7}{36}$ by 21 | 29. $85\frac{3}{8}$ by 66 | 42. $735\frac{3}{8}$ by 21 |

Find products:

- | | | |
|---------------------------------|--------------------------------|----------------------------------|
| 43. $213 \times 609\frac{3}{8}$ | 48. $596 \times 56\frac{1}{4}$ | 53. $379 \times 49\frac{7}{10}$ |
| 44. $612 \times 48\frac{3}{8}$ | 49. $972 \times 32\frac{5}{8}$ | 54. $49 \times 465\frac{1}{4}$ |
| 45. $842 \times 95\frac{7}{8}$ | 50. $96 \times 325\frac{3}{8}$ | 55. $786 \times 49\frac{7}{8}$ |
| 46. $728 \times 34\frac{3}{10}$ | 51. $856 \times 98\frac{3}{8}$ | 56. $9872 \times 36\frac{7}{12}$ |
| 47. $96 \times 207\frac{3}{8}$ | 52. $54 \times 657\frac{1}{4}$ | 57. $4398 \times 94\frac{3}{8}$ |

Find:

- | | | |
|---------------------------|---------------------------|---------------------------|
| 58. $\frac{1}{16}$ of 320 | 60. $\frac{2}{10}$ of 600 | 62. $\frac{1}{16}$ of 182 |
| 59. $\frac{1}{20}$ of 375 | 61. $\frac{1}{10}$ of 480 | 63. $\frac{1}{18}$ of 738 |

Multiplying a fraction by a fraction.**Oral Work**

Find:

1. $\frac{1}{3}$ of $\frac{3}{4}$ 3. $\frac{7}{9}$ of $\frac{27}{35}$ 5. $\frac{5}{8}$ of $\frac{16}{21}$ 7. $\frac{5}{8}$ of $\frac{3}{4}$
 2. $\frac{2}{3}$ of $\frac{3}{10}$ 4. $\frac{6}{7}$ of $\frac{21}{25}$ 6. $\frac{3}{5}$ of $\frac{15}{16}$ 8. $\frac{5}{8}$ of $\frac{3}{10}$

Observe that $\frac{1}{4}$ of $\frac{3}{5} = \frac{1 \times 3}{4 \times 5}$, or $\frac{3}{20}$; $\frac{3}{4}$ of $\frac{3}{5} = \frac{3 \times 3}{4 \times 5}$, or $\frac{9}{20}$.

A fractional part of a fraction equals *a fraction times a fraction*.

Written Work1. Find $\frac{7}{8} \times \frac{16}{21}$.

$$\frac{7}{8} \times \frac{16}{21} = \frac{7 \times 16}{8 \times 21} = \frac{112}{168}, \text{ or } \frac{2}{3}$$

Or,

$$\frac{7}{8} \times \frac{16}{21} = \frac{2}{3}$$

To find $\frac{7}{8} \times \frac{16}{21}$ means to find $\frac{7}{8}$ of $\frac{16}{21}$.
 $\frac{1}{8}$ of $\frac{16}{21} = \frac{2}{21}$; $\frac{7}{8}$ of $\frac{16}{21} = 7 \times \frac{2}{21} = \frac{14}{21}$,
 or $\frac{2}{3}$.

A fractional part of a fraction is found by multiplying the numerators for the numerator of the product and the denominators for the denominator of the product.

Indicate the operation, and cancel when possible.

An integer may be expressed in fractional form, thus:

$$8 = \frac{8}{1} \quad \frac{3}{4} \text{ of } 8 = \frac{3}{4} \text{ of } \frac{8}{1} = \frac{6}{1}, \text{ or } 6.$$

Find products:

2. $\frac{3}{8} \times \frac{16}{25}$ 6. $\frac{25}{36} \times \frac{14}{15}$ 10. $\frac{25}{36} \times \frac{37}{38}$ 14. $\frac{22}{35} \times \frac{13}{100}$
 3. $\frac{5}{16} \times \frac{32}{45}$ 7. $\frac{12}{60} \times \frac{3}{19}$ 11. $\frac{22}{45} \times \frac{15}{16}$ 15. $\frac{15}{28} \times \frac{14}{75}$
 4. $\frac{12}{20} \times \frac{4}{25}$ 8. $\frac{25}{36} \times \frac{13}{50}$ 12. $\frac{12}{36} \times \frac{34}{35}$ 16. $\frac{22}{25} \times \frac{6}{45}$
 5. $\frac{9}{50} \times \frac{2}{3}$ 9. $\frac{25}{32} \times \frac{41}{100}$ 13. $\frac{71}{74} \times \frac{37}{38}$ 17. $\frac{62}{100} \times \frac{14}{90}$

18. Find $\frac{3}{4}$ of $12\frac{5}{8}$.

NOTE. — Change the mixed number to an improper fraction.

Find:

19. $\frac{2}{3}$ of $7\frac{1}{2}$

23. $3\frac{3}{4} \times \frac{1}{5}$

27. $2\frac{3}{4} \times 7\frac{5}{8}$

20. $\frac{5}{8}$ of $12\frac{3}{4}$

24. $7\frac{3}{8} \times \frac{5}{8}$

28. $8\frac{1}{4} \times 5\frac{1}{2}$

21. $\frac{8}{9}$ of $13\frac{5}{8}$

25. $5\frac{5}{12} \times \frac{1}{6}$

29. $7\frac{8}{9} \times 3\frac{1}{4}$

22. $\frac{7}{12}$ of $10\frac{1}{8}$

26. $8\frac{3}{8} \times \frac{5}{8}$

30. $4\frac{7}{12} \times 3\frac{3}{4}$

Find the value of:

31. $4\frac{3}{8} \times 6\frac{5}{7}$

$$4\frac{3}{8} \times 6\frac{5}{7} = \frac{35}{8} \times \frac{48}{7} = 30.$$

32. $6\frac{3}{8} \times 2\frac{3}{11} \times 7\frac{8}{9}$

38. $35\frac{1}{16} \times 22\frac{1}{32} \times 12$

33. $8\frac{7}{10} \times 2\frac{1}{5} \times 2\frac{3}{8}$

39. $51\frac{3}{7} \times 19\frac{9}{15} \times 12\frac{1}{8}$

34. $10\frac{1}{2} \times 1\frac{1}{8} \times 7\frac{3}{4}$

40. $6\frac{3}{7} \times 5\frac{1}{2} \times \frac{7}{8} \times 2\frac{3}{10}$

35. $14\frac{1}{7} \times 17\frac{3}{8} \times 8\frac{3}{8}$

41. $20\frac{2}{3} \times 20\frac{4}{7} \times 20\frac{1}{6} \times 10$

36. $27\frac{1}{6} \times 42\frac{3}{8} \times 6\frac{1}{4}$

42. $7\frac{3}{8} \times 15 \times 2\frac{1}{6} \times 12\frac{3}{17}$

37. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{1}{5} \times 5\frac{1}{8}$

43. $172\frac{1}{2} \times 2\frac{7}{15} \times 3\frac{7}{8}$

DIVISION

Oral Work

1. How many half inches are there in 1 in.? in 2 in.? in 3 in.? in 4 in.? Then how many times is $\frac{1}{2}$ contained in 1? in 2? in 3? in 4?

2. If $1 \div \frac{1}{4} = 4$, what does $2 \div \frac{1}{4}$ equal? $3 \div \frac{1}{4}$? $12 \div \frac{1}{4}$?

3. What does $1 \div \frac{1}{6}$ equal? $3 \div \frac{1}{6}$? $6 \div \frac{1}{6}$?

To divide any number by a fractional unit, multiply the number by the denominator. Thus, $12 \div \frac{1}{4} = 12 \times \frac{4}{1} = \frac{48}{1}$, or 48.

4. $10 \div \frac{1}{4} = ?$ 7. $10 \div \frac{1}{12} = ?$ 10. $20 \div \frac{1}{9} = ?$
 5. $15 \div \frac{1}{2} = ?$ 8. $9 \div \frac{1}{12} = ?$ 11. $25 \div \frac{1}{4} = ?$
 6. $16 \div \frac{1}{8} = ?$ 9. $9 \div \frac{1}{15} = ?$ 12. $18 \div \frac{1}{3} = ?$
 13. $1 \div \frac{2}{4} = \text{how many?}$ 17. $1 \div \frac{5}{10} = \text{how many?}$
 14. $1 \div \frac{3}{8} = \text{how many?}$ 18. $1 \div \frac{4}{16} = \text{how many?}$
 15. $1 \div \frac{7}{14} = \text{how many?}$ 19. $1 \div \frac{2}{3} = \text{how many?}$
 16. $1 \div \frac{4}{8} = \text{how many?}$ 20. $1 \div \frac{3}{4} = \text{how many?}$

Observe that the number of times each of the above fractions is contained in 1 equals the number of times the numerator is contained in the denominator.

The number of times a fraction is contained in 1 is called the **reciprocal** of the fraction. Thus, $\frac{2}{3}$ is contained in 1, $\frac{3}{2}$ times. Hence $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$.

1 divided by any fraction equals the fraction **inverted**.

Find quotients :

21. $8 \div \frac{2}{3}$ 23. $4 \div \frac{3}{8}$ 25. $10 \div \frac{2}{6}$ 27. $25 \div \frac{5}{6}$
 22. $6 \div \frac{3}{4}$ 24. $15 \div \frac{2}{5}$ 26. $12 \div \frac{6}{7}$ 28. $14 \div \frac{2}{3}$

Any number is divided by a fraction by inverting the terms of the divisor and multiplying.

Written Work

1. $\frac{2}{3} \div \frac{2}{4} = \text{how many?}$

$$\frac{2}{3} \div \frac{2}{4} = \frac{2}{3} \times \frac{4}{2} = \frac{8}{3}. \quad \text{Since } 1 \div \frac{1}{4} = 4, \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \text{ of } 4, \text{ or } \frac{8}{3}.$$

NOTE. — The process should be shortened by *cancellation* when possible

Find quotients:

2. $\frac{3}{4} \div 5$ 6. $\frac{11}{18} \div 5$ 10. $\frac{2}{17} \div 5$
 3. $\frac{7}{8} \div 9$ 7. $\frac{31}{120} \div 25$ 11. $\frac{21}{65} \div 8$
 4. $\frac{15}{16} \div 7$ 8. $\frac{16}{28} \div 7$ 12. $\frac{3}{11} \div 9$
 5. $\frac{12}{30} \div 18$ 9. $\frac{24}{25} \div 8$ 13. $\frac{2}{21} \div 24$

14. $1\frac{2}{3} + 16$ 19. $1\frac{1}{10} + 42$ 24. $1\frac{6}{7} + 48$
 15. $1\frac{1}{2} + 22$ 20. $2\frac{4}{5} + 18$ 25. $1\frac{1}{2} + 22$
 16. $1\frac{5}{8} + 45$ 21. $7\frac{2}{5} + 36$ 26. $2\frac{4}{5} + 36$
 17. $2\frac{1}{10} + 62$ 22. $1\frac{5}{8} + 60$ 27. $1\frac{7}{8} + 51$
 18. $1\frac{1}{8} + 12$ 23. $1\frac{3}{8} + 38$ 28. $1\frac{7}{8} + 68$
 29. Divide $3\frac{1}{2}$ by 5.

$$3\frac{1}{2} = \frac{25}{8}; \quad \frac{25}{8} \div 5 = \frac{25}{8} \times \frac{1}{5} = \frac{5}{8}.$$

NOTE.—Small mixed numbers are frequently reduced to improper fractions and then divided in the same manner as proper fractions.

30. $2\frac{4}{7} + 6$ 36. $28\frac{4}{7} + 25$ 42. $10\frac{2}{13} + 24$
 31. $4\frac{1}{5} + 7$ 37. $28\frac{1}{8} + 18$ 43. $29\frac{2}{5} + 42$
 32. $4\frac{8}{11} + 13$ 38. $32\frac{8}{11} + 21$ 44. $22\frac{3}{11} + 58$
 33. $2\frac{2}{5} + 48$ 39. $21\frac{7}{8} + 25$ 45. $30\frac{3}{5} + 23$
 34. $5\frac{3}{5} + 23$ 40. $18\frac{7}{5} + 26$ 46. $19\frac{1}{3} + 26$
 35. $17\frac{7}{9} + 8$ 41. $19\frac{1}{11} + 45$ 47. $21\frac{7}{9} + 56$
 48. Divide $1286\frac{2}{3}$ by 9.

$$\begin{array}{r} 9 \overline{)1286\frac{2}{3}} \\ \underline{1428\frac{2}{3}} \end{array}$$

NOTE.—The division may be performed by changing both numbers to sevenths, but it is a much shorter process to perform the work as indicated, changing the remaining mixed number, $8\frac{2}{3}$, to an improper fraction, $\frac{62}{3}$, and dividing it by 9. Thus, $\frac{1}{9}$ of $\frac{62}{3} = \frac{62}{27}$.

49. $25681\frac{2}{11} \div 7$ 55. $8002\frac{7}{13} + 12$
 50. $9863\frac{7}{16} \div 6$ 56. $44281\frac{3}{5} + 12$
 51. $65321\frac{1}{12} + 8$ 57. $10935\frac{5}{8} \div 4$
 52. $6879\frac{9}{10} \div 5$ 58. $9720\frac{3}{5} + 11$
 53. $36370\frac{5}{8} \div 11$ 59. $7090\frac{9}{10} \div 9$
 54. $36480\frac{1}{4} \div 12$ 60. $8048\frac{8}{9} \div 8$

Change the mixed numbers to improper fractions before dividing. Divide:

- | | | |
|--------------------------------------|---|--|
| 61. 18 by $\frac{4}{9}$ | 78. $\frac{7}{12} \div \frac{2}{3}$ | 95. $\frac{31}{365} \div \frac{62}{75}$ |
| 62. 12 by $\frac{4}{5}$ | 79. $\frac{15}{16} \div \frac{11}{24}$ | 96. $\frac{25}{144} \div \frac{7}{2}$ |
| 63. 25 by $\frac{8}{9}$ | 80. $\frac{13}{18} \div \frac{26}{27}$ | 97. $\frac{29}{506} \div \frac{58}{75}$ |
| 64. 32 by $1\frac{2}{3}$ | 81. $\frac{5}{22} \div \frac{3}{8}$ | 98. $\frac{13}{144} \div \frac{13}{48}$ |
| 65. 40 by $2\frac{5}{8}$ | 82. $\frac{27}{32} \div \frac{9}{16}$ | 99. $\frac{39}{100} \div \frac{13}{15}$ |
| 66. 48 by $1\frac{5}{6}$ | 83. $\frac{25}{36} \div \frac{15}{16}$ | 100. $\frac{17}{72} \div \frac{51}{144}$ |
| 67. $172 \div \frac{8}{9}$ | 84. $\frac{27}{50} \div \frac{42}{65}$ | 101. $\frac{39}{64} \div \frac{13}{24}$ |
| 68. $235 \div \frac{4}{5}$ | 85. $\frac{62}{75} \div \frac{3}{5}$ | 102. $22\frac{3}{4} \div 2\frac{5}{8}$ |
| 69. $770 \div 1\frac{1}{6}$ | 86. $\frac{133}{144} \div \frac{19}{24}$ | 103. $45\frac{3}{8} \div 5\frac{1}{25}$ |
| 70. $882 \div \frac{3}{7}$ | 87. $\frac{99}{125} \div \frac{209}{250}$ | 104. $6\frac{3}{16} \div 8\frac{5}{8}$ |
| 71. $1035 \div \frac{5}{6}$ | 88. $\frac{81}{100} \div \frac{27}{50}$ | 105. $152 \div 3\frac{1}{25}$ |
| 72. $984 \div \frac{3}{8}$ | 89. $\frac{14}{25} \div \frac{7}{8}$ | 106. 160 by $\frac{5}{8}$ of $\frac{3}{4}$ |
| 73. $\frac{3}{4} \div \frac{2}{3}$ | 90. $\frac{13}{50} \div \frac{13}{25}$ | 107. $\frac{5}{8}$ of 640 by $6\frac{2}{18}$ |
| 74. $\frac{3}{8} \div \frac{3}{4}$ | 91. $\frac{23}{75} \div \frac{23}{25}$ | 108. 198 by $12\frac{3}{8}$ |
| 75. $\frac{7}{8} \div \frac{3}{4}$ | 92. $\frac{61}{100} \div \frac{13}{50}$ | 109. $3\frac{3}{4} \div 4\frac{1}{6}$ |
| 76. $\frac{9}{10} \div \frac{3}{5}$ | 93. $\frac{35}{144} \div \frac{5}{12}$ | 110. $10\frac{1}{8} \div 6\frac{3}{4}$ |
| 77. $\frac{13}{14} \div \frac{6}{7}$ | 94. $\frac{27}{100} \div \frac{8}{100}$ | 111. $10\frac{1}{2} \div 3\frac{1}{5}$ |

A **complex fraction** is a fraction that has a fraction or a mixed number in either or both of its terms.

$$\text{Thus, } \frac{21\frac{1}{6}}{\frac{5}{8}} = \frac{9}{4} \div \frac{5}{6} = \frac{9}{4} \times \frac{6}{5} = \frac{27}{10} = 2\frac{7}{10}$$

Simplify:

- | | | | |
|---|---|--|---|
| 112. $\frac{\frac{7}{8}}{7\frac{1}{3}}$ | 113. $\frac{\frac{3}{5} \text{ of } 21\frac{1}{6}}{1\frac{5}{7}}$ | 114. $\frac{\frac{3}{4} \times 6\frac{2}{3}}{3\frac{1}{3} \div \frac{3}{8}}$ | 115. $\frac{8 - (\frac{1}{4} \text{ of } \frac{3}{4})}{\frac{3}{4} \div (6\frac{1}{2} - 3\frac{1}{4})}$ |
|---|---|--|---|

FRACTIONAL RELATIONS. RATIO

The **ratio** of one number to another is the quotient of the first divided by the second.

Finding what part one number is of another.

Oral Work

1. What part of 12 is 4? What is the ratio of 12 to 4? Express the answers also in the form of division. Thus, $\frac{4}{12} = 4 \div 12 = \frac{1}{3}$; $\frac{12}{4} = 12 \div 4 = 3$.

2. What part of $\frac{5}{8}$ is $\frac{3}{8}$? of $\frac{4}{5}$ is $\frac{2}{5}$? How, then, can we find what part one number is of another?

To find what part the second of two numbers is of the first, *divide the second by the first.*

To find the ratio of two numbers, *divide the first number by the second number.*

Written Work

1. What part of 208 is 96?

$$96 \div 208 = \frac{96 \div 16}{208 \div 16} = \frac{6}{13}$$

Divide 96 by 208, and reduce the resulting fraction $\frac{96}{208}$ to its lowest terms, $\frac{6}{13}$.

2. What part of $\frac{7}{8}$ is $\frac{3}{16}$?

$$\frac{3}{16} \div \frac{7}{8} = \frac{3}{16} \times \frac{8}{7} = \frac{3}{14}$$

Divide $\frac{3}{16}$ by $\frac{7}{8}$ by inverting the divisor of $\frac{7}{8}$ and multiplying. The result shows that $\frac{3}{16}$ is $\frac{3}{14}$ of $\frac{7}{8}$.

What part of

3. 90 is 16?

4. 120 is 50?

5. 200 is 18?

6. 125 is 25?

What is the ratio of

7. $\frac{8}{9}$ is $\frac{2}{3}$?

8. $\frac{7}{8}$ is $\frac{5}{8}$?

9. 4 is $3\frac{1}{5}$?

10. 2 is $1\frac{1}{4}$?

11. 30 to 48?

12. 48 to 30?

13. $\frac{3}{5}$ to $\frac{2}{10}$?

14. $\frac{3}{10}$ to $\frac{2}{5}$?

Finding a number when a fractional part

of, or the

Oral Work

1. If $\frac{3}{4}$ of a number is 30, what is the

If *three* fourths of the number is 30, *one* fourth of 30, or 10, and *four* fourths of the number is 4×10 , or 40. Hence 30 is $\frac{3}{4}$ of 40.

Written Work

1. 360 is $\frac{6}{5}$ of what number?

$\frac{1}{5}$ of the number = $\frac{1}{6}$ of 360, or 60

$\frac{6}{5}$ of the number = 25×60 , or 1500

Since 360 is $\frac{6}{5}$ of a number,
 $\frac{1}{5}$ of it is $\frac{1}{6}$ of 360, or 60, and $\frac{6}{5}$
 of it is 25×60 , or 1500.

2. If $\frac{7}{8}$ is $\frac{3}{4}$ of a number, what is the number?

$\frac{1}{4}$ of the number = $\frac{1}{3}$ of $\frac{7}{8}$, or $\frac{7}{24}$

$\frac{3}{4}$ of the number = $4 \times \frac{7}{24} = \frac{7}{6}$, or $1\frac{1}{6}$

Since $\frac{3}{4}$ of the number is $\frac{7}{8}$,
 $\frac{1}{4}$ of it is $\frac{1}{3}$ of $\frac{7}{8}$, or $\frac{7}{24}$; and $\frac{3}{4}$ of
 the number is $4 \times \frac{7}{24}$, which
 equals $\frac{7}{6}$, or $1\frac{1}{6}$.

Find the number of which:

3. 84 is $\frac{7}{8}$ 5. $3\frac{1}{2}$ is $\frac{7}{9}$ 7. $\frac{7}{9}$ is $\frac{7}{16}$ 9. $1\frac{1}{2}$ is $2\frac{2}{3}$
 4. 196 is $\frac{4}{11}$ 6. $12\frac{2}{3}$ is $\frac{1}{20}$ 8. $1\frac{2}{3}$ is $\frac{6}{7}$ 10. $5\frac{2}{3}$ is $1\frac{1}{4}$

11. If $\frac{2}{3}$ of a man's salary is \$900, what is his salary?

12. A man lost in speculation \$2700, which was $\frac{3}{16}$ of his entire fortune. What was his fortune?

13. If $1\frac{1}{2}$ of the number of books in a library is 9922, how many books are there in the library?

14. There are 30,205 women in a certain town, which is $\frac{7}{15}$ of the number of men. How many men are there?

15. An author spent \$2100 for a piece of property, which was $\frac{7}{15}$ of what he was paid for a novel. How much did he receive for the novel?

PROBLEMS

TO THE TEACHER. — Require pupils to interpret each problem before attempting to solve it. Encourage them, whenever practicable, to estimate results mentally. Also lead them to choose the most economical method of solution.

1. Find the cost of 11 library rugs at \$59.90 each.

SOLUTION. — $10 \times \$59.90 = \599.00 , cost of 10 rugs	SHORT FORM \$599.00
<u>59.90, cost of 1 rug</u>	<u>59.90</u>
$11 \times \$59.90 = \658.90 , cost of 11 rugs	\$658.90

2. Find the cost of 11 head of cattle averaging 1175 lb. at 9¢ a pound.

SOLUTION. — 11750 lb., weight of 10 head	\$1292.50, cost at 10¢ per lb.
<u>1175 lb., weight of 1 head</u>	<u>129.25, cost at 1¢ per lb.</u>
12925 lb., total weight	\$1263.25, cost at 9¢ per lb.

In stating the work, simply write the numbers, estimating all the other work mentally.

3. Find the cost of $24\frac{3}{4}$ lb. of coffee at 29¢ a pound. In estimating, think of the cost of 25 lb. at 30¢ a pound and subtract the cost of 1 lb.

SOLUTION. — 25 lb. at 30¢ = \$7.50.
 25 lb. at 29¢ = \$7.50 — \$.25, or \$7.25, estimated cost.

4. A lady buys \$1.55 worth of fish at the store. How many pounds does she purchase at 15¢ a pound?

5. Estimate quickly the cost of $3\frac{1}{2}$ doz. apples at 23¢ a dozen and $4\frac{1}{2}$ lb. of cheese at 27¢ a pound.

SOLUTION —	\$0.69	}	cost of apples
+	.11 $\frac{1}{2}$		
$4 \times 25¢ =$	1.00		
$+ 4 \times 2¢ =$.08		
$+ \frac{1}{2}$ of 27¢ =	.13 $\frac{1}{2}$		
	<u>\$2.02</u>		
	total cost		

Explain the meaning of each step in the work

6. Which method do you prefer, the short method, or the long multiplication method?

7. The coal for the school buildings in a certain town cost \$94.50. How many tons were purchased at \$6 $\frac{1}{2}$ a ton?

8. A shoe dealer buys shoes at \$21 a dozen pairs and retails them at \$2.50 a pair. How much does he gain on each pair?

9. If oranges are bought at the rate of 4 dozen for \$1.08 and sold at 30 cents a dozen, what will be the gain on 3 dozen?

10. A man buys a farm of 102 acres and divides it into lots of 6 to the acre. How many lots are there if $\frac{1}{6}$ of the farm is laid out in streets?

11. How many sheep, at \$15 a head, may be purchased with the money from the sale of 10 head of cattle at \$90 a head?

12. A contractor buys 8000 bricks at \$12.50 a thousand. Find the amount of his bill.

13. If I buy goods at \$1.37 a yard and sell them at \$1.50 a yard, how much do I gain on 12 yards?

14. A boy earns 75 cents a day and pays $\frac{3}{8}$ of his wages for board. At that rate how much can he save in 26 days?

15. A lady purchased 2 handkerchiefs at 35 cents each; 6 yards of ribbon at 12 cents a yard; 6 yards of cloth at \$1.12 a yard. How much change should she receive from a ten-dollar bill?

16. A farmer buys fertilizer at \$28 a ton and retails it at \$1.90 a hundred pounds. How much does he gain on 35 tons?

17. At 16 cents a pound, how many pounds of fish does a woman get for 83 cents?

18. A huckster bought $6\frac{1}{2}$ pounds of grapes at 16 cents a pound, and $6\frac{1}{2}$ dozen bananas at 18 cents a dozen. How much did he pay for both?

19. A grocer buys peaches at $16\frac{1}{2}$ cents a can and sells them at $20\frac{1}{2}$ cents a can. How many cans must he purchase and sell in order to gain \$25?

SOLUTION. — On each can he gains 4 cents.

To gain \$1 he must sell 25 cans.

To gain \$25 he must sell 25 times 25 cans, or 625 cans.

20. A street car conductor earns $23\frac{1}{2}$ cents an hour. How much does he earn in 10 hours?

21. If the conductor averages 10 hours a day, how much will he earn in 30 days?

22. If a box of 40 dozen oranges is purchased for \$5 and retailed at 40 cents a dozen, what is the entire gain?

23. What fraction of a gallon is a pint? a gill? a quart?

24. A man sold $\frac{2}{3}$ of his farm for \$1800. At the same rate, how much would he receive for the whole farm?

SOLUTION. — $\$1800 = \frac{2}{3}$ of the amount received for the farm.

$\frac{1}{3}$ of \$1800 = \$900, or $\frac{1}{3}$ of the amount received for the farm.

3 times \$900 = \$2700, or $\frac{3}{2}$ of the amount received for the farm.

25. In a certain school there are 40 pupils in the grammar grade, which are $\frac{2}{3}$ of the number in the intermediate grade. How many pupils are there in both grades?

26. A man sold a piano for \$360, which was $\frac{4}{5}$ of what it cost him. How much did it cost him?

27. If $\frac{1}{4}$ of a ton of hay costs \$16, how much will $2\frac{1}{2}$ tons cost?

28. If it takes 4 men 9 days to do a piece of work, how long will it take 6 men, at the same rate, to do the work?

29. If $\frac{3}{4}$ of a farm is valued at \$1240, what is the value of $\frac{1}{4}$ of the farm?

30. A lady spends $\frac{2}{5}$ of her income for board, and $\frac{1}{5}$ of the remainder for clothes and travel. If she saves \$160 per year, what is her income?

31. A farmer has 600 bushels of wheat, which is $\frac{2}{3}$ of $\frac{3}{4}$ of what his neighbor has. How many bushels has his neighbor?

32. A house was sold for \$1800, which was $\frac{3}{4}$ of its cost. What was the cost?

33. If 20 men can dig a ditch in 25 days, how long will it take 5 men?

34. A farmer sold 36 head of cattle, which was 6 more than $\frac{3}{4}$ of all he owned. How many had he remaining?

35. If a man receives \$1.50 for $\frac{3}{4}$ of a day's work, how much should he receive for $\frac{1}{2}$ of a day's work?

36. I purchased an overcoat for \$45, and found I had $\frac{1}{5}$ of my money left. How much had I at first?

37. A contractor employs 6 men for $27\frac{1}{2}$ days and pays them \$363. Find the average daily wages.

38. A hardware merchant bought a bill of hardware at auction for $1\frac{1}{2}$ of its value, and retailed it for $\frac{3}{4}$ of its value. If his gain was \$48.75, how much did he pay for the goods?

39. A boy bought lemons at the rate of 4 for 5 cents, and sold them at the rate of 3 for 5 cents. If he made \$6 in 2 weeks of 6 days each, what were his daily average sales?

40. A path leading to the top of a hill has a rise of 9 feet in 90 feet. What is the elevation of the hill in feet, if the path is $\frac{3}{4}$ of a mile long?

41. A teacher taught $8\frac{1}{2}$ months, and after spending $\frac{2}{5}$ of his salary for board had left \$204. How much did he earn per month?

42. Two men, A and B, each bought farms, A's farm costing $2\frac{3}{4}$ times as much as B's. Find the cost of each, if both cost \$58,000.

43. A ship is worth \$120,000, and the owner of $\frac{3}{4}$ of it sells $\frac{1}{4}$ of his share. Find the value of the part he has remaining.

44. I purchased 160 acres of land at \$60 an acre, and sold $\frac{5}{8}$ of it at \$70 an acre. Find the number of acres I had left, and my gain on the number of acres sold.

45. A real estate agent bought land for \$7200, and sold it so as to gain $\frac{1}{10}$ of the cost. If the gain was \$6 an acre, how many acres did he buy?

46. A merchant buys 36 seal bags at \$2 $\frac{1}{4}$ each; 24 velvet bags at \$1 $\frac{3}{4}$ each; and 30 silk bags at \$ $\frac{3}{4}$ each. What is the amount of his bill?

47. If the seal bags in example 46 are sold at \$3 each, the velvet bags at \$2 $\frac{1}{2}$ each, and the silk bags at \$1 $\frac{1}{4}$ each, how much does the merchant make?

48. A cubic foot of water weighs 62 $\frac{1}{2}$ pounds. A barrel contains about 4 $\frac{1}{2}$ cubic feet. Find the weight of a barrel of water.

49. Find the perimeters of each of five rooms, the dimensions being as follows: 22 $\frac{1}{2}$ ft. by 16 ft., 14 $\frac{3}{4}$ ft. by 15 $\frac{1}{2}$ ft., 12 $\frac{3}{4}$ ft. by 17 ft., 18 $\frac{1}{4}$ ft. by 15 $\frac{3}{4}$ ft., 30 $\frac{1}{2}$ ft. by 24 ft.

50. From a certain number $18\frac{7}{8} + 27\frac{5}{12}$ was subtracted, leaving a remainder of $9\frac{5}{8}$. What was the number?

51. A hall is 21 $\frac{3}{4}$ feet long and 10 $\frac{1}{4}$ feet wide. At 12 cents a foot, how much will it cost to put a molding around this hall?

REVIEW OF DECIMAL FRACTIONS

1. What is the largest common fractional unit?
2. Show that an integral unit may be divided into any number of fractional units.
3. Name the different fractional units from $\frac{1}{2}$ to $\frac{1}{20}$ in order of their size.
4. What fractional unit divides the integral unit into 10 equal parts? into 100 equal parts? into 1000 equal parts?

The divisions of an integral unit into 10ths, 100ths, 1000ths, etc., are called **decimal divisions**.

There are three ways by which decimal divisions may be expressed :

- (1) by words; as, nine tenths ;
- (2) by common fractions; as, $\frac{9}{10}$, $\frac{9}{100}$;
- (3) by decimals; as, .9, .75.

A **decimal fraction** is any number of 10ths, 100ths, 1000ths, etc., of an integral unit. When expressed after a decimal point and without a written denominator, it is usually called a **decimal**.

A **decimal point** is a period placed after ones' place and before tenths' place.

5. What is the *largest* decimal unit? the *second* largest? the *third* largest?

In any decimal system 10×1 unit of any place = 1 unit of the next higher place.

6. Show that United States money is a *decimal system*.

NOTATION AND NUMERATION OF DECIMALS

1. Express decimally $\frac{6}{10}$, $\frac{75}{100}$, $\frac{6}{1000}$, $\frac{64}{1000}$, $\frac{6}{100}$, $\frac{72}{1000}$.

Every decimal contains as many decimal places as there are naughts in the denominator of the equivalent fraction.

Table of places and names of *integral* and *fractional* units:

Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths
5	3	5	7	8	4	5	5	.	0	0	7	4	8	9	8

The decimal is read as an integer and the last figure is given the required denomination.

Read:

- | | | |
|---------|----------|-------------|
| 2. .25 | 6. .101 | 10. .60745 |
| 3. .05 | 7. .0045 | 11. .678705 |
| 4. .005 | 8. .4045 | 12. .0065 |
| 5. .375 | 9. .0002 | 13. .60005 |

14. Since $.5 = .50 = .500$, does canceling naughts from the right of a decimal change the value of the decimal?

15. Observe that canceling naughts from the right of a decimal really means canceling naughts from the numerator and the denominator. Thus, $.50 = \frac{50}{100} = .5\cancel{0} = \frac{5\cancel{0}}{10\cancel{0}}$.

Read first as given, then as if the naughts at the right of the decimal were canceled:

- | | | |
|-------------|------------|-------------|
| 16. .040 | 18. .7500 | 20. 10.0057 |
| 17. 26.0050 | 19. 8.0090 | 21. 20.0900 |

ADDITION AND SUBTRACTION OF DECIMALS

In adding or subtracting decimals, tenths must be placed under tenths, hundredths under hundredths, etc.

Written Work

1. Add .9084 and .004.

$$\begin{array}{r} .9084 \\ .004 \\ \hline .9124 \end{array}$$

2. Subtract .004 from .9084.

$$\begin{array}{r} .9084 \\ .004 \\ \hline .9044 \end{array}$$

Add, as indicated, and test :

	3.		4.		5.		6.		
7.	.85	+	62.8	+	9.556	+	97.	=	_____
8.	9.06	+	3.09	+	0.038	+	6.4	=	_____
9.	<u>82.073</u>	+	<u>99.08</u>	+	<u>26.908</u>	+	<u>31.3</u>	=	_____
10.		+		+		+		=	_____

Subtract examples 11 to 14 and add the remainders :

11.	30.276	12.	8.0089	13.	318.76	14.	48.9
	<u>29.008</u>		<u>7.3846</u>		<u>7.96</u>		<u>22.6</u>
15.	+		+		+	=	—

Add :

16. 3.7	17. .001	18. .65
5.06	12.3	.001
8.023	15.0248	10.1
<u>9.04</u>	<u>18.0149</u>	<u>25.004</u>

Find differences :

19. .96	21. .4217	23. 5.206	25. 15.
<u>.2827</u>	<u>.375</u>	<u>4.7869</u>	<u>4.008</u>
20. 97.	22. \$55.68	24. 225.5	26. 849 $\frac{3}{4}$
<u>28.278</u>	<u>32.09</u>	<u>97.439</u>	<u>678.625</u>

MULTIPLICATION OF DECIMALS

Multiplying a decimal or a mixed decimal by a decimal or a mixed decimal.

Oral Work

1. Multiply .05 by .5.
2. What is the numerator in .05? in .5? the denominator?
3. What shows the denominator in a decimal?
4. Multiply the numerators in $.5 \times .05$; thus, $5 \times 5 = 25$. Multiply the denominators in $.5 \times .05$; thus, $10 \times 100 = 1000$. Write the result as a common fraction; thus, $\frac{25}{1000}$. What term of the fraction is expressed by the decimal point?
5. $\frac{5}{10} = .5$; $\frac{5}{100} = .05$; $\frac{5}{1000} = .005$; $\frac{5}{10000} = .0005$. Observe that there are as many decimal places in each decimal as there are *noughts* in the denominator of the equivalent common fraction.

Find the product of the following decimals by multiplying the numerators and the denominators, separately, and expressing the result as a decimal; thus,

$$.04 \times .7 = \frac{4}{100} \times \frac{7}{10} = \frac{28}{1000}, \text{ or } .028.$$

- | | | |
|----------------------|-----------------------|-----------------------|
| 6. $.2 \times .06$ | 12. $.35 \times .05$ | 18. $.02 \times .42$ |
| 7. $.04 \times .5$ | 13. $.07 \times .05$ | 19. $.06 \times .004$ |
| 8. $.25 \times .05$ | 14. $.03 \times .02$ | 20. $.05 \times .009$ |
| 9. $.7 \times .06$ | 15. $.15 \times .003$ | 21. $.019 \times .4$ |
| 10. $.06 \times .07$ | 16. $.7 \times .004$ | 22. $.05 \times .002$ |
| 11. $.30 \times .02$ | 17. $.6 \times .008$ | 23. $.07 \times .008$ |

The product of two decimals contains as many decimal places as the sum of the decimal places in both factors.

Written Work

1. Multiply .26 by .14.

$$\begin{array}{r}
 (b) \quad .26 \\
 \quad .14 \\
 \hline
 104 \\
 \quad 26 \\
 \hline
 .0364
 \end{array}$$

1. What is the sum of the decimal places in the two factors?

2. The product, then, must contain how many places?

When the product has not enough decimal places, supply the deficiency by prefixing naughts.

Find products:

- | | | |
|---------------------------------|---------------------------|--------------------------|
| 2. $.35 \times .23$ | 19. $.4296 \times .565$ | 36. $.5986 \times .584$ |
| 3. $.29 \times .48$ | 20. $.3592 \times 7$ | 37. $.007 \times 40$ |
| 4. $.028 \times .04$ | 21. $75.95 \times .64$ | 38. $88.2 \times .81$ |
| 5. $46 \times .13$ | 22. 85.004×26.09 | 39. $.0095 \times .05$ |
| 6. $.36 \times 56$ | 23. $376 \times .008$ | 40. $7.876 \times .784$ |
| 7. 8.9×9.21 | 24. $.0013 \times 920$ | 41. $662.9 \times .0075$ |
| 8. $.089 \times 60$ | 25. $5500 \times .005$ | 42. 419.8×2.74 |
| 9. $.037 \times 38$ | 26. $424 \times .059$ | 43. $30.08 \times .007$ |
| 10. $2.005 \times .021$ | 27. $426 \times .083$ | 44. $.375 \times 3.028$ |
| 11. 36.8×44 | 28. $727 \times .62$ | 45. 99.63×4.095 |
| 12. 48.25×72 | 29. $647 \times .5$ | 46. $.059 \times 8.92$ |
| 13. $625.6 \times 9\frac{1}{2}$ | 30. $59.4 \times .09$ | 47. 5.32×2.36 |
| 14. 49.10×17.4 | 31. 8.88×4.7 | 48. $14.37 \times .003$ |
| 15. $.0914 \times .015$ | 32. $59.79 \times .61$ | 49. $7.08 \times .0005$ |
| 16. $72.184 \times .0025$ | 33. 7.707×9.1 | 50. 2.004×2.004 |
| 17. 35.683×7 | 34. 268.6×8.4 | 51. $.05 \times .005$ |
| 18. $.4234 \times 448$ | 35. $3005 \times .007$ | 52. 7.875×4.97 |

DIVISION OF DECIMALS

Dividing a decimal or a mixed decimal by an integer.

Oral Work

1. Find $\frac{1}{2}$ of 45 hundredths; of 60 hundredths.
2. Find $\frac{1}{4}$ of .24; of .36; of .44.
3. Find $\frac{1}{8}$ of 8 and 48 hundredths; 24 and 24 hundredths.
4. Find $\frac{1}{6}$ of 18.36; 24.12; 30.06; 48.06.

Observe that in each problem a decimal or a mixed decimal when divided by an integer is simply separated or *partitioned*.

A decimal or a mixed decimal is divided by an integer by placing a decimal point above or below the decimal point in the dividend, before beginning to divide, and dividing as in the division of integers.

Find quotients:

- | | | |
|--------------------|----------------------|----------------------|
| 5. $48.66 \div 6$ | 10. $5.1250 \div 25$ | 15. $100.25 \div 25$ |
| 6. $545.5 \div 5$ | 11. $.93093 \div 31$ | 16. $64.064 \div 32$ |
| 7. $60.84 \div 12$ | 12. $24.624 \div 54$ | 17. $3510.5 \div 35$ |
| 8. $19.5 \div 15$ | 13. $4.50 \div 30$ | 18. $64.064 \div 64$ |
| 9. $56.56 \div 14$ | 14. $806.4 \div 16$ | 19. $48.048 \div 48$ |

Dividing a decimal or a mixed decimal by a decimal or a mixed decimal.

Oral Work

Study of Problems

1. $6.25 \div 1.25 = 5$
 2. $62.5 \div 12.5 = 5$
 3. $625 \div 125 = 5$
1. What is the first quotient? the second? the third?
 2. What was done to the first problem to make the second? to the second to make the third?

3. How is a decimal affected by moving the decimal point one place to the right? two places?

4. How did moving the decimal point to the right the same number of places in both dividend and divisor of each problem affect the quotient?

Multiplying both dividend and divisor by the same number does not change the quotient.

Written Work

1. Divide 5.49 by .9.

$$(a) \begin{array}{r} 6.1 \\ .9 \overline{)5.49} = 9 \overline{)54.9} \end{array}$$

$$(b) \begin{array}{r} 6.1 \\ .9 \overline{)5.49} \end{array}$$

In (a) both dividend and divisor are multiplied by 10 to make the divisor a whole number. This is done by moving the decimal point in the divisor and the dividend one place to the right.

In (b) the example is written in its original form and the changed position of the decimal point in the dividend is indicated by a caret, placed as many places to the right of the decimal point as there are decimal places in the divisor. As there is one decimal point in the divisor, place the caret one place to the right of the decimal point in the dividend. Then divide as in integers, placing the decimal point in the quotient immediately after all the numbers to the left of the caret have been used in the process of division.

2. Divide 1.7535 by .035.

$$\begin{array}{r} 50.1 \\ .035 \overline{)1.7535} \\ \underline{175} \\ 35 \\ \underline{35} \\ 5 \end{array}$$

As there are three decimal places in the divisor, place the caret three places to the right of the point. Place the decimal point in the quotient immediately after all the numbers to the left of the caret have been used.

Mark off by a caret the same number of decimal places from the right of the decimal point in the dividend as there are decimal places in the divisor. Divide as in integers, placing the decimal point in the quotient immediately after all the numbers to the left of the caret have been used in the process of division.

NOTE. — This method is called the **Austrian method**. It retains the identity of the example and saves the time required to restate it.

Find quotients:

- | | |
|------------------------|---------------------------|
| 3. $8.10 \div .54$ | 19. $4000 \div .004$ |
| 4. $.126 \div .07$ | 20. $.475 \div .019$ |
| 5. $4.199 \div 1.9$ | 21. $88.9014 \div 36.435$ |
| 6. $43.46 \div 21.2$ | 22. $291.816 \div 2.52$ |
| 7. $432 \div .36$ | 23. $.1968 \div .012$ |
| 8. $.432 \div 36$ | 24. $.0567 \div 21$ |
| 9. $15.68 \div .028$ | 25. $171.50 \div .0098$ |
| 10. $.20956 \div .169$ | 26. $.076923 \div .2331$ |
| 11. $9.375 \div .75$ | 27. $.0011 \div 100$ |
| 12. $.3105 \div .15$ | 28. $1100 \div .011$ |
| 13. $.5 \div .125$ | 29. $.025 \div .00625$ |
| 14. $13.410 \div .018$ | 30. $.0009 \div 4.5$ |
| 15. $27.839 \div 2.87$ | 31. $2.10 \div .01$ |
| 16. $232.5 \div .9375$ | 32. $.137 \div 200$ |
| 17. $.00261 \div .145$ | 33. $.02116 \div 92$ |
| 18. $.003410 \div .62$ | 34. $25.125 \div 1.005$ |

The division is frequently not exact. In such cases the sign \div may be placed after the decimal to show that the division is not complete. Thus, $1 \div 7 = .142 \div$

Find the sum of the quotients:

- | 35. | 36. | 37. |
|------------------|------------------|-------------------|
| $1 \div .1 =$ | $3 \div .03 =$ | $.18 \div 72 =$ |
| $.1 \div 10 =$ | $30 \div .3 =$ | $.04 \div 50 =$ |
| $.25 \div 50 =$ | $6 \div .006 =$ | $2 \div .025 =$ |
| $2.5 \div .5 =$ | $16 \div .04 =$ | $20 \div .002 =$ |
| $25 \div 2.5 =$ | $60 \div 300 =$ | $200 \div 12.5 =$ |
| $.15 \div .15 =$ | $.6 \div 30 =$ | $64 \div .016 =$ |
| $1.5 \div 15 =$ | $.66 \div 1.1 =$ | $64 \div 160 =$ |
| $.15 \div 2.5 =$ | $.9 \div .009 =$ | $.4 \div 400 =$ |

COMPARISON OF COMMON FRACTIONS AND DECIMALS**Changing decimals to common fractions.****Oral Work**

Write as common fractions and change to lowest terms :

1. .25 3. .045 5. .50 7. .0025

2. .7 4. .025 6. .75 8. .0775

9. Give the steps in changing a decimal to its fractional equivalent.

10. Change $.16\frac{2}{3}$ to a common fraction.

$$.16\frac{2}{3} = \frac{16\frac{2}{3} \times 3}{100 \times 3} = \frac{50}{300}, \text{ or } \frac{1}{6}.$$

11. Explain why multiplying both terms of the fraction $\frac{16\frac{2}{3}}{100}$ by 3 does not change the value of the fraction.**Written Work**

Change to common fractions :

1. $.37\frac{1}{2}$

5. $.83\frac{1}{3}$

9. $.62\frac{1}{2}$

13. $.12\frac{1}{2}$

2. $.41\frac{2}{3}$

6. $.06\frac{1}{4}$

10. $.33\frac{1}{3}$

14. $.14\frac{2}{3}$

3. $.66\frac{2}{3}$

7. $.04\frac{1}{6}$

11. $.31\frac{1}{4}$

15. $.58\frac{1}{3}$

4. $.24\frac{1}{6}$

8. $.08\frac{1}{3}$

12. $.03\frac{1}{3}$

16. $.88\frac{1}{3}$

Changing common fractions to decimals.**Written Work**

Since $\frac{4}{5} = 4 \div 5$, to change a fraction to a decimal, consider it as a problem in division of decimals. Thus, $\frac{4}{5} = 4 \div 5 = 5 \overline{)4.0}$.

0.8

Change to decimals and test:

- | | | | | |
|------------------|-------------------|-------------------|-------------------|--------------------|
| 1. $\frac{5}{8}$ | 3. $\frac{3}{20}$ | 5. $\frac{7}{8}$ | 7. $\frac{2}{16}$ | 9. $\frac{7}{16}$ |
| 2. $\frac{3}{5}$ | 4. $\frac{5}{16}$ | 6. $\frac{1}{25}$ | 8. $\frac{1}{32}$ | 10. $\frac{1}{25}$ |

When the division does not terminate, the quotient may be expressed thus: $\frac{3}{7} = 7 \overline{)3.000}$, or thus: $7 \overline{)3.000}$
 $\quad\quad\quad 0.428\overline{4}$ $\quad\quad\quad 0.428 +.$

Change to decimals of not more than four places:

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 11. $\frac{7}{11}$ | 15. $\frac{1}{27}$ | 19. $\frac{3}{57}$ | 23. $\frac{1}{80}$ |
| 12. $\frac{9}{31}$ | 16. $\frac{9}{19}$ | 20. $\frac{3}{49}$ | 24. $\frac{5}{13}$ |
| 13. $\frac{4}{73}$ | 17. $\frac{1}{7}$ | 21. $\frac{7}{13}$ | 25. $\frac{2}{17}$ |
| 14. $\frac{2}{53}$ | 18. $\frac{2}{17}$ | 22. $\frac{5}{11}$ | 26. $\frac{1}{25}$ |

Parts of \$1

$1\text{¢} = \frac{1}{100}$ of \$1.	$12\frac{1}{2}\text{¢} = \frac{1}{8}$ of \$1.	$50\text{¢} = \frac{1}{2}$ of \$1.
$2\text{¢} = \frac{1}{50}$ of \$1.	$16\frac{2}{3}\text{¢} = \frac{1}{6}$ of \$1.	$62\frac{1}{2}\text{¢} = \frac{5}{8}$ of \$1.
$4\text{¢} = \frac{1}{25}$ of \$1.	$20\text{¢} = \frac{1}{5}$ of \$1.	$66\frac{2}{3}\text{¢} = \frac{2}{3}$ of \$1.
$5\text{¢} = \frac{1}{20}$ of \$1.	$25\text{¢} = \frac{1}{4}$ of \$1.	$75\text{¢} = \frac{3}{4}$ of \$1.
$6\frac{1}{4}\text{¢} = \frac{1}{16}$ of \$1.	$33\frac{1}{3}\text{¢} = \frac{1}{3}$ of \$1.	$80\text{¢} = \frac{4}{5}$ of \$1.
$8\frac{1}{3}\text{¢} = \frac{1}{12}$ of \$1.	$37\frac{1}{2}\text{¢} = \frac{3}{8}$ of \$1.	$83\frac{1}{3}\text{¢} = \frac{5}{6}$ of \$1.
$10\text{¢} = \frac{1}{10}$ of \$1.	$40\text{¢} = \frac{2}{5}$ of \$1.	$87\frac{1}{2}\text{¢} = \frac{7}{8}$ of \$1.

Finding the total cost when the quantity purchased is given and the price of a unit is an even part of \$1.

Written Work

Business computations may be shortened by knowing the relation that the price of a unit bears to \$1 or to \$100.

1. How much will 44 dozen oranges cost at \$.25 a dozen?

DECIMAL METHOD

$$\begin{array}{r}
 \text{\$.25, price} \\
 44, \text{ no. of dozen} \\
 \hline
 100 \\
 100 \\
 \hline
 \text{\$11.00, total cost}
 \end{array}$$

SHORT METHOD

$$\begin{array}{r}
 4 \overline{) \$44} \\
 \underline{\$11}
 \end{array}$$

At \$1 a dozen, 44 doz. would cost \$44.
 At $\frac{1}{4}$ a dozen, they cost $\frac{1}{4}$ of \$44, or \$11.

Find the cost of the quantity at \$1. Divide this by the number representing the quantity that can be purchased for \$1.

Find the cost of:

2. 60 lb. cheese at $33\frac{1}{3}$ ¢ a pound.
3. 25 lb. coffee at 25¢ a pound.
4. 960 yd. calico at $6\frac{1}{4}$ ¢ a yard.
5. 50 lb. prunes at $12\frac{1}{2}$ ¢ a pound.
6. 120 yd. ribbon at $37\frac{1}{2}$ ¢ a yard.
7. 600 lb. beef at $33\frac{1}{3}$ ¢ a pound.

Finding the quantity purchased when the total cost is given and the price of a unit is an even part of \$1.

Written Work

1. How many yards of calico, at $6\frac{1}{4}$ ¢ a yard, can be purchased for \$100?

DECIMAL METHOD

$$\begin{array}{r}
 1600. \\
 \text{\$.0625) \$100.0000}^{\wedge}
 \end{array}$$

SHORT METHOD

$$100 \times 16 \text{ yd.} = 1600 \text{ yd.}$$

Since $6\frac{1}{4}$ ¢ = $\frac{1}{16}$ ¢, \$1 pays for 16 yd.;
 and \$100 pays for 100×16 yd., or
 1600 yd.

Multiply the number representing the quantity purchased for \$1 by a number equal to the number of dollars invested.

Find the quantity of each article if a grocer invested :

2. \$160 in sugar at $6\frac{1}{4}$ ¢ a pound.
3. \$120 in cornmeal at 4¢ a pound.
4. \$6 in rice at 10¢ a pound.
5. \$100 in cloth at 50¢ a yard.
6. \$13 in ribbon at 5¢ a yard.
7. \$3 in tape at 2¢ a yard.
8. \$800 in sugar at $6\frac{1}{4}$ ¢ a pound.
9. \$100 in fish at $12\frac{1}{2}$ ¢ a pound.
10. \$4 in collars at $12\frac{1}{2}$ ¢ each.
11. \$1 in bananas at 20¢ a dozen. (Find the number of bananas.)

PROBLEMS

1. If the price of gas is \$1.00 a thousand cubic feet, how much is the average gas bill per month, when 85,620 cubic feet are consumed in 6 months?
2. How many bushels of apples, at \$1.65 a bushel, can be bought for \$82.50?
3. A teacher paid .35 of his salary for board, .18 for clothing, .07 for travel, .1 for incidentals, and had \$300 left. How much was his salary?
4. From $3\frac{3}{4}$ take the sum of $3\frac{3}{4}$ thousandths and $3\frac{3}{4}$ millionths.
5. If seventy-five hundredths of a number is 372, what is eighty-three and one third hundredths of it?
6. A man who owned $\frac{3}{4}$ of a store sold $\frac{3}{8}$ of his share for \$1406.25. What was the value of the store? What part had he left?

7. A real estate agent purchased four lots. He sold two of them at a loss of \$67.25. If he sells the other two lots at a profit of \$75.75, how much does he gain on his investment?

8. A school board paid \$144 for books: $\frac{2}{3}$ of the amount was paid for General Histories at \$1.20 each; $\frac{1}{3}$ of it for Algebras at \$1 each; and the remainder for Rhetorics at \$.96 each. How many books were purchased?

9. A man invested .32 of his money in mining stock, .48 in railroad stock, and had \$12,000 remaining. How much had he at first?

10. An engineer took a contract to build a bridge for \$18,500. The material cost \$10,575; he employed 40 men for $7\frac{1}{2}$ weeks of 6 days each at \$2.25 a day, and 30 men for $4\frac{1}{2}$ weeks of 5 days each at \$2.75 a day. How much did he gain?

11. A merchant bought 140 boys' suits at \$6.75 a suit. He sold $\frac{3}{4}$ of them at \$9.50 a suit and the remainder at \$8.50 a suit. How much did he gain?

12. Four men purchased an oil property, the first paying for .3 of it, the second for .375 of it, the third for $\frac{1}{5}$ of it, and the fourth the remainder, which was \$3000. How much did the property cost them?

13. A merchant bought 975 pounds of sugar for \$48.75. He sold $\frac{2}{3}$ of it at \$.055 a pound, $\frac{1}{3}$ of it at \$.06 a pound, and the remainder at \$.065 a pound. Find his gain.

14. I invested .4 of my money in a farm and deposited .75 of the remainder in a bank. If the amount paid for the farm was \$300 less than the amount deposited in the bank, how much money had I?

Find the sum of the quotients:

15.	$5 \div 5$	16.	$.6 \div .2$
	$5 \div .5$		$66 \div .22$
	$5 \div .05$		$11 \div .022$
	$5 \div .005$		$.08 \div .2$
	$.005 \div 5$		$.088 \div 22$
	$.5 \div 5$		$.16 \div .004$
	$500 \div .5$		$.078 \div .013$
17.	$.001 \div .01$	18.	$3.24 \div .18$
	$.72 \div .004$		$.5 \div .0125$
	$.096 \div .32$		$675 \div .75$
	$.198 \div 6.6$		$9 \div .225$
	$.05 \div .125$		$.288 \div 32$
	$8.1 \div .09$		$100 \div .001$
	$216 \div .036$		$39.2 \div .14$
	$.8 \div 160$		$4.4 \div 55$
		19.	$.002 \div 20$
			$31.11 \div 6.1$
			$.003 \div 37.5$
			$16 \div .016$
			$.4 \div 250$
			$40.04 \div 1.43$
			$.576 \div .8$
			$43.3 \div 100$

20. A merchant bought 250 barrels of apples at \$6 a barrel. He sold 10 barrels, which were damaged, for $\frac{1}{4}$ of the cost. On the sale of 150 barrels he gained 50 cents a barrel. If his total gain was \$110, at what price did he sell the remaining number of barrels?

21. A boy bought apples at the rate of 4 for 10 cents, and sold them at the rate of 3 for 10 cents. If he made \$3 in a week of 6 days, what were his daily average sales?

22. The average wages of a steel mill employing 3000 men were \$2.50 a day. If a $\frac{1}{10}$ reduction in wages is made, how much per day will the company's pay roll be reduced?

23. In a certain school the salary of the teacher for the term is \$600. The books and supplies cost \$120.60; fuel, \$100; repairs and other expenses, \$115.30. There are 35 pupils in the school. Find the average cost per pupil for the term.

PERCENTAGE

In percentage we have only one unit of measure, — the decimal .01, or $\frac{1}{100}$.

The term **per cent** means *hundredths* or *by the hundred*. The sign for it is %.

Thus, five hundredths may be written $\frac{5}{100}$, .05, 5 per cent, or 5%. These are called **equivalents**.

Percentage is the process of computing by hundredths. It is simply an application of decimals.

Oral and Written Work

Write both as a *decimal* and as a *common fraction* in its lowest terms each of the following per cents. Thus, 10% = .10 = $\frac{10}{100}$, or $\frac{1}{10}$.

- | | | | |
|---|--|-----------|---------------------|
| 1. 10 % | 5. $37\frac{1}{2}$ % | 9. 120 % | 13. 75 % |
| 2. $6\frac{1}{2}$ % | 6. $14\frac{1}{2}$ % | 10. 250 % | 14. 300 % |
| 3. $8\frac{1}{2}$ % | 7. $33\frac{1}{3}$ % | 11. 43 % | 15. $\frac{1}{2}$ % |
| 4. $12\frac{1}{2}$ % | 8. 125 % | 12. 65 % | 16. $\frac{3}{4}$ % |

Write the following decimals as fractions and per cents :

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| 17. .05 | 21. $.08\frac{1}{2}$ | 25. 1.50 | 29. .25 |
| 18. .20 | 22. 2.50 | 26. $.11\frac{1}{2}$ | 30. $.16\frac{2}{3}$ |
| 19. $.33\frac{1}{3}$ | 23. 1.25 | 27. $.03\frac{1}{2}$ | 31. $.37\frac{1}{2}$ |
| 20. .50 | 24. 1.20 | 28. $.06\frac{2}{3}$ | 32. .45 |

Write the following as decimals and as per cents :

- | | | | |
|-------------------|-------------------|--------------------|--------------------|
| 33. $\frac{1}{2}$ | 37. $\frac{1}{5}$ | 41. $\frac{1}{9}$ | 45. $\frac{1}{12}$ |
| 34. $\frac{1}{3}$ | 38. $\frac{1}{6}$ | 42. $\frac{1}{10}$ | 46. $\frac{5}{8}$ |
| 35. $\frac{1}{4}$ | 39. $\frac{1}{7}$ | 43. $\frac{1}{16}$ | 47. $\frac{5}{6}$ |
| 36. $\frac{2}{3}$ | 40. $\frac{1}{8}$ | 44. $\frac{3}{8}$ | 48. $\frac{7}{8}$ |

Memorize the following equivalents :

$1\% = \frac{1}{100}$	$14\frac{2}{3}\% = \frac{1}{7}$
$2\% = \frac{1}{50}$	$16\frac{2}{3}\% = \frac{1}{6}$
$3\frac{1}{3}\% = \frac{1}{30}$	$20\% = \frac{1}{5}$
$4\% = \frac{1}{25}$	$25\% = \frac{1}{4}$
$5\% = \frac{1}{20}$	$33\frac{1}{3}\% = \frac{1}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$37\frac{1}{2}\% = \frac{3}{8}$
$6\frac{2}{3}\% = \frac{1}{15}$	$50\% = \frac{1}{2}$
$8\frac{1}{3}\% = \frac{1}{12}$	$62\frac{1}{2}\% = \frac{5}{8}$
$9\frac{1}{11}\% = \frac{1}{11}$	$66\frac{2}{3}\% = \frac{2}{3}$
$10\% = \frac{1}{10}$	$75\% = \frac{3}{4}$
$11\frac{1}{9}\% = \frac{1}{9}$	$83\frac{1}{3}\% = \frac{5}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$

49. What is 50% of 100? of 40? of 10? 2? $\frac{1}{2}$? $\frac{1}{4}$?

50. What is $16\frac{2}{3}\%$ of 90? of 150? of 120? 9? 12? 3?

51. What is $33\frac{1}{3}\%$ of 3000? of 2000? of 50? 75? 100?

52. What is $12\frac{1}{2}\%$ of 200? of 32? of 96? 4? 6? 1?

53. What is $37\frac{1}{2}\%$ of 72? of 56? of 800? 2000? 40? 8?

54. What is 20% of 400? of 600? of $\frac{1}{2}$? $\frac{1}{4}$? 16? 20?

55. Find $12\frac{1}{2}\%$ of 16; of 48; of 72; 96; 168; 800; 220; 404.

56. Find $66\frac{2}{3}\%$ of 60; of 480; of 318; 131; 827; 945; 2154; 2721.

57. Find 5% of 25; of 50; of 75; 100; 125.

In each example in 57 we have two terms, a per cent and a *number*. In each case we are to find 5% of the number. The number of which we take the $\frac{5}{100}$ (viz. 25, 50, etc.) is called the *base*. The number of hundredths (5) to be taken is called the *rate*, and the number of hundredths *actually taken*, that is, the answer, is called the *percentage*.

The **base** is the number on which the percentage is computed.

The **rate** or **rate per cent** is the number of hundredths. We generally express the rate as a decimal.

The **percentage** is the product obtained by taking a certain per cent of the base.

The **sum** or **amount** is the base plus the percentage.

The **difference** is the base minus the percentage.

Finding a given per cent of any number.**Oral Work**

1. What is 20 % of 300? *Think of 20 % of 300 as $\frac{1}{5}$ of 300, or 60.*

Find :

- | | | |
|-----------------|------------------------------|------------------------------|
| 2. 2 % of 100 | 12. 6 % of 150 | 22. $33\frac{1}{3}$ % of 333 |
| 3. 5 % of 400 | 13. 7 % of 800 | 23. $62\frac{1}{2}$ % of 864 |
| 4. 10 % of 500 | 14. 5 % of 440 | 24. $66\frac{2}{3}$ % of 724 |
| 5. 20 % of 800 | 15. 6 % of 550 | 25. 75 % of 968 |
| 6. 50 % of 1200 | 16. $3\frac{1}{3}$ % of 900 | 26. $87\frac{1}{2}$ % of 568 |
| 7. 40 % of 1000 | 17. $8\frac{1}{3}$ % of 480 | 27. 6 % of \$60 |
| 8. 25 % of 360 | 18. $9\frac{1}{6}$ % of 660 | 28. 8 % of \$560 |
| 9. 30 % of 90 | 19. $11\frac{1}{3}$ % of 729 | 29. 10 % of 350 |
| 10. 3 % of 420 | 20. $12\frac{1}{2}$ % of 648 | 30. 30 % of 960 |
| 11. 2 % of 500 | 21. $16\frac{2}{3}$ % of 366 | 31. $62\frac{1}{2}$ % of 856 |

Written Work

1. What is 17 % of 245? 2. $66\frac{2}{3}$ % of 300?

(1) 245, number

.17, rate

41.65, percentage

17 % of a number equals .17 of it.
 .17 times 245 = 41.65.

(2) $\frac{2}{3}$ of $\frac{100}{3} = 200$

$66\frac{2}{3}$ % of a number equals $\frac{2}{3}$ of it.
 $\frac{2}{3}$ of 300 = 200.

A given per cent of any number is found by taking that number of hundredths of the number.

Find :

- | | | |
|------------------|--------------------|------------------------------|
| 3. 18 % of 328 | 6. 38 % of \$77.50 | 9. $14\frac{2}{3}$ % of 105 |
| 4. 19 % of \$126 | 7. 72 % of 328 | 10. 75 % of $\frac{4}{5}$ |
| 5. 21 % of 263 | 8. 80 % of 6.75 | 11. $87\frac{1}{2}$ % of 168 |

Merchants at different times in the year offer special discounts from their regular prices. These sales are frequently called "Anniversary sales," "Bargain sales," "January sales," etc.

Find the selling price from the following sales as advertised :

REGULAR PRICE		PER CENT OF DISCOUNT	SPECIAL SELLING PRICE
12.	\$45 overcoats @	33 $\frac{1}{3}$ % off	—
13.	\$12 boys' suits @	25 % off	—
14.	\$60 bedroom suite @	16 $\frac{2}{3}$ % off	—
15.	\$100 parlor suite @	25 % off	—
16.	\$50 ladies' suits @	40 % off	—
17.	\$25 ladies' suits @	30 % off	—
18.	\$5, \$10, and \$15 ladies' trimmed hats @	50 % off	—
19.	\$1.80 carpet @	33 $\frac{1}{3}$ % off	—
20.	\$.30 matting @	25 % off	—
21.	\$.30 wall paper @	33 $\frac{1}{3}$ % off	—
22.	\$25 men's suits @	20 % off	—

23. James has a paper route which averages him \$16 a month. If he increases his sales 25 % in June, how much are his sales for the month of June?

24. My natural gas meter for March, 1912, showed 10,000 cu. ft. of gas used. The price is 27¢ per thousand cubic feet and a discount of 10 % is given for payment before the 10th of the month. Find the gas bill if paid April 9.

25. Notice is posted in a steel mill January 1, of an increase of 10 % in wages. If the wages in six different departments are \$1.60, \$1.70, \$1.85, \$1.90, \$2.00, and \$2.40 per day before January 1, what are the wages after January 1?

26. In 1916, Mr. Sanial raised 240 bu. of potatoes per acre on five acres. In 1917 he succeeded in increasing the number of bushels per acre on five acres of ground 15 %. Find the number of bushels raised in 1917.

27. A merchant marked a lot of goods costing \$ 12,000, at 25 % above cost. How much did he gain?

28. 40 % of a western farm containing 600 acres is in wheat, 30 % of the remainder in corn, $66\frac{2}{3}$ % of the remainder in oats and grass. How many acres are there in each crop, and how much remains not cultivated?

29. How much does an architect receive, at $4\frac{1}{2}$ %, for the plans of a house that cost \$ 8350?

30. A man's salary is \$ 150 per month. He spends 40 % of it for clothing and other expenses. How much does he save in a year?

31. A man purchases 80 acres of land for \$ 6400, and sells them at 25 % gain. How much does he receive per acre?

Find the gain or loss:

32. Cost \$ 75, gain 20 %. **36.** Cost \$ 356, gain 21 %.

33. Cost \$ 96, gain $33\frac{1}{3}$ %. **37.** Cost \$ 132.50, gain 28 %.

34. Cost \$ 115, loss 15 %. **38.** Cost \$ 485.60, loss 5 %.

35. Cost \$ 227, loss 19 %. **39.** Cost \$ 880.80, gain $12\frac{1}{2}$ %.

40. In a school of 400 pupils, 45 % are girls. How many girls are there? how many boys?

41. A clerk who received \$ 50 a month had his wages increased 15 %. How much were his wages increased?

42. If iron ore yields 63 % of pure metal to the ton, how much iron is there in 40 tons of ore?

43. A merchant, failing in business, paid 85 % of his debts. How much should a creditor receive whose claim is \$ 2850?

44. A bill of goods cost \$137.50. How much was gained by selling the goods at a profit of 12 %?

45. Compare 48 % of \$45 with 45 % of \$48.

46. I owe a debt of \$246.50. If I pay 40 % of it at one time, and 50 % of the remainder at another time, how much do I still owe?

47. An automobile cost \$3500 and the repairs for 2 years were 10 % of the cost. If the automobile was sold at 40 % reduction from the cost, find the entire loss.

48. The operating expenses of a factory are 45 % of the sales. If the sales for a year amount to \$650,450, how much are the operating expenses?

49. 400 men were employed in a factory at daily wages averaging \$1.95. If 100 of these men receive $33\frac{1}{3}$ % of the entire daily wages, find their average daily wages. Find the average daily wages of the other 300 men.

50. Three newsboys, John, James, and Henry, earned together \$550 in a year. John earned 40 %, James 60 % of the remainder, and Henry what remained. Find how much each earned.

Finding what per cent one number is of another number.

Oral Work

1. What part of \$10 is \$5? What per cent of \$10 is \$5?
Think \$5 is $\frac{1}{2}$ of \$10, or 50 % of \$10.

What per cent of :

2. 20 is 10?

7. $1\frac{1}{2}$ yd. is $\frac{1}{2}$ yd.?

3. 35 ft. is 7 ft.?

8. $3\frac{3}{4}$ in. is $1\frac{1}{4}$ in.?

4. 100 is $16\frac{2}{3}$?

9. 25 gal. is $6\frac{1}{4}$ gal.?

5. 500 lb. is 100 lb.?

10. 60 rods is 20 rods?

6. $87\frac{1}{2}$ is $12\frac{1}{2}$?

11. 1 mile is 80 rods?

Written Work

1. What per cent of 75 is 15?

$15 \div 75 = .20$, or 20 % Since 15 is $\frac{1}{5}$ of 75, reduce the fraction $\frac{1}{5}$ to per cent by dividing 15 by 75.

The rate equals the percentage divided by the base.

What per cent of :

- | | | |
|-------------------------------------|--|----------------------------|
| 2. 25 is 10? | 10. 4 is $\frac{3}{5}$? | 18. \$ 3.00 is 18¢? |
| 3. 32 is 12? | 11. 125 is 75? | 19. 32 qt. is 8 qt.? |
| 4. 65 is 39? | 12. 4 bu. is 1 pk.? | 20. 1 gal. is 1 pt.? |
| 5. 96 is 72? | 13. $\frac{7}{8}$ is .125? | 21. 2 bu. is 1 pk.? |
| 6. 25 is $6\frac{1}{4}$? | 14. $2\frac{1}{2}$ is $1\frac{1}{4}$? | 22. 1 yd. is 4 in.? |
| 7. $\frac{8}{9}$ is $\frac{5}{9}$? | 15. 36 qt. is 10 qt.? | 23. 75 min. is 15 min.? |
| 8. 25 is 5? | 16. 20 is 6? | 24. 16 is 4? |
| 9. 100 is $33\frac{1}{3}$? | 17. 9 is 5? | 25. 25 is $8\frac{1}{3}$? |

26. John is 5 ft. 5 in. in height, and his father is 6 ft. 3 in. in height. What per cent of the father's height is John's height?

27. Ben's father by improved farming increased his oat crop from $37\frac{1}{2}$ bu. to 45 bu. per acre. What per cent of the second crop was the first crop?

28. Mrs. Brown kept an itemized account of her receipts and expenditures. Her expenses in August were \$60, and in September, \$10 less than in August. What per cent less were her expenses in September than in August?

29. Mary picked 75 qt. of berries during the season; Henry picked 90 qt., and Ben picked 125 qt. What per cent of the number of quarts Ben picked did each of the others pick?

30. May lives 270 rods from the schoolhouse, and Helen, 180 rods. What per cent of the distance May walks to school does Helen walk?

31. Mr. Bohn was receiving \$3.50 per day and got an increase to \$3.85 per day. Find the per cent of increase in his wages.

$\$3.85 - \$3.50 = \$.35$, increase
in wages per day.

$\$.35 \div \$3.50 = .10$, or 10 %
increase.

The amount of increase is the difference between \$3.85 and \$3.50, or \$.35. The percentage of increase is reckoned on \$3.50 as base. To find what part \$.35 is of \$3.50, divide \$.35 by \$3.50 and express the result, .10, as 10%.

32. A huckster buys lemons at 20¢ a dozen and sells them at 24¢ a dozen. Find his per cent of profit.

20¢, cost per dozen.

24¢, selling price per dozen.

4¢, profit per dozen.

$\frac{4}{20} = \frac{1}{5}$, or 20 %, profit.

The profit is 4¢, the difference between the cost, 20¢, and the selling price, 24¢. 4¢ is $\frac{1}{5}$ of 20¢, the cost. $\frac{1}{5} = \frac{1}{5}$, or 20 %.

33. A huckster purchases 10 bu. of apples and retails them for \$13.20, which is at a loss of \$1.80 on the cost of the apples. What is the huckster's per cent of loss?

\$13.20, selling price of apples.

1.80, loss on apples.

\$15.00, cost of apples

$\$1.80 \div \$15 = .12$, or 12 % loss.

\$13.20, the selling price, + \$1.80, the loss, = \$15, the cost. To find what part \$1.80 is of \$15, divide \$1.80 by \$15 and express the result, .12, as 12 %.

Increase or decrease must be reckoned on the original number or amount taken.

Profit or loss must be reckoned on the original cost.

34. Berries quoted in the market on Monday at 12¢ a basket were sold on Wednesday at 15¢ a basket. Find the per cent of increase on the cost.

Have each pupil give the steps in the solution before doing the actual work.

	BOUGHT FOR	SOLD FOR	GAIN	Loss	PER CENT OF GAIN OR LOSS
35.	\$1.20	\$1.60	\$.40		—
36.	?	2.40		\$.40	—
37.	4.00	6.00	?		—
38.	?	5.00	1.50		—
39.	1.20	1.50	?		—
40.	?	80.00		20.00	—
41.	12.50	15.00	?		—
42.	1.05	.90		?	—
43.	15.00	12.00		?	—
44.	.90	.60		?	—
45.	7.50	10.00	?		—
46.	1.75	2.25	?		—

47. A house rents for \$240 a year. The taxes and insurance are \$30. If the property is valued at \$3500, what per cent does the owner realize on the value of the property?

48. A farmer raised 150 bu. of potatoes from 6 bu. of seed. What per cent of the crop was the seed?

49. If a table is bought for \$4.50 and sold for \$6, what per cent is gained?

50. A farm was bought for \$3500 and sold for \$4200. What was the gain per cent?

51. If hats are bought for \$27 a dozen and retailed at \$2.75 each, what per cent gain is realized?

52. A grain dealer bought 500 bushels of oats at 54 cents a bushel, 500 bushels at 50 cents a bushel, and sold it all at 60 cents a bushel. What per cent did he gain or lose?

REVIEW PROBLEMS

1. Ruth's temperature on Monday rose from 97.6 degrees to 100.4 degrees. Find to three decimal places the per cent of increase in Ruth's temperature.

2. On Tuesday Ruth's temperature fell to 99.7 degrees. Find to three decimal places the per cent of fall in temperature from the highest on Monday.

3. The thermometer on Monday, September 2, registered 85 degrees, and on Tuesday, September 3, 90 degrees. Find the per cent of increase over Monday.

4. The thermometer on Wednesday in the same city showed 70 degrees. Find the per cent of fall in temperature from each of the other days.

5. A man made a fire in his peach orchard one night, which raised the temperature from 32 degrees to 35 degrees. What per cent did the fire raise the temperature in the orchard?

6. A boy's salary was increased from \$45 to \$55 a month. Find the per cent of the increase.

7. Roy raised 80 bu. ears of corn on one acre in 1916. By studying the soil and using better methods of farming, he succeeded in raising 100 bu. ears of corn on the same acre in 1917. Find the per cent of increase over 1916.

8. By paying cash Mrs. Burns gets cheese for 20¢ a pound instead of 22¢ a pound. What per cent does she save by paying cash?

9. Mrs. Adams's milk man left this notice: "Milk 11¢ a quart after Monday, September 1." If the milk now costs 10¢ a quart, find the per cent of increase.

10. Mr. Wilson buys a house and lot for \$6400. The average expenses per year for taxes are \$80; for insurance \$12; and for repairs \$24. What must be the annual rent that he may have an income of 6% net on the original cost of the property?

11. I sold goods for \$8.75. My actual loss was \$1.25. What was the cost and the per cent of loss?

12. A man owns a farm valued at \$9600. His annual taxes are \$86.40. How much must he make each year to clear 8% on the cost of his property?

13. 10 is what per cent of 20? 15 is what per cent of 100?

14. What per cent of 10 days are 30 days? What per cent of 30 days are 10 days?

15. $\frac{5}{8}$ of 72 is how many per cent of 120?

16. I paid \$7200 for a house, \$150 for repairs, and \$50 for taxes. I then sold the house for \$9000. What per cent did I make on my money?

17. What is the difference between $\frac{1}{4}\%$ of \$7000 and 25% of \$7000?

18. If my property sells for \$7433.25 and I owe \$8745, what per cent of my debts can I pay?

19. I bought 50 bu. of apples at \$1 a bushel, but lost 10 bu. by freezing. At what price per bushel did I sell the remainder, if my entire loss was 4% of the cost of the apples?

20. A merchant has invested in his building and store \$6000. After paying all the expenses, he finds that he has cleared for the year \$1200. What per cent does he make on the cost of the building and store?

21. The material for a sidewalk costs \$225 and the labor $33\frac{1}{3}\%$ as much. If the completed work costs \$350, what per cent of the cost of material and labor is the profit?

Find the per cent of reduction in the following:

REGULAR PRICE	SELLING PRICE	PER CENT OF DISCOUNT
22. Ladies' coats, \$27.75	\$18.75	—
23. Ladies' coats, \$16	12.00	—
24. Boys' suits, \$10	7.50	—
25. Boys' suits, \$15	12.50	—
26. Girls' shoes, \$3	2.40	—
27. Boys' caps, \$1	.60	—
28. Bedroom suite, \$60	42.50	—
29. Bedroom suite, \$65	47.50	—
30. Men's overcoats, \$32	24.50	—
31. Men's overcoats, \$21	16.50	—
32. Black lynx fur scarf, \$90	67.50	—
33. Black lynx muff, \$40	32.50	—
34. Men's shirts, \$1.50	1.20	—
35. Men's shirts, \$2	1.40	—
36. Men's shirts, \$2.50	2.00	—
37. Ladies' trimmed hats, \$15	10.00	—
38. Ladies' trimmed hats, \$6	4.85	—
39. Ladies' shoes, \$5	3.75	—

NOTE.—In business it is seldom required to find the whole when a per cent of it is given. This case in percentage is therefore given on pages 338–341, so that it may be omitted without interrupting the sequence of the lessons, but may be taught at the option of the teacher.

CHAPTER II

BUSINESS PRACTICES

RECEIPTS

John Bentz rents a house in Detroit, Mich., from James Smith for one year for \$240, rent payable the first day of each month in advance.

1. Every receipt should state (1) the *place* and the *date* of payment; (2) who *pays* the money; (3) who *receives* the money; (4) *for what* the money is paid; (5) the *amount*, both in figures and in writing.

2. Every receipt in full should state *in full to date*.

Write the receipt given Mr. Bentz for September's rent.

\$	_____

Received from	_____

For	_____

1. John Bentz fails to pay the rent for August when due, but pays on September 1 the rent for both August and September. Write the proper receipt.

2. Write the receipt in full to date for each of the following bills which I owe:

John Thompson for milk, \$6.75; Frank Jones for coal, \$16.85; The Smith Company for books, \$3.75.

RECEIPTED BILLS

JOLIET, ILL., May 1, 1917.				
Mr. James Grant, 24 Broad St.				
Bought of FRANK COLT AND SONS, 92 PARK PLACE, FANCY GROCERS.				
TERMS: <i>Cash</i>				
1917				
Apr.	1	1 bbl. Salt,	\$ 1.50	\$ 1 50
"	20	20 lb. Granulated Sugar,	.06 $\frac{1}{2}$	1 30
"	26	10 bu. Apples,	1.00	10 00
"	27	6 lb. Honey,	.20	1 20
				14 00
Received payment, May 10, 1917 Frank Colt and Sons Per Watson.				

Observe:

The *place* and the *date* of the sale; the names of the *buyer* and the *seller*; the *name*, the *quantity*, and the *price* of each article; the *entire amount* of each separate item; the *total amount* of the bill; the *receipt* of the bill.

1. Ask your grocer for one of his billheads and make out and receipt a bill for a purchase made for your mother.

2. Take blank paper and make out from memory the bill-head of your local grocer.

3. Consider your teacher as the grocer, and make out purchases and receipt the bills.

4. Bring to class as many bills for purchases as you can.

A **bill** is a written statement, in detail, of goods sold or of services rendered.

A bill is **receipted** when the words "Received payment" are written at the bottom, either by the *seller* or by a person authorized by him.

NOTE. — When the person authorized signs the name of the seller, he should always write on the next line below the word "by" or "per" and his own name or initials.

When a person purchases anything on time, the purchaser is called a **debtor**.

When the seller extends the time of payment to any one, the seller is said to give credit, and therefore is called a **creditor**.

Some abbreviations used in business :

Acct. %,	account	mdse.,	merchandise
Amt.,	amount	No. (#),	number
bal.,	balance	paym't,	payment
Co.,	company	pd.,	paid
Cr.,	creditor	per,	by
Dr.,	debtor	pc.,	piece
do. ("),	the same	rec'd,	received

The symbol # means *pounds* when placed *after* a number ; but *number* when placed *before* a number.

Thus 6# means 6 pounds, but #6 means Number 6.

Make receipted bills for the following transactions, performing all necessary operations:

1. Carter Bros., Elkins, W. Va., purchased from The Baldwin Company, Topeka, Kansas, the following: 3 dozen locks @ \$4.80, 67 kegs of nails @ \$4.10, 6 dozen lanterns @ \$6.25, 650 feet steel tracks @ 32¢, and 7 lawn mowers @ \$4.25.

2. Mr. John Dunn, Lima, Ohio, bought from Arnold and Blake, Lima, Ohio, 36 barrels of apples @ \$4.80, 4 boxes of prunes @ \$1.65, 500 pounds of coffee @ 11 $\frac{3}{8}$ ¢, 7 boxes of yeast @ 75¢, 50 pounds of Huyler's cocoa @ 32¢.

3. Mr. James Brown, Tampa, Florida, purchased from Aaron Ward and Sons, Tampa, Florida, 22 bunches of bananas @ \$1.75, 32 boxes of oranges @ \$3.15, 17 boxes of lemons @ \$2.80, 29 crates of cranberries @ \$2.25, 6 boxes of grape fruit @ \$2.90, and 35 bbl. of apples @ \$2.75.

4. James Freeman, Salem, Oregon, bought from The Loft Company, Chicago, Illinois, 1600 pounds of dressed beef @ 12 $\frac{1}{2}$ ¢, 267 pounds of mutton @ 10¢, 933 pounds of pork @ 14¢, and 180 pounds of lard @ 16¢.

5. Lane and Bruce, Mobile, Alabama, bought of the Brown Lumber Company, Flint, Michigan, 28,215 feet lumber at \$32 per M, 147,820 feet at \$27 per M, 92,629 feet at \$60 per M, 63,605 feet at \$35 per M.

NOTE. — \$32 per M equals \$.032 per board foot.

6. Mrs. James Thorpe of Freeport, Illinois, bought of The Snow Company, Chicago, Illinois, 1 pair gloves at \$2.75, 5 yd. ribbon at 39¢ a yard, $\frac{1}{2}$ dozen handkerchiefs at 25¢ each.

7. Gould Brothers, Buffalo, N. Y., shipped to Greyson Brothers, Baltimore, Maryland, 50 barrels apples at \$1.90 a barrel, and 20 barrels at \$1.20 a barrel. Make out the bill to be sent to Greyson Brothers for the fruit.

8. The Avon Fruit Farm, Bangor, Maine, shipped to The Brook Company, Portland, Maine, 6 crates red raspberries at \$3.20 a crate; 10 crates black raspberries at \$2.70 a crate, and 5 crates cherries at \$3 a crate. Make out the bill the Avon Fruit Farm would mail to the purchaser.

ACCOUNTS

In the study of bills we simply found how much the debtor owed to the seller, or what one party owed to another for services rendered.

In an **account** we have a business transaction covering a period of time in which there is both a debtor's *bill* and a debtor's *payments*.

Form of Account

CHICAGO, ILL., May 1, 1918.					
Mr. Samuel Bond,					
Oak Park, Ill.					
To THE CLARKE COMPANY, Dr.					
Dr.					
Apr.	1	To Account rendered	\$ 110 29		
"	10	" 20 yd. silk @ \$1.50	30 00		
"	25	" 2 Ladies' suits @ 45.00	90 00		
"	29	" Mdse.	7 80	238 09	
Cr.					
"	15	By Cash	100 00		
"	24	By Cash	60 00	160 00	
Balance due The Clarke Co.				78 09	

NOTE.—If the above balance were paid in full May 1, the words "Received Payment" would be followed by the signature

THE CLARKE COMPANY,

Per _____

It is customary for the creditor to send an itemized account to the debtor. If it is not paid, another form of bill called a *statement* is sent, which contains only these words: "To account rendered" or "To Mdse." followed by the amount.

Render the following statements :

1. Jan. 31, 1918, the debits and credits of George Weil in account with H. M. Broadhurst, Detroit, Mich., were as follows :

DEBITS

Jan. 1, To account rendered, \$295.63,
Jan. 7, To 3 overcoats @ \$32,
Jan. 12, To 7 yd. dress goods @ \$4.75,
Jan. 31, To 1 suite of furniture, \$185.

CREDITS

Jan. 5, By cash, \$250,
Jan. 19, By check, \$200.

Find the balance due Broadhurst.

2. Nov. 30, 1918, the debits and credits of R. D. McClurg with Stevenson & Company, Memphis, Tenn., were as follows :

DEBITS

Nov. 1, To account rendered, \$86.25,
Nov. 11, To 12 cases corn @ \$1.65,
Nov. 18, To 4 bbl. sugar, 1692 lb., @ 4¼¢,
Nov. 22, To 75 lb. dry beef @ 19¢.

CREDITS

Nov. 6, By cash, \$85,
Nov. 30, By note for balance due.

3. On Oct. 31, the account of Wm. B. Eager with H. A. Soltori, Dayton, Ohio, was as follows :

DEBITS

Oct. 2, To mdse., \$93.37,
Oct. 9, To mdse., \$107.62,
Oct. 21, To mdse., \$21.58.

CREDITS

Oct. 10, By cash, \$80,
Oct. 21, By check, \$100,
Oct. 31, By cash, \$25.

LEDGER ACCOUNTS

The orders or payments received by a firm are first put in a **day book** in the order of their receipt. Each customer's business is then copied in a book called the **ledger**, which is ordinarily *balanced* each month or when an account is paid.

A ledger account is headed by the name of the customer and is so arranged that the *purchases* appear on the *left* side as *debits*, and the *payments* or *services rendered* on the *right* side as *credits*. Dollar signs are usually omitted.

The statement of account on page 77 is simply a copy of Mr. Bond's ledger account with The Clarke Company.

The following form shows a ledger account with James Roberts, balanced at the close of the month, the balance being brought down to continue the account to the next month.

Dr.				James Roberts				Cr.			
1918								1918			
Feb.	1	Bal. bro't f'w'd		19	30	Feb.	6	Cash	6*	30	00
"	2	Lumber	2*	19	80	"	11	Drayage	45	15	75
"	4	Cement	8	40	50	"	18	Drayage	60	8	50
"	5	Sand	15	9	50	"	22	Drayage	70	26	50
"	9	Tile	30	15	70	"	25	Cash	74	82	50
"	13	Plaster	50	56	30	"	28	Balance		38	75
"	18	Sewer pipe	60	20	90						
"	26	Lumber	75	20	00						
				202	00					202	00
Mch.	1	Bal. bro't f'w'd		38	75						

1. What debts in this ledger account did Mr. Roberts incur during the month? What payments did he make?

*These numbers refer to the pages in the *day book* in which the accounts are found.

2. Find the sum of the debits and the sum of the credits. Does the difference equal the balance, \$38.75?

3. Is the balance in favor of or against Mr. Roberts?

4. Had the balance been in favor of Mr. Roberts, on which side would it have been entered?

5. How do you determine on which side to enter the balance? on which side to bring down the balance?

To **foot** a ledger is to add and set down (usually in pencil) the total debits and credits of the accounts.

TEST. — When the footing of one side equals the footing of the other side, the account is balanced.

Written Work

The day book shows the following sales and receipts. Make a ledger for the year, enter each item, foot and close the accounts.

1. William Stone.

DEBITS. — Feb. 1, cook stove, \$22; Feb. 4, nails, \$2.40; Feb. 5, heater, \$75.70; Feb. 12, tin roofing, \$79.08; Feb. 19, hardware, \$14; Feb. 23, lime and cement, \$50.70; Feb. 28, tile, \$22.

CREDITS. — Feb. 1, sand, \$15; Feb. 4, drayage, \$9.50; Feb. 12, cash, \$50; Feb. 16, lumber, \$74.25; Feb. 25, cash, \$20; Feb. 28, cash, \$105.

2. Morris Brown & Co.

DEBITS. — Apr. 4, mdse., \$15.90; Apr. 5, lumber, \$190.72; Apr. 21, mdse., \$23.15; Apr. 12, wagon, \$90; Apr. 27, mdse., \$20.70; surrey, \$129.70; May 7, mdse., \$40.05; May 12, cash \$100; May 20, lumber \$189.

CREDITS. — Apr. 10, labor, \$129.71; Apr. 14, cash, \$75; Apr. 21, labor, \$29.70; Apr. 28, cash, \$147; May 7, labor, \$270.20; May 19, stone work, \$175.39; May 28, cash, \$70.

COMMERCIAL OR TRADE DISCOUNT**Oral Work**

1. I owe a bill of \$50 due in 60 days. As the creditor needs the money, he offers to take \$40 if I pay at once. What per cent is the reduction? On what is the reduction reckoned?

2. A catalogue lists goods at \$1, \$.50, \$.25, subject to a discount of 20 %. Find the net price of each article.

The *fixed* or *list* price of an article, or the *amount* of an obligation, is always considered as the *base*.

Commercial discount is a reduction from the fixed or list price of an article, or from the amount of a bill or obligation. It differs from a "bargain sale" discount, since trade discount is given at all times and not for special sales.

1. **Trade discounts** are regular reductions from the fixed or list price of an article at the time of sale.

2. **Time discounts** are reductions from a bill or other obligation for payment within a certain time.

3. **Cash discounts** are reductions made for the immediate payment of a bill of goods sold on time.

The **net price** is the price after trade discounts have been deducted.

Written Work

1. Neckties listed at \$6 per dozen are sold at 50 % discount. Find the net price. What is the *kind* of discount?

2. An agent buys \$100 worth of goods on 60 days' time, or 5 % off, if paid immediately. What *kind* of discount is he offered?

3. A merchant offers \$100 worth of goods for \$90. What is the *kind* of discount? What is the per cent of discount?

4. I pay cash for a bill of goods sold on 60 days' time, and thereby get 10 % discount, or \$12.50. Find the amount of the purchase. Why is this a cash discount?

5. A suit marked \$40 is offered for \$28. Find the per cent of discount. What is the *kind* of discount?

6. A merchant buys boys' suits at \$60 a dozen list price, less 20%. What is the *kind* of discount?

7. A dealer sells \$75 worth of goods at 5% discount, if paid within 60 days, or 10%, if paid in cash. Explain the different kinds of discount, and tell the amount saved by paying cash.

8. What discount is made without reference to *time*?

Business houses print on their billheads their terms of credit. For example:

(1) "Terms: 60 days net; 3% off 10 da."

(2) "Terms: 90 days net; 60 days 2%; 10 days 5%."

(3) "Terms: 30 days net; cash 5%, etc."

9. Johnston Brothers, Lima, Ohio, purchase of The Utility Company, Chicago, Illinois, \$300 worth of merchandise. Terms: 60 da. net; 5% off 10 da. Find the discount if paid within 10 days.

10. I buy \$600 worth of goods. Terms: 30 da. net; 5% off for cash. Find the amount I save by paying cash.

11. Explain the meaning of a billhead which reads as follows: "Terms: 60 da. net; 10% cash."

12. \$200 worth of goods are bought Aug. 10, 1917. Terms: 60 da. net; 5% off 30 da.; 10% off 10 da. Find the amount saved by payment Sept. 5.

13. Jamison and Son bought \$1500 worth of merchandise from Brown and Company. Terms: 90 days net; 2% off 60 da.; 5% off 30 da.; 10% off cash. What was the amount of the bill if cash was paid?

14. A bill of goods for \$200 is sold at 10% for cash, 5% for 60 days; or 2% for 90 days. Find each discount.

Successive trade discounts.

Wholesale merchants, publishers, and manufacturers usually have fixed price lists for their goods from which the retailer gets a certain **trade discount**. If a reduction from these prices is to be made, an extra discount is taken from the former discount price.

1. A music dealer buys an organ on 60 days' time, list price \$100 at 40 %, 20 % off. Find the net price.

$.40 \times \$100 = \40 , 1st discount	Observe that when more than one discount is given, each successive discount is reckoned on the last discount price.
$\$100 - \$40 = \$60$, 1st discount price	
$.20 \times \$60 = \12 , 2d discount	
$\$60 - \$12 = \$48$, net cost	

2. If the music dealer gets a further discount of 10 % for cash, what is the price of the organ?

3. On what discount price was the last discount reckoned?

NOTE. — The *trade* discount is first deducted; then the *cash* discount is taken from the remainder.

Find the net cost of articles listed at :

4. \$90, discount 30 %, 10 %.
5. \$120, discount 25 %, 20 %.
6. \$240, discount 10 %, $33\frac{1}{3}$ %.
7. \$35, discount 20 %, 5 %.
8. \$12.50, discount 20 %, 20 %.
9. \$348, discount 20 %, 10 %, 5 %.
10. \$100, discount 10 %, 5 %, 2 %.
11. \$425, discount $37\frac{1}{2}$ %, 16 %.
12. \$400, discount 20 %, 10 %, 5 %.

13. What is the net price of a bill of goods for \$5700 after discounts of 25 %, 20 %, and 5 % are allowed?

14. What is the cost of a bill for \$42 with discounts of 20 % and 5 % ?

SUGGESTION. — The total discount expressed in per cent is 20 % + (5 % of 80 %), or 24 %. 24 % of \$42 = ?

15. What is the cost of a bill of farming implements listed at \$480, discounts 50 %, 5 %, and freight \$3.60 ?

16. Goods marked \$50 were bought at 10 % trade discount, 2 % off for cash. If sold at \$55, what was the rate of gain?

17. A dealer bought 50 gross of buttons @ \$1.20, with 25 %, 10 %, and 5 % off. How much did he pay for them ?

18. A grocer is offered by one firm a discount of 10 % on a bill of goods of \$1000, by another firm two successive discounts of 5 % on the same bill. Which is the better offer and how much ?

19. What is the difference between a discount of 15 %, 5 %, on a bill of \$2000 and a discount of 5 %, 15 %, on the same bill ? Show that the same result is obtained from any number of discounts on a bill in whatever order they are taken.

20. Three firms bid on the glass for a building as follows: (1) \$2000, 60 %, 20 % off. (2) \$2100, 70 %, 10 % off. (3) \$2400, 80 %, 10 % off. Which offer is the best and how much ?

21. A jobber buys merchandise listed at \$1500, at 20 %, 15 % off, and sells at 15 %, 10 %, 5 % off. Find his profit.

22. I purchase \$2000 worth of goods, trade discount 20 %, cash 5 %. I pay cash plus \$5.95 freight. What is the entire cost of the goods ?

23. Find a single discount equal to successive discounts of 40 %, 20 %, 10 %.

COMMERCIAL BILLS

NOTE. — A commercial bill differs from an ordinary bill in introducing the element of discount.

1. On June 10, 1918, James Boydson, Atlanta, Georgia, bought of Trude and Banister, 35 dozen bronze locks at \$5.50 a dozen, less 20%, 10%. Terms: 30 days net; 5% off 10 days.

Form of Bill

ATLANTA, GEORGIA, June 10, 1918.					
Mr. James Boydson,					
Atlanta, Georgia.					
Bought of TRUDE AND BANISTER,					
HARDWARE MERCHANTS.					
TERMS: 30 days net; 5% off 10 days.					
	35 doz. Bronze Locks @ \$ 5.50	\$ 192 50			
	Less $\frac{1}{5}, \frac{1}{10}$	53 90			
		138 60			
	Cash, less 5%	6 93	\$ 131 67		
	Received payment, June 15, 1918				
	Trude and Banister.				

In the above problem, since Trude and Banister received payment within 10 days, a cash discount of 5% was deducted from the \$138.60, making the payment \$131.67. Had the bill not been paid for thirty days, the net amount would have been \$138.60.

2. Harmon and Co., Ogden, Utah, ordered from Weston and Sons, Chicago, Illinois, 10 doz. Acme lawn mowers, list \$10 each, less 40%, 20%. Terms: 60 days net; 5% off 10 days. Make out receipted bill if paid in 10 days.

3. Howard Johnston, Boise, Idaho, ordered from the Abbot Company, Topeka, Kansas, 12 buggies, list \$100, less 40%, 20%. Terms: 30 days net; 2% off 10 days. Make out receipted bill if paid in 10 days.

4. Fisher Brothers, Auburn, Maine, ordered from the Alton Company, Barre, Vermont, the following; discount, $33\frac{1}{3}\%$: 1 gas range, \$32.00; 100 ft. hose, \$.15; 24 garden rakes, \$.50. Terms: 30 days net; 2% off 10 days. Make out receipted bill if paid in 30 days.

5. The M. L. Jones Company, Boone, Iowa, bought of the Sampson Company, Dayton, Ohio, July 3, 1917, on account 30 days net; 2% off 10 days: 1500 ft. of galvanized pipe at 30¢ a foot less 30% and 10%; 200 valves at 50¢ each; 200 elbows at 15¢ each, less 10% and 10%. Find the net amount of the bill if paid July 20, 1917.

6. William Greyson and Sons, Akron, Ohio, sold to James Gamble, Detroit, Michigan, April 9, 1917, on account 30 days net; 2% off 10 days: 50 cultivators at \$25 each, less 40% and 10%; 13 mowers at \$75 each, less 30% and 10%. Find the net amount of the bill if paid April 25, 1917.

7. The Owen Company of Butte, Montana, bought from The Mason Company, Denver, Colorado, July 9, 1917, on account 60 days net, 5% off 10 days: 3 pianos at \$450 each, less 40%; 5 pianos at \$575 each, less 30%; and 2 pianos at \$300 each, less 40%. Find the net amount of the bill if paid Sept. 1, 1917.

8. J. B. Anderson, Athens, Georgia, sold to James Smith, Athens, Georgia, July 2, 1918, on account 60 days net; 4% off 10 days: 10 sewing machines at \$60 each, less $33\frac{1}{3}\%$; 20 sewing machines at \$70 each, less 25%; and 3 sewing machines at \$75 each, less 40% and 10%. Find the net amount of the bill if paid within ten days.

COMMISSION AND BROKERAGE

A person who buys or sells goods or transacts business for another is called an **agent, collector, commission merchant, or commission broker**, according to the nature of the business transacted.

A commission merchant *actually* receives the goods which he buys and sells for another. A commission broker simply makes the *contract* between the buyer and the seller for whatever is to be bought or sold, the goods being delivered directly from the seller to the buyer.

The **commission or brokerage** is a certain per cent of the amount of money involved in the transaction.

A commission merchant gets a certain per cent on the amount of his sales; a collector gets a certain per cent on the amount *collected*; a broker gets a certain per cent of the *cost* or the *selling price*.

If a man buys or sells goods, the **net proceeds** is the amount left after commission and all other charges have been paid.

The one who sends the merchandise to be sold is the **principal, the shipper, or the consignor**.

1. Mr. Beard sells 60 bbl. apples, at \$ 2.25 a barrel, through his commission merchant, who charges 10 %. Find the merchant's commission.

2. A real estate agent sold four lots for \$ 250, \$ 325, \$ 395, and \$ 405, respectively. How much was his commission at 5 % ?

3. A commission merchant sold 320 bbl. apples at \$ 3.25 a barrel and 16 bbl. at \$ 4.80 a barrel. Find his commission at 7 %.

4. A lawyer collected an account of \$ 385 for a client, charging 5 %. How much should he remit ?

5. An agent sold a house for me for \$ 4250 at 5 % commission. How much money should he remit to me ?

6. Mary is a clerk in a store at a salary of \$8 a week, and 2% commission. If her average sales are \$300 a week, how much does she earn in a week?

7. An agent rents a house for \$37.50 a month on 5% commission. How much has the owner left at the end of the year, if repairs cost him \$49 a year?

The following problems are taken from various commission merchant's bills.

PRODUCE	AMOUNT	PRICE	RATE OF COMMISSION	COMMISSION
8. Apples	400 crates	\$ 1.80	10%	?
9. Raspberries	600 crates	\$ 3.20	5%	?
10. Butter	3050 lb.	\$.35	5%	?
11. Peaches	550 bu.	\$ 1.50	12½%	?
12. Eggs	2590 doz.	\$.23	10%	?
13. Corn	350 bu.	\$.80	5%	?
14. Cantaloupes	50 boxes	\$ 3.00	12½%	?

15. A Philadelphia commission merchant sold 275 bbl. apples at \$4 a barrel, on a commission of 10%. The freight was \$67.50 and the drayage was \$11.25. Find the commission and the net proceeds of the sale.

$$\begin{array}{r}
 275 \times \$4 = \$1100.00, \text{ amount of sale} \\
 10\% \text{ of } \$1100 = \$110, \text{ commission} \\
 \quad 67.50, \text{ freight} \\
 \quad 11.25, \text{ drayage} \\
 \hline
 188.75 \\
 \hline
 188.75 \\
 \hline
 \$911.25, \text{ net proceeds}
 \end{array}$$

16. A cotton broker sold 200 bales of cotton of 225 pounds each, at 12½¢ a pound, charging a commission of 2½%. Find the net proceeds.

17. My agent bought 180 bbl. apples at \$4.80 a barrel. He paid \$50 freight and \$6 storage. I sent him \$937.28. What was his rate of commission?

18. A fruit grower shipped to his commission merchant 600 bbl. apples, which were sold at \$3.50 a barrel. The agent deducted \$43.90 freight charges, \$27.75 cartage, 12¢ a barrel for cold storage, and 5% commission. Find the amount remitted.

19. James Amidon and Company offered one of their salesmen \$2400 a year, traveling expenses, and 2% on all sales over \$40,000; or 8% on all sales if he paid his own expenses of \$1800. He chose the first offer, and sold \$72,000 worth of goods in the year. Did he gain or lose, and how much, by accepting the first offer?

20. Find the amount of commission, at $2\frac{1}{2}\%$, received by a real estate agent for selling a house and lot for \$3036.

21. I telegraphed my agent at Chicago to buy me 10,000 bu. wheat at \$1.50 or less. He bought at \$1.44 and charged me $\frac{1}{8}\%$ a bushel brokerage. Find the amount of the check I should send him.

22. My broker bought 50,000 lb. cotton at $11\frac{7}{8}\%$ a pound. His commission was $\frac{7}{8}\%$, and freight, storage, and cartage amounted to \$95.80. How much should I remit?

23. A house bought for \$8500 was sold afterwards for 80% of the purchase price. If the agent received 2% on each transaction, find his commission.

24. An agent bought a property for \$36,790 at 2% commission. The owner put \$674.20 in repairs and afterwards sold the property through the same agent for \$43,000, at 2% commission. Find the agent's total commission, and the amount gained by the owner.

25. An attorney invested for his client \$8750 in a mortgage. The owner of the property paid the attorney 2% commission for getting the money and \$35.75 for examining the title and for docket fees. How much did it cost the owner to secure the money?

26. My Chicago broker sells for me 82,560 bu. corn at $83\frac{3}{4}\text{¢}$ a bushel, charging $\frac{1}{8}\text{¢}$ a bushel brokerage. Find the amount remitted to me.

27. An agent for the Diamond Pneumatic Tool Company received 20% on the sales of 13 pneumatic drills averaging \$737.50 each. If his expenses for traveling and freight on machines were \$697.50, find his net profits.

LOCAL OR MUNICIPAL TAXES

Every township, borough, county, or city must have money in order to build and repair public roads, to maintain public schools, boards of health, to pay officers to look after the business of the municipality, etc. In cities, there are also fire departments, police departments, and many other public services that must be paid by public funds.

The money necessary to carry on such public services is raised by a **tax** on people's property.

Property is of two kinds :

FIRST : Real property, such as land with the buildings, timber, mines, etc. on it.

SECOND : Personal property or things that can be moved from place to place, such as cattle, horses, sheep, money, household goods, etc.

Most states levy also a **poll tax** of one dollar or more on each male inhabitant over twenty-one years of age.

Taxes are collected *yearly* by a person called a **tax**

collector in townships and boroughs, and a **treasurer** in counties and cities. School appropriations are raised partly by state appropriations and partly by taxes.

Taxes are levied as so many *mills on the dollar*; thus, 2 mills on \$1.00, 2.3 mills on a dollar, etc.; or, as so many *dollars on a hundred dollars*; thus, \$.02 on \$100.

Steps in levying and collecting taxes.

Assessors elected by the people place a valuation on every person's real and personal property that is subject to taxation.

The amount of tax necessary for the public needs is determined for each year. Upon the total assessed valuation the rate necessary to raise the needed tax is then computed.

School boards generally levy all school taxes.

The **tax book** is placed in the hands of the **tax collector** or **treasurer** to whom the people pay their taxes.

NOTE.—The methods of collecting taxes vary in different places. Pupils should ascertain the methods in their own localities.

Oral Work

1. Find out what kinds of property in your community are taxed for local public needs.

2. Find out who are the assessors in your community and get the assessed valuation of the community property and the rate of taxation. Then find the assessed valuation of your parents' property and see if the amount corresponds with the collector's tax book.

3. From what source are the superintendent, the principal, and the teachers of your school paid?

4. Where does the money come from to pay the mayor, or burgess; the policemen; for the street repairing or paving; for the electric light, etc.

5. Find out from your city or borough or township treasurer the amount of money collected each year for public needs.

6. A man's property is assessed on the tax book at \$2500, and the tax rate is 2 mills. Find the tax including \$1 poll tax.

SOLUTION. — $.002 \times \$2500 = \5 , 2 mill tax rate.

$\$5 + \$1 = \$6$, total tax.

7. Find my taxes on property assessed at \$5000 on which I pay 5 mills on the dollar and \$1 poll tax.

Find the tax on property assessed at :

- | | |
|--|--|
| 8. \$2000, tax levy 3 mills. | 15. \$6000, tax levy 2 mills. |
| 9. \$3000, tax levy 5 mills. | 16. \$1200, tax levy 3 mills. |
| 10. \$5000, tax levy 12 mills. | 17. \$8000, tax levy $3\frac{1}{2}$ mills. |
| 11. \$8000, tax levy 11 mills. | 18. \$6000, tax levy $2\frac{1}{2}$ mills. |
| 12. \$3200, tax levy $2\frac{1}{2}$ mills. | 19. \$3200, tax levy 2 mills. |
| 13. \$5000, tax levy $3\frac{1}{2}$ mills. | 20. \$1000, tax levy 4 mills. |
| 14. \$20000, tax levy $\frac{1}{2}$ mill. | 21. \$7000, tax levy $\frac{1}{2}$ mill. |

Written Work

1. Frank Keller's real property is assessed at \$5200, and his personal property at \$800. The tax rate in his city is 13 mills. Find his tax, including \$1 poll tax.

$\$5200 + \$800 = \$6000$, assessed valuation.

$.013 \times \$6000 = \78 , 13 mill tax rate.

$\$78 + \$1 = \$79$, total tax.

Study of Problem

What is the *first* step in this problem?

What is the *second* step in this problem?

What is the easiest method of solution?

Find the missing terms :

	ESTIMATED VALUATION	ASSESSED VALUATION	RATE	TAXES	POLL TAX
2.	\$ 23000	\$ 19000	.002	()	3 polls, \$ 1 each
3.	147500	80%	.002	()	2 polls, \$ 2 each
4.	120000	\$ 95000	()	\$ 332.50	

5. The assessed valuation of a town is \$ 900,000 and the amount of taxes to be raised is \$ 16,200. What is the rate of taxation and what is Mr. Owen's tax who owns property assessed at \$ 10,000 and personal property assessed at \$ 2500 ?

6. The assessor's book shows the following: Samuel Johnston assessed for 50 acres of land at \$ 50 per acre; 3 cows at \$ 40 each; 2 horses at \$ 100 each; poll tax \$ 1; tax levy $7\frac{1}{2}$ mills. Find the amount of Mr. Johnston's tax.

7. The real estate of a town is valued at \$ 985,470 and the personal property at \$ 195,645. A tax of \$ 7866.69 is to be raised. There are 780 persons, each assessed a poll tax of \$ 1. How much tax will be paid by Mr. Cowperthwait, a non-resident (paying no poll tax), whose property is assessed at \$ 12,460 ?

8. A town whose property is assessed at \$ 1,750,000 needs \$ 5150 for improvements. There are 620 persons who pay a poll tax of \$ 1.25 each. What is the rate of taxation, and what is Mr. Randolph's tax, whose property is valued at \$ 6500 and who pays his own and his son's poll tax ?

9. Mr. Bell's property has an actual valuation of \$ 4800. If he pays 17 mills city tax and $3\frac{1}{2}$ mills county tax on a $\frac{2}{3}$ valuation, in a certain year, find the amount of his tax for that year.



DENOMINATE NUMBERS

We measure the quantity of anything by finding how many times it contains some unit of the same kind, called the **unit of measure**.

Thus, the number of bushels in a load of apples is found by seeing how many times it contains the unit of measure, 1 *bushel*.

A **denominate number** is a concrete number whose *unit* is a *measure* established by custom or law ; as, 5 yards, or 8 bushels. In these numbers 1 yard and 1 bushel are the units of measure.

A **simple denominate number** is a number of one denomination ; as, 3 feet, 5 pecks.

A **compound denominate number** is a number composed of two or more denominations that express one quantity ; as, 8 yards, 2 feet, 3 inches (length):

REDUCTION

Oral Work

Reduction of denominate numbers is changing their form without changing their value. Thus,

$$2 \text{ bu.} = 8 \text{ pk.} = 64 \text{ qt.}$$

$$\text{Or, } 16 \text{ qt.} = 2 \text{ pk.} = .5 \text{ bu.}$$

1. Recite from memory the following tables : Liquid Measures ; Dry Measures ; Measures of Length, Surface, and Volume ; Avoirdupois Weight ; Time Measures ; Stationers' Measures ; Counting.

NOTE. — The tables of denominate numbers are found on pages 365-368

2. From your experience name several common articles sold by weight, capacity, and length.

NOTE. — Measures are often expressed as mixed numbers. Thus, $1\frac{1}{2}$ gal. or 1.75 gal. = $1\frac{1}{2} \times 8$ pt., or 14 pt.

Change the numbers in column (a) to the denominations indicated in column (b):

(a) NUMBER	(b) DENOMINATION	(a) NUMBER	(b) DENOMINATION	(a) NUMBER	(b) DENOMINATION
3. 6 pt.	quarts	16. 4000 lb.	tons	29. 2 sq. yd.	sq. feet
4. 16 gal.	quarts	17. $3\frac{1}{4}$ T.	pounds	30. 2 acres	sq. rods
5. $4\frac{1}{2}$ gal.	quarts	18. 6 yd.	feet	31. 54 cu. ft.	cu. yards
6. 24 qt.	gallons	19. $5\frac{1}{2}$ yd.	feet	32. 360 sec.	minutes
7. 14 qt.	pints	20. 6 ft.	yards	33. $15\frac{1}{2}$ min.	seconds
8. 32 qt.	pecks	21. 24 ft.	yards	34. 180 min.	hours
9. 36 pt.	bushels	22. 36 in.	feet	35. $6\frac{1}{2}$ hr.	minutes
10. $8\frac{1}{2}$ pk.	quarts	23. 24 in.	feet	36. 72 hr.	days
11. $4\frac{1}{2}$ bu.	quarts	24. 33 rd.	feet	37. $3\frac{1}{2}$ da.	hours
12. 32 oz.	pounds	25. 640 rd.	miles	38. 21 da.	weeks
13. $3\frac{1}{2}$ lb.	ounces	26. $3\frac{1}{2}$ mi.	rods	39. $3\frac{1}{2}$ wk.	days
14. 500 lb.	hundred-weight	27. 5.5 ft.	inches	40. $1\frac{1}{2}$ yr.	days
15. 200 cwt.	pounds	28. 288 sq. in.	sq. feet	41. $3\frac{1}{2}$ da.	hours

Written Work

1. Change 32 gal. 3 pt. to pints.

SOLUTION. — $32 \text{ gal.} = 32 \times 8 \text{ pt.} = 256 \text{ pt.}$
 $\quad \quad \quad 3 \text{ pt.} \quad \quad \quad = 3 \text{ pt.}$
 $\quad \quad \quad \hline 32 \text{ gal. 3 pt.} \quad \quad = 259 \text{ pt.}$

2. Change 85 qt. to gallons, etc.

SOLUTION. — 21, no. gal.

4 qt.) 85 qt.

84

1 qt. remainder

Ans. 21 gal. 1 qt.

Since 4 qt. = 1 gal., 85 qt. = 21 gal. with 1 qt. remaining.

Change the numbers in column (a) to the denominations in column (b).

(a) NUMBER	(b) DENOMINATION	(a) NUMBER	(b) DENOMINATION
3. 16 gal. 2 qt.	quarts	41. 40,600 lb.	tons and pounds
4. $16\frac{1}{4}$ gal.	quarts	42. 6000 lb. oats *	bushels
5. 15 gal. 1 qt.	pints	43. 5000 lb. wheat *	bushels
6. 26 pk. 3 qt.	quarts	44. 5600 lb. barley *	bushels
7. 48 bu. 3 qt.	pecks	45. 4000 lb. potatoes *	bushels
8. 26 bu. 1 pt.	quarts	46. 5600 lb. corn in ear *	bushels
9. 13 qt.	pecks	47. 10,000 lb. shelled corn *	bushels
10. 19 qt.	bushels	48. $16\frac{1}{2}$ lb.	ounces
11. 75 qt.	bushels, etc.	49. 18.75 lb.	ounces
12. 35 pk.	bushels, etc.	50. 19.6 cwt.	ounces
13. 39 qt.	bushels, etc.	51. 14.75 cwt.	ounces
14. 57 mi.	feet	52. $84\frac{3}{4}$ lb.	ounces
15. 46 ft.	yards, etc.	53. 1.8 hr.	minutes
16. 240 ft.	rods, etc.	54. $2\frac{1}{2}$ hr.	minutes
17. 760 rd.	miles, etc.	55. $15\frac{1}{2}$ min.	seconds
18. 350 ft.	inches	56. $23\frac{1}{2}$ min.	seconds
19. 40 yd.	inches	57. $14\frac{1}{2}$ wk.	days
20. 57 in.	feet and inches	58. 215 da.	weeks and days
21. $117\frac{1}{4}$ ft.	inches	59. 5.6 yr.	days
22. $56\frac{1}{4}$ yd.	feet	60. 1025 da.	years, etc.
23. 40,198 ft.	rods	61. $340\frac{1}{2}$ min.	seconds
24. $13\frac{1}{2}$ sq. yd.	sq. feet	62. $210\frac{1}{2}$ hr.	minutes
25. 2080 sq. in.	sq. feet	63. 316 min.	hours, etc.
26. $14\frac{1}{2}$ A.	sq. rods	64. 342 hr.	days, etc.
27. $16\frac{1}{2}$ cu. yd.	cu. feet	65. 350 da.	weeks
28. $24\frac{1}{2}$ T.	pounds	66. 720 da.	years, etc.
29. 4.5 cwt.	pounds	67. 1464 da.	leap years
30. 3 cwt. 5 lb.	pounds	68. $27\frac{1}{2}$ doz.	things
31. $5\frac{1}{2}$ L. T.	pounds	69. 1125	dozens
32. 482 oz.	pounds and ounces	70. 15 doz.	gross, etc.
33. 3 L. T. 50 lb.	pounds	71. $25\frac{1}{2}$ gross	dozens
34. 8.5 cwt.	pounds	72. 240	score
35. $\frac{7}{8}$ T.	pounds	73. 55 score	things
36. 7.1 T.	pounds	74. 42 quires	reams, etc.
37. 4300 lb.	L. T., etc.	75. $40\frac{1}{2}$ reams	quires
38. 5400 lb.	L. T., etc.	76. 460 sheets	quires, etc.
39. 750 cwt.	L. T., etc.	77. $56\frac{1}{2}$ quires	sheets
40. 75,000 lb.	T., etc.	78. $58\frac{1}{2}$ gross	dozens
		* See page 367	

OPERATIONS WITH DENOMINATE NUMBERS

NOTE.—Operations in compound denominate numbers are usually performed by reducing all the numbers to simple numbers of like denomination and then adding, subtracting, etc.

Written Work

1. Find the sum of 1 hr. 15 min. and 3 hr. 45 min.

$$\begin{aligned}\text{SOLUTION. — } 1 \text{ hr. } 15 \text{ min.} &= 1\frac{1}{4} \text{ hr.} \\ 3 \text{ hr. } 45 \text{ min.} &= \frac{3\frac{1}{2} \text{ hr.}}{5 \text{ hr.}}\end{aligned}$$

2. John sold a fish that weighed 12 lb. 8 oz. at 16¢ a pound. How much did he receive for it?

$$\begin{aligned}\text{SOLUTION. — } 12 \text{ lb. } 8 \text{ oz.} &= 12\frac{1}{2} \text{ lb.} \\ 12\frac{1}{2} \times 16\text{¢} &= \$2.\end{aligned}$$

3. A farmer sold 4 lb. 10 oz. of butter for \$1.48. What was the price per pound?

$$\begin{aligned}\text{SOLUTION. — } \$1.48 \div 74 &= \$.02, \text{ price per ounce.} \\ 16 \times \$.02 &= \$.32, \text{ price per pound.}\end{aligned}$$

Find the missing numbers :

WEIGHT	PRICE PER POUND	AMOUNT	WEIGHT	PRICE PER POUND	AMOUNT
4. 2 lb. 6 oz.	8¢	?	12. ?	32¢	48¢
5. 3 lb. 12 oz.	12¢	?	13. ?	16¢	49¢
6. 4 lb. 14 oz.	16¢	?	14. 3 lb. 8 oz.	?	68¢
7. 3 lb. 9 oz.	32¢	?	15. 16 lb. 10 oz.	16¢	?
8. 8½ lb. 6 oz.	20¢	?	16. 4 lb. 12 oz.	?	96¢
9. 5 lb. 10 oz.	30¢	?	17. 5 lb. 8 oz.	?	44¢
10. 7 lb. 12 oz.	18¢	?	18. 10 lb. 10 oz.	10¢	?
11. 6 lb. 13 oz.	24¢	?	19. ?	24¢	78¢

The most important application of subtraction of denominate numbers is in finding the *difference* between two dates.

20. How long a time elapsed from April 19, 1775 to April 14, 1861?

yr.	mo.	da.	The later date is written in the minuend as the 14th day of the 4th month of 1861; and the earlier date in the subtrahend as the 19th day of the 4th month of 1775. In borrowing, consider 30 days to a month and 12 months to a year. The difference will be the time in years, months, and days.		
1861	4	14			
1775	4	19			
<hr/>					
85	11	25			

21. Washington was born February 22, 1732; he was inaugurated president April 30, 1789. How old was he when he became president?

22. Find the time from the signing of the Declaration of Independence, July 4, 1776 to the beginning of the Civil War, April 14, 1861.

23. General Ulysses S. Grant was born April 27, 1822. How old was he when the Civil War closed April 9, 1865?

24. Abraham Lincoln was born Feb. 12, 1809, and died April 15, 1865. How old was he at the time of his death?

25. The treaty of peace between Russia and Japan was signed at Portsmouth, N. H., September 5, 1905. How much time elapsed from that date to Jan. 1, 1918?

26. Ex-president Grover Cleveland was born at Caldwell, N.J., March 18, 1837, and died at Princeton, N.J., June 24, 1908. How old was he at the time of his death?

27. Washington Irving was born April 3, 1783, and died Nov. 28, 1859. How old was he when he died?

28. Charles Darwin was born Feb. 12, 1809, and died April 19, 1882. How old was he when he died?

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20. John takes 2 ft. and 2 in. at a step. How many steps does he take in going to school, if he lives 286 rd. from the schoolhouse?

21. William is 10 years old and steps on an average 1 ft. 4 in. How many steps does he take in going to school, if he lives 190 rd. 2 yd. from the schoolhouse?

22. Mr. Jones puts a wire fence around a field 40 rd. 10 ft. long, and 130 rd. 12 ft. wide. How many feet of wire are there?

23. A teamster uses a wagon, the hind wheels of which are 4 ft. 6 in. in diameter and the front wheels 3 ft. 8 in. in diameter. The circumference is 3.1416 times the diameter. Find the circumference of each wheel, and the difference in the length of the tires.

NOTE. — The **circumference** of a circle is its bounding line. A **diameter** is a straight line passing through the center with both ends terminating in the circumference.

24. Fred's bicycle goes 7 ft. 4 in. in making one turn of the wheel. How many turns of the wheel does it make in traveling $1\frac{1}{2}$ miles?

25. Mr. James builds a sign for city advertising 40 ft. long and 6 ft. high. How many square feet of boards are required to build the sign?

26. Fred and Leonard pick 3 bu. 3 pk. 6 qt. of chestnuts, and sell them at 5¢ a pint. How much do they realize from the sale of the chestnuts?

27. A grocer buys a barrel containing 3 bu. of apples at \$2.75 a bushel. They cost 25¢ for delivery. If 3 pk. become decayed and unsalable, how much does he gain on the barrel, when he retails the good apples at \$1.05 per peck?

28. Mrs. Johnston pays 45¢ a hundred pounds for ice, but by paying in advance she gets 1500 lb. for \$5. How much does she gain by paying in advance in a summer season when she uses in all $1\frac{1}{2}$ T. of ice?

29. Mr. Jones orders his coal bin filled for the winter, and receives a bill for 6 T. 9 cwt. Find the cost of the coal at \$6.25 per ton.

30. How many boxes, each holding 1 qt., can be filled from 3 bu. 1 pk. 7 qt. of blackberries?

31. A rural mail carrier's route is 21 mi. 176 rd. 4 yd. in length. Find the number of feet he travels in one delivery of mail.

32. A fruit grower sold in one season 23 bu. crates of cherries at 10 cents per basket; and 45 bu. crates and 17 baskets of strawberries at 13 cents per basket. Find the amount of the sales. (A crate contains 32 baskets.)

33. A milk dealer put his milk in pint bottles. Find the number of bottles delivered in one evening if he sold 23 gal. 3 qt. 1 pt.

34. How many 4-ounce packages of soda can be put up from 1 T. 3 cwt. 75 lb. of soda?

35. A huckster bought 3 bbl. apples, each containing $2\frac{1}{4}$ bu. for \$8.25, and retailed them at 15¢ a half peck. Find his profits.

36. Find the length of a double-track railroad laid with 1640 rails, each 30 ft. in length.

37. An ocean steamer in making a certain trip consumed 1920 T. of coal. If the time was 6 da. 5 hr. and 8 min., find the average number of pounds consumed per minute.

38. The report of a cannon was heard 1 min. 5 sec. after it was discharged. If sound travels 1120 ft. per second, how many miles, rods, etc., was the hearer from the cannon?

39. A pupil pays \$45 tuition in a term of 9 mo. of 20 da. each, and is absent from school 16 days. Find the amount of tuition lost to him by his absence.

40. A gardener sold on an average 6 bushel crates and 17 baskets of blackberries each day for 8 days. Find the total number of bushels and baskets sold.

41. 57 students at a ball game each wore a badge $5\frac{1}{2}$ in. long. How many yards and inches of ribbon were needed to make the badges?

42. A wire fence inclosing a square field 40 rd. 5 yd. on a side has 4 wires. Find the total length of wire in feet.

43. A 45 horse-power automobile used on an average 3 gal. 3 qt. 1 pt. of gasoline daily on a certain trip of 11 days. Find the number of gallons, etc., used.

44. A confectioner bought 12 bu. of chestnuts at \$2.50 per bushel and retailed them at 5¢ per pint. Find his gain.

45. A ball room is $46\frac{1}{2}$ ft. long and $30\frac{3}{4}$ ft. wide. Find the cost of a picture molding around it at 9¢ per foot.

46. A horse is fed $1\frac{1}{2}$ pk. of oats per day. How much will the oats for the horse cost for 60 days at 50¢ a bushel?

47. Find $\frac{7}{8}$ of 275 ft. 4 in.; $\frac{3}{4}$ of 11 lb. 8 oz.; $\frac{3}{4}$ of 36 bu. 3 pk. 4 qt.

48. A dairyman's sales for each of 4 weeks were as follows: 1st week, 115 gal. 3 qt. 1 pt.; 2d week, 105 gal. 3 qt. 1 pt.; 3d week, 113 gal. 1 qt.; 4th week, 103 gal. 1 qt. 1 pt. If $\frac{1}{8}$ of the total sales were uncollectible, find the cash amount of his sales at $7\frac{1}{2}$ ¢ per quart.

49. A city weighmaster one morning recorded the weight of 4 loads of hay as follows: 1 T. 5 cwt. 19 lb.; 1 T. 7 cwt. 29 lb.; 1 T. 98 lb.; 1 T. 9 cwt. 3 lb. Find the total value of the hay at \$14 $\frac{1}{2}$ per ton.

UNITED STATES POSTAL RATES

The postage on all **sealed matter** or matter containing written information is 3 ¢ an ounce or fraction thereof.

NOTE. — On drop letters (letters mailed for delivery from the post office from which they are posted) the postage is 2 ¢ an ounce. None of the letters in the following examples are drop letters. The postage on **post cards** is 2 ¢.

The postage on **newspapers** and **periodicals**, when sent by the publishers or news agents, is 1 ¢ for each pound or fraction thereof. When sent by any one else the rate is 1 ¢ for each four ounces or fraction thereof.

The postage on **miscellaneous printed matter** weighing four pounds or less, including photographs, maps, proof sheets, etc., but excluding books, newspapers, and periodicals, is 1 ¢ for each two ounces or fraction thereof.

1. Mary mails five letters weighing 1 oz. each and a sealed package weighing 1 lb. 5 oz. How much postage is required ?

2. John mails newspapers weighing $2\frac{1}{2}$ lb. ; miscellaneous printed matter weighing 1 lb. 7 oz. ; and 1 sealed package weighing 19 oz. Find the amount of postage necessary.

3. Mary receives maps through the mail on which the postage is 14 ¢. What is the weight of the maps ?

4. Registration costs 10 ¢. How much does it cost James to mail and register newspapers weighing 2 lb. 9 oz. ?

5. A newsdealer mails a package of periodicals weighing 3 lb. 11 oz. Find the cost.

6. Susan sends her picture weighing 10 oz. to Los Angeles, unsealed. Find the postage, including registration.

7. Martha mails 15 letters weighing $\frac{3}{4}$ oz. each, twelve post cards, a newspaper weighing 1 lb. 1 oz., and a sealed package weighing 12 oz. Find the cost of the postage.

8. A news agent sent to Seattle, Wash., newspapers weighing $3\frac{1}{2}$ lb. What was the cost of the postage ?

UNITED STATES PARCEL POST RATES

Fourth-class mail matter includes all matter not now embraced by law in the first three classes, which is not greater in size than 84 in. in length and girth combined, nor in form or kind likely to cause injury or damage, and not of a character perishable within a reasonable period.

The limit of weight for local delivery and for delivery within the *first three zones* is 70 pounds; in *other zones* it is 50 pounds.

Rates of postage on domestic parcel post matter are as follows:

(a) Parcels weighing 4 oz. or less, except books, seeds, plants, etc., 1 ¢ for each ounce or fraction thereof, for any distance.

(b) Parcels weighing 8 oz. or less containing books, seeds, plants, etc., 1 ¢ for each 2 oz. or fraction thereof, for any distance.

(c) Parcels weighing more than 8 oz. containing books, seeds, etc., parcels of miscellaneous printed matter weighing more than 4 lb., and all other parcels of fourth-class matter weighing more than 4 oz. are chargeable according to distance or zone, at the pound rates shown in the table below, a fraction of a pound being considered a full pound.

In addition, parcels on which the postage amounts to 25 ¢ or more are subject to a tax of 1 ¢ for each 25 ¢ or fractional part thereof. This tax must be paid by *special internal revenue stamps* affixed to the parcels.

TABLE OF RATES

Weights	1st zone		2d zone, 50 to 150 miles	3d zone, 150 to 300 miles	4th zone, 300 to 600 miles	5th zone, 600 to 1000 miles	6th zone, 1000 to 1400 miles	7th zone, 1400 to 1800 miles	8th zone, all over 1800 miles
	Local Rate	Zone rate, 50 miles							
1 pound	\$0.05	\$0.05	\$0.05	\$0.06	\$0.07	\$0.08	\$0.09	\$0.11	\$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 pounds	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 pounds	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 pounds	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 pounds	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 pounds	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 pounds	.10	.15	.15	.26	.47	.68	.89	1.11	1.32

The sender of a C. O. D. parcel on which the postage is prepaid may have the price (up to \$100) collected of the addressee on payment of a fee of 10 ¢ in stamps for amounts up to \$50, or 25 ¢ for greater amounts up to \$100. This fee includes *insurance*.

NOTE. — For packages weighing more than 11 lb. use the following table :

Local rate. — 5¢ for the first pound and 1¢ for each additional 2 pounds.

Zone	1st pound	Each extra pound	Zone	1st pound	Each extra pound
1st	\$.05	\$.01	5th	\$.08	\$.06
2d	.05	.01	6th	.09	.08
3d	.06	.02	7th	.11	.10
4th	.07	.04	8th	.12	.12



Find the cost of mailing each parcel :

WEIGHT	MILES	WEIGHT	MILES
1. 4 oz.	500	8. 5 oz.	950
2. 1 lb. 5 oz.	140	9. 1 lb. 1 oz.	275
3. 8 lb. 2 oz.	45	10. 4½ lb.	575
4. 7¾ lb.	750	11. 8 oz.	25
5. 9 oz.	200	12. 8 lb. 5 oz.	175
6. 1 lb. 11 oz.	750	13. 10 lb.	450
7. 11 lb.	900	14. 11 lb.	990

NOTE. — There is no extra tax on packages subject to less than 25¢ postage; for packages subject to 25¢ postage, the tax is 1¢; for 26¢ to 50¢, the tax is 2¢; for 51¢ to 75¢, it is 3¢; and so on.

15. Mrs. May receives an 8 lb. turkey from her brother 75 miles distant. Find the cost of mailing it by parcel post.

16. A store sends out by parcel post a 6¾ lb. package to Mrs. Johnson 25 miles distant. Find the cost of mailing it.

17. Find the cost of mailing by parcel post for a distance of 950 miles a package of garden seed weighing 3 lb. 10 oz.

18. A girl in Lancaster, Pa. receives by parcel post a package from San Francisco (8th zone) weighing 9 lb. Find the cost of mailing it.

19. Find the cost of mailing a 10-pound package of chestnuts sent 75 miles by parcel post.

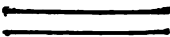
20. Find the cost of mailing an umbrella weighing 15 oz. sent 70 miles.



SCHOOL SHOP WORK


Oral and Written Work

NOTE. — Each pupil should be supplied with a ruler showing half inches, quarter inches, eighth inches, and sixteenth inches, a straight-edge, a pair of compasses, and a protractor.

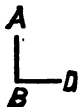
1. Observe the two lines. How do they compare in direction? 

Parallel lines are lines that cannot meet, however far they may be extended.

An **angle** is the difference in direction of two lines that meet.

When two lines meet each other, making a square corner, they form a **right angle**; thus, 

Lines drawn at right angles to each other are **perpendicular**; Thus, AB and BD are perpendicular to each other.



2. Draw a line on the blackboard one yard in length. Then draw five lines parallel to the first line, each line two inches below the former line. Then draw five lines perpendicular to these lines, 2 inches apart.

3. Divide a line, 18 inches in length, into half inches; into quarter inches; into eighth inches.

4. Take a board three feet in length and, beginning four inches from each end, divide the space into four equal parts.

5. The end of a house is 28 ft. Show the spaces for frames each $3\frac{1}{2}$ ft. wide if the windows are to be four feet from each end.

6. Draw a diagram showing spaces for hat hooks on a board 12 ft. in length if the hooks are set 6 in. apart.

7. The front of a house is 40 ft. in length. It has one door $3\frac{1}{2}$ ft. in width and two windows each 60 in. in width. Draw a diagram showing the door in the center and the windows in the centers of the spaces on each side of the door.

8. A hallway is 8 ft. in width and 16 ft. in length. Make a diagram showing the hallway on a scale of 1 inch to the foot.

9. The end of a room is 14 ft. 6 in. Draw a 4-foot window in the center of the side.

10. A bedroom 12 ft. by 16 ft. has windows on one side and one end. Draw a diagram showing a 4-foot window, 5 ft. from the end of the side and a $3\frac{1}{2}$ -foot window in the center of the end.

11. On one side of a chicken house 24 ft. in length James puts four small windows each 18 in. in width. Draw a diagram showing the windows placed equal distances apart.

12. The library is 15 ft. in width. Draw a diagram showing the mantle 5 ft. wide and the fireplace 3 ft. wide placed in the center.

13. A floor is 10' by 24'. How many tiles 6" square will it take to lay it? Draw a diagram 1 inch to the foot to illustrate this.

14. Draw on a scale of $\frac{1}{8}$ " to the foot lines to represent 16 ft.; 24 ft.; 60 ft.; 100 ft.; 120 ft.

CUTTING FROM PATTERNS

NOTE.—In the following exercises, no allowance is to be made for waste.

1. A decorator uses a burlap pattern $12'' \times 18''$. Make a diagram on a suitable scale to show how many such pieces can be cut without waste from a piece of burlap 2 yd. wide and 6 yd. long.

2. Mrs. Long needs 15 pieces of cheese cloth, each $12'' \times 14''$. Make a diagram to show how many pieces can be cut from a piece of cloth 1 yd. in width by 3 yd. in length. How much will be wasted?

3. A calendar firm makes blotters for calendars $3\frac{3}{4}'' \times 9''$. The blotter paper from which they are cut is 1 yard square. How many blotters can be cut from each piece?

4. The industrial school makes school bags from patterns $12'' \times 18''$. Show the number of patterns that can be cut from a piece of cloth 1 yd. wide and 10 yd. long.

5. How many book covers $7'' \times 24''$ can be cut from paper $36'' \times 48''$?

6. Colored rectangles $3\frac{1}{2}'' \times 6\frac{1}{4}''$ are to be cut from cardboard for decorations. How many rectangles can be cut from a piece 1 yard square?

7. The side design on a book bag is $4\frac{1}{4}'' \times 7''$. How many such designs can be cut from a piece of cloth 1 yd. square?

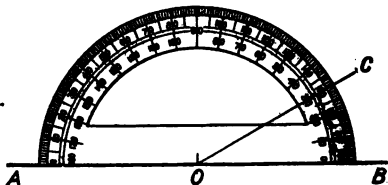
8. A burlap design is $16'' \times 60''$. How many such designs can be cut from a piece of burlap $2\frac{1}{2}$ yd. wide by 3 yd. long?

9. Mrs. Grey makes burlap plate holders to sell at a charity bazaar. How many $8'' \times 12''$ holders can she cut from a yard square of burlap? How many $7\frac{1}{2}'' \times 12''$ holders?

ANGLES

Angles are measured by an instrument called a **protractor**.

When the center O of the protractor is placed at the vertex of the angle to be measured, the size of the angle may be seen on the scale between the lines that form the angle. Thus, BOC is an angle of 30° , and AOO is an angle of 150° .



Every circumference contains 360 degrees (360°); each degree, 60 minutes ($60'$); and each minute, 60 seconds ($60''$).

Table of Angular Measure

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^\circ$)
360 degrees	= 1 circumference (C)

The length of a degree at the equator is $69\frac{1}{4}$ miles.

Draw an angle of 90° ; 45° ; 60° ; 120° ; 30° .

Kinds of Angles



Which one of these angles is a right angle? Why? Which is less than a right angle? Which is greater than a right angle?

A **right angle** is an angle of 90° .

An **acute angle** is an angle less than 90° .

An **obtuse angle** is an angle greater than 90° .

MAKING MORTISE JOINTS

1. On a piece of wood 4 in. wide and 15 in. long, make a mortise 1 in. wide and 2 in. long, equal distances from the ends and sides.

2. The four legs of a stand are 15 in. long and 2 in. square. A brace 1 in. by 3 in. is mortised into the legs 4 in. from the foot of the legs, and 1 in. in depth. Draw a diagram to show the mortise joint if the mortise is $\frac{1}{2}$ in. from each edge.

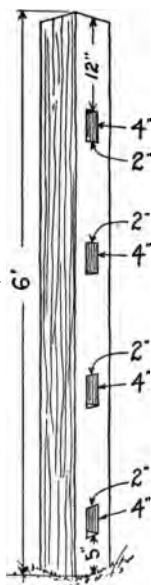
3. A piece of wood is 4 in. square and 6 ft. long. Draw a diagram to show 4 mortise joints each 1 inch by 8 inches if they are equal distances apart, and the top and bottom joint are 1 ft. from the ends.

4. Show by a diagram the location of mortise joints 1 in. by 2 in. at equal distances from the sides and ends of a piece of wood 4 in. wide and 12 in. long.

5. The legs of a plant stand are 2 in. square and 18 in. high. Show a mortise of 1 in. by 2 in. in the center of one side of the leg. Make a diagram.

6. Space on a post 6 in. square and 8 ft. high, four mortise joints 2 in. by 4 in., beginning 1 ft. from the top and bottom. Space the mortises on paper.

7. Read this diagram of a fence post with four holes mortised for rails. How far is the first hole from the ground? the last hole from the top of the post? How high is the post? Give the dimensions of each mortise hole.



MARKET GARDENING



NOTE. — The planting, in all cases, is begun 2 ft. from the edge.

1. This plot is 300' by 100'. James planted the entire length of the plot in 10 rows of sweet corn each 2 ft. apart. The grains were planted 8 in. apart. Estimating an average of one ear to a grain, about how many ears of sweet corn did he raise?

2. James sold the sweet corn at 15¢ a dozen ears. How much did he receive for 1500 ears?

3. James planted 2 rows of cabbage, each 2 ft. apart, the length of the plot. He set the cabbage plants 18 in. apart. How many dozen plants did he plant?

4. He sold the cabbage at an average of 8¢ a head. How much did he receive for 60 heads?

5. James planted 2 rows, $2\frac{1}{2}$ ft. apart, in early potatoes. He raised 3 bu. and 3 pk. of potatoes, which he retailed at 45¢ a peck. How much did he get for the potatoes?

6. James planted 3 rows, 14 in. apart, in mammoth onion seed. He raised $2\frac{1}{2}$ bu. onions, which he retailed at 10¢ a quart. How much did he receive for the onions?

7. He planted 1 row, the length of the lot, in celery. How much did he get for 100 bunches at 8¢ a bunch?

8. He planted 1 row, the length of the lot, in peas, and 1 row in lima beans. He sold 104 qt. peas at 14¢ a quart and 2 bu. 18 qt. of lima beans at 15¢ a quart. How much did he receive for them?

9. He planted 1 row in beets which yielded 25 bunches. How much did he receive for them at 6¢ a bunch?

10. He cut the corn after taking off the ears, and sowed the ground in turnips. He retailed 16 half pecks of turnips at 80¢ a bushel. How much did he receive for them?

11. He planted 2 doz. tomato plants, which yielded 3 bu. tomatoes. These he sold at an average of 20¢ a quart. For how much did he sell the tomatoes?

12. James paid for onion seed 25¢; for cabbage plants 50¢; for potatoes \$1.40; for tomato plants 40¢; for peas, beans, celery, turnips, and beets 10¢ each; for sweet corn 15¢; for fertilizer \$2.50; for sprays to kill cabbage worms and potato bugs 25¢; for plowing and harrowing \$1; for hose and hand cultivator \$3.75. What was the entire outlay?

13. James estimated his labor in caring for the garden and marketing as 20 days of 8 hours each, at 75¢ a day. Make out James's account January 1, 1918, showing the amount paid and what it was paid for, the amount realized and for what it was received. Find the balance.



HOUSEHOLD PROBLEMS

1. It takes $2\frac{1}{2}$ yd. of material for a girl's waist. How many waists can be cut from a piece of lawn measuring $17\frac{1}{2}$ yd.? If the lawn costs 15¢ a yard, what is the cost per waist?

2. The material for a ruffle is 12 in. deep. It is to have a 1-in. hem, with $\frac{1}{8}$ in. for first turn over, and 5 quarter-inch tucks. How wide should the ruffle be when finished?

3. Edith wished to cover two cushions, one 27 in. and the other 21 in. long. The covers required twice the length of the cushions, plus 1 in. for seams, and the narrow pieces for the edge of the cushion covers. The material which Edith bought was wide enough to allow for these small strips. The price per yard was 50¢. How much did the material cost?

4. Mrs. Drake found that she saved 60% on her fuel bills by combining different processes of cooking and laundry work at a given time. At this rate how much was saved for each \$8 spent for fuel?

5. If her fuel bill before the saving was \$130 a year, how much did she save per year by her economy?

6. Mrs. Thorne purchased 2 tablecloths at a sale. The regular price for the cloths was \$10 each. The sale price was 50% off the regular price. The tablecloths were in daily service for 18 months and were then unfit for use. How much did the tablecloths cost per month?



7. Mrs. Noble bought 2 tablecloths at the same sale. She selected a firm, good half-bleached linen. The regular price for these cloths was \$4, and the discount was 25%. These cloths were in constant use for 36 months. How much did they cost per month?

8. Mrs. May required 4 yards of serge for her daughter's dress. The regular cost of the material was \$1 per yard. She found three remnants containing the required number of yards. The remnants were of such lengths that each one could be used to advantage, one for sleeves, one for waist, and the third for the skirt of the dress. Mrs. M. got the three remnants for \$2.75. What per cent did she save on the regular price basis?

9. Mrs. Grant, who needed 4 yd. of serge for a dress, bought 4 remnants amounting to $4\frac{1}{4}$ yd. for \$3.25, but finding that the remnants would not give the skirt lengths for the pattern, was obliged to buy 1 additional yard of serge at \$1. How much more did she pay for her material than if she had bought the 4 yards at \$1 per yard?

10. If soap is left unwrapped in a dry place, it loses at least 25% of the moisture which it contains, and the same amount of soap becomes 25% more efficient. On this basis, figure the saving on soap costing \$7.50 which had been thoroughly dried before using.

11. How much material is taken up by a $\frac{1}{2}$ -inch hem with
 "st turn over and six $\frac{1}{8}$ -inch tucks?

to be finished with a 1-inch hem. Allowing
 "st turn over and $\frac{1}{4}$ inch for the gathering,
 the ruffle be cut to finish 7 inches?

f lawn 27 in. wide was tucked. When fin-
 i in. wide. How many $\frac{1}{8}$ -inch tucks were

14. If it requires one and a half times the width of a skirt for a full ruffle, what will be the length of a ruffle for a skirt measuring $2\frac{1}{2}$ yards around, allowing 4 in. for seams?

Table of Equivalents

3 teaspoonfuls (tsp.) equal 1 tablespoonful (tbsp.).

4 tablespoonfuls equal $\frac{1}{2}$ cup (c.), or $\frac{1}{4}$ gill.

2 gills = 1 cup.

2 cups solid meat = 1 pound.

2 cups = 1 pint.

2 tablespoonfuls sugar = 1 ounce.

4 cups flour = 1 pound.

2 tablespoonfuls flour = 1 ounce.

2 cups solid butter = 1 pound.

2 tablespoonfuls liquid = 1 ounce.

3 cups meal = 1 pound.

2 cups granulated sugar = 1 pound.

15. A class made up of 20 girls used the following supplies. Compute the total amount of each commodity used, at the prevailing market prices.

16. Each girl used 3 teaspoonfuls of sugar. How much was used by the class? What was the cost?

17. Each girl used 4 tablespoonfuls of butter. How much was used by the class? What was the cost?

18. Each girl used 1¢ worth of milk. How much was used by the class? What was the cost?

19. Each girl used $1\frac{1}{2}$ ¢ worth of flour. How much was used by the class? What was the cost?

20. Each girl used $\frac{1}{3}$ ¢ worth of meal. How much was used by the class? What was the cost?

21. Each girl used $\frac{1}{4}$ ¢ worth of solid meat. How much was used by the class? What was the cost?

22. Each girl used 2 tablespoonfuls of vinegar. How much was used by the class? What was the cost?

23. Allowing 43¢ for other materials used by this class, calculate the sum total for the lesson; the cost per person.

SIMPLE INTEREST

TO THE TEACHER. — Before beginning this work, review carefully the preceding work. Throughout the grade, continue drills to increase skill in interpretation of problems, in estimating results, and in choice of method in solution and calculation.

Interest is money paid for the use of money.

The **principal** is the sum of money on which the interest is paid.

The **rate of interest** is a certain number of hundredths of the principal paid for the use of the principal for 1 year.

The **amount** is the sum of the principal and the interest.

Simple interest is generally computed on the basis of a year of 12 months of 30 days each, or 360 days.

In finding interest it is always necessary to know the *time*. The interest for 2 yr. equals 2 times the interest for 1 yr. Since 6 mo. = $\frac{6}{12}$ yr. or $\frac{1}{2}$ yr., the interest for 6 mo. equals $\frac{1}{2}$ the interest for 1 yr. Since 5 mo. = $\frac{5}{12}$ yr., the interest for 5 mo. equals $\frac{5}{12}$ the interest for 1 yr.

Oral Work

1. Mr. Adams borrows \$200 from Mr. Crane for 2 yr. at 6%. What is the principal? the rate of interest? the interest for 1 yr.? the interest for 2 yr.? the amount at the end of 2 yr.?

2. What part of a year is 1 mo.? 2 mo.? 3 mo.? 4 mo.? 5 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 10 mo.? 11 mo.? 12 mo.?

How much is the interest at 6% on :

- | | |
|--|--|
| 3. \$100 for $2\frac{1}{2}$ yr.? | 9. \$150 for 3 mo. ($\frac{1}{4}$ yr.)? |
| 4. \$200 for $3\frac{1}{2}$ yr.? | 10. \$80 for $2\frac{1}{2}$ yr.? |
| 5. \$300 for $1\frac{1}{2}$ yr.? | 11. \$200 for $\frac{3}{4}$ yr.? |
| 6. \$250 for 6 mo. ($\frac{1}{2}$ yr.)? | 12. \$100 for $2\frac{1}{4}$ yr.? |
| 7. \$400 for $1\frac{3}{4}$ yr.? | 13. \$900 for $1\frac{3}{4}$ yr.? |
| 8. \$800 for $2\frac{3}{4}$ yr.? | 14. \$750 for $1\frac{1}{2}$ yr.? |

INTEREST FOR YEARS AND MONTHS

Written Work

1. What is the interest on \$400 for $3\frac{1}{2}$ years at 6%?

\$400, principal
 .06, rate
\$24.00, interest for 1 year
 $3\frac{1}{2}$
\$84.00, interest for $3\frac{1}{2}$ years

The interest for 1 year is .06 of the principal, or \$24. The interest for $3\frac{1}{2}$ years is $3\frac{1}{2} \times \$24$, or \$84.

Multiply the principal by the rate and the product by the number of years.

Find the interest on :

- | | |
|--|---|
| 2. \$216 for 1 year at 5%. | 9. \$150 for $4\frac{1}{2}$ years at 5%. |
| 3. \$950 for 2 years at 8%. | 10. \$190 for $2\frac{1}{2}$ years at 6%. |
| 4. \$150 for 3 years at 6%. | 11. \$600 for $2\frac{1}{3}$ years at 4%. |
| 5. \$740 for 4 years at 4%. | 12. \$950 for 9 mo. at 6%. |
| 6. \$150 for $1\frac{1}{2}$ years at 4%. | 13. \$940 for $2\frac{3}{4}$ years at 7%. |
| 7. \$96 for 2 years at 8%. | 14. \$650 for $\frac{3}{4}$ year at 6%. |
| 8. \$75 for $3\frac{2}{3}$ years at 7%. | 15. \$225 for $\frac{7}{8}$ year at 8%. |

METHOD BY ALIQUOT PARTS

Written Work

1. Find the amount of \$960 for 3 yr. 7 mo. 18 da. at 5%.

$$\begin{array}{r}
 \$960, \text{ principal} \\
 \underline{.05, \text{ rate}} \\
 \$48.00, \text{ int. for 1 yr.} \\
 3 \times \$48 = \$144.00, \text{ int. for 3 yr.} \\
 \frac{1}{2} \text{ of } \$48 = 24.00, \text{ int. for } 6 \text{ mo.} \\
 \frac{1}{12} \text{ of } \$48 = 4.00, \text{ int. for } 1 \text{ mo.} \\
 \frac{1}{4} \text{ of } \$4.00 = 2.00, \text{ int. for } 15 \text{ da.} \\
 \frac{1}{12} \text{ of } \$4.00 = .40, \text{ int. for } 3 \text{ da.} \\
 \hline
 \$174.40, \text{ int. for 3 yr. 7 mo. 18 da.} \\
 960.00, \text{ principal} \\
 \hline
 \$1134.40, \text{ amount}
 \end{array}$$

Find the amount of:

2. \$1400 for 3 yr. 3 mo. 12 da. at 6%.
3. \$975 for 5 yr. 8 mo. 24 da. at $4\frac{1}{2}\%$.
4. \$360 for 4 yr. 5 mo. 10 da. at $5\frac{1}{2}\%$.
5. \$480 for 2 yr. 9 mo. 15 da. at 7%.

Find the interest and amount at 6% on:

- | | |
|------------------------------|---------------------------------|
| 6. \$500 for 6 mo. 15 da. | 14. \$175.50 for 105 da. |
| 7. \$250 for 8 mo. 20 da. | 15. \$150 for 9 mo. 12 da. |
| 8. \$360 for 5 mo. 10 da. | 16. \$387.50 for 6 mo. 25 da. |
| 9. \$475 for 90 da. | 17. \$125.50 for 10 mo. 21 da. |
| 10. \$900 for 6 mo. 25 da. | 18. \$345.50 for 6 mo. 15 da. |
| 11. \$125 for 4 mo. 19 da. | 19. \$755 for 1 yr. 9 mo. 6 da. |
| 12. \$325 for 6 mo. 23 da. | 20. \$544 for 5 yr. 3 mo. 5 da. |
| 13. \$25.50 for 3 mo. 29 da. | 21. \$80.80 for 2 yr. 15 da. |

THE ONE DOLLAR SIX PER CENT METHOD

The interest on \$1 for 30 da. (1 mo.) = \$.005.

The interest on \$1 for 1 da. ($\frac{1}{30}$ of \$.005) = \$.000 $\frac{1}{6}$.

Change the time to months and days. Since the interest on \$1 for 1 mo. is $\frac{1}{2}$ of a cent and for 1 da. $\frac{1}{6}$ of a mill, the interest on one dollar will be $\frac{1}{2}$ as many cents as there are months and $\frac{1}{6}$ as many mills as there are days. Multiply the result by a number equal to the number of dollars in the principal.

Written Work

1. What is the interest on \$240.60 for 2 yr. 3 mo. 13 da. at 6%?

2 yr. 3 mo. = 27 mo.

\$.135, interest on \$1 at 6% for 27 mo.

\$.002 $\frac{1}{6}$, interest on \$1 at 6% for 13 da.

\$.137 $\frac{1}{6}$, interest on \$1 at 6% for 2 yr. 3 mo. 13 da.

$240.60 \times $.137\frac{1}{6} = \$33.00$, interest on \$240.60

Find the interest at 6% on:

- | | |
|-----------------------|-----------------------------|
| 2. \$7450 for 93 da. | 14. \$8790 for 5 mo. |
| 3. \$8400 for 65 da. | 15. \$8250 for 6 mo. |
| 4. \$9800 for 40 da. | 16. \$150 for 3 mo. 6 da. |
| 5. \$8440 for 72 da. | 17. \$180 for 5 mo. 9 da. |
| 6. \$5500 for 5 da. | 18. \$195 for 6 mo. 10 da. |
| 7. \$6750 for 8 da. | 19. \$250 for 8 mo. 12 da. |
| 8. \$4765 for 25 da. | 20. \$340 for 2 yr. 4 mo. |
| 9. \$6245 for 110 da. | 21. \$275 for 3 yr. 8 mo. |
| 10. \$8425 for 52 da. | 22. \$450 for 1 yr. 5 mo. |
| 11. \$5150 for 3 mo. | 23. \$675 for 3 yr. 7 mo. |
| 12. \$8465 for 4 mo. | 24. \$64.60 for 2 yr. 9 mo. |
| 13. \$9640 for 7 mo. | 25. \$78.40 for 4 yr. 3 mo. |

SHORT METHODS

Written Work

1. Find the interest at 6% on \$345 for 8 mo.

$$6\% \text{ int. for 12 mo.} = 1\% \text{ int. for 2 mo.}$$

$$1\% \text{ of } \$345 = \$3.45$$

$$\text{Int. for 8 mo.} = 4 \times \text{int. for 2 mo.} = 4 \times \$3.45, \text{ or } \$13.80$$

Find the interest at 6% on :

- | | |
|------------------------------|----------------------------------|
| 2. \$125 for 120 da. | 24. \$805 for 75 da. |
| 3. \$190 for 4 mo. | 25. \$425 for $2\frac{1}{2}$ mo. |
| 4. \$325.50 for 8 mo. | 26. \$500 for 30 da. |
| 5. \$62.50 for 240 da. | 27. \$280 for 45 da. |
| 6. \$550 for 22 mo. | 28. \$345.50 for 1 yr. 6 mo. |
| 7. \$640 for 18 mo. | 29. \$850 for 1 yr. 2 mo. |
| 8. \$435 for 180 da. | 30. \$392 for 1 yr. 4 mo. |
| 9. \$552 for 21 mo. | 31. \$362 for 1 yr. 2 mo. |
| 10. \$632 for 120 da. | 32. \$563.25 for 1 yr. |
| 11. \$562 for 16 mo. | 33. \$147.50 for 1 yr. |
| 12. \$335 for 8 mo. | 34. \$150 for 90 da. |
| 13. \$222 for 9 mo. | 35. \$650 for 120 da. |
| 14. \$375 for 1 yr. 4 mo. | 36. \$160 for 100 da. |
| 15. \$387.50 for 1 yr. 6 mo. | 37. \$60.20 for 10 mo. |
| 16. \$500 for 6 mo. 15 da. | 38. \$175.50 for 105 da. |
| 17. \$250 for 8 mo. 20 da. | 39. \$150 for 9 mo. 10 da. |
| 18. \$360 for 5 mo. 10 da. | 40. \$385.50 for 6 mo. |
| 19. \$475 for 90 da. | 41. \$125.50 for 10 mo. |
| 20. \$900 for 6 mo. 20 da. | 42. \$345.50 for 6 mo. |
| 21. \$125 for 4 mo. 20 da. | 43. \$755 for 1 yr. 9 mo. |
| 22. \$325 for 6 mo. 20 da. | 44. \$544 for 5 yr. 3 mo. |
| 23. \$25.50 for 3 mo. 20 da. | 45. \$80.80 for 2 yr. 15 da. |

Find the interest from the following conditions, solving by the shortest method:

	PRIN.	TIME	RATE		PRIN.	TIME	RATE
46.	\$350	105 da.	7%	49.	\$129	23 mo.	9%
47.	\$685	4 mo.	7%	50.	\$750	13 mo.	$9\frac{1}{2}\%$
48.	\$850	87 da.	6%	51.	\$492	97 da.	$7\frac{1}{4}\%$

52. At 5% on \$237.50 from Jan. 3, 1916, to Sept. 11, 1918.

53. At 7% on \$309.75 from May 5, 1915, to Jan. 12, 1917.

54. At $7\frac{1}{2}\%$ on \$7500 from June 12, 1915, to Nov. 1, 1916.

55. At $6\frac{1}{2}\%$ on \$6225 from Oct. 11, 1915, to March 1, 1918.

56. At $4\frac{1}{2}\%$ on \$750 from Feb. 12, 1914, to Aug. 9, 1918.

57. At 8% on \$2900 from July 1, 1914, to May 10, 1918.

58. At 4% on \$3675 from June 4, 1915, to Apr. 1, 1918.

59. At 3% on \$290.80 from Nov. 12, 1912, to Apr. 10, 1917.

60. At 5% on \$875 from June 11, 1913, to March 11, 1918.

61. At $5\frac{1}{2}\%$ on \$8000 from Apr. 1, 1913, to May 9, 1918.

62. Find the amount of \$1275.80 for 1 yr. 7 mo. 24 da. at 7%.

63. Find the interest of \$4780 from April 1, 1916, to Sept. 18, 1918, at $6\frac{1}{2}\%$.

64. On September 16, 1916, I borrowed \$3600 at 8%. How much will settle the loan April 1, 1918?

65. July 28, 1916, a broker borrows \$3200 at 5% interest and on the same day loans it at $7\frac{1}{2}\%$ interest. If full settlement is made April 1, 1918, how much will the broker make by reloaning the money?

NOTE. — To find difference in time, see page 99.

66. A man borrowed \$6500 for 2 yr. 6 mo. at 4%. How much did he have to pay, including principal and interest?

67. Find the interest on \$5400 for 3 yr. 6 mo. 10 da. at 6%.



SAVING MONEY

TO THE TEACHER. — In Book Two, pupils were taught how to keep simple accounts, showing the *receipts* and payments of their earnings and expenditures. Review this subject carefully. Discuss with the class the need and the value of saving money; the evils arising from extravagance; the relation of thrift to success in life; the importance of system in keeping accounts; the need of exercising caution in making investments, etc.

Interest on the principal and on its unpaid interest, combined at regular intervals, is called **compound interest**.

NOTE. — Compound interest is allowed on savings bank accounts, as any one might draw his interest and invest it. Example 14 on p. 127 is worked by compound interest.

Every pupil should make a careful study of the following table, which shows the amount of \$1 deposited annually at 2% to 6% compound interest, for periods from 1 to 5 years.

HOW YOUR SAVINGS WILL GROW

Periods	2 per cent	3 per cent	4 per cent	5 per cent	6 per cent
1	1.02	1.03	1.04	1.05	1.06
2	2.06	2.09	2.12	2.15	2.18
3	3.12	3.18	3.24	3.31	3.37
4	4.20	4.30	4.41	4.52	4.63
5	5.30	5.46	5.63	5.80	5.97

SAVINGS BANK ACCOUNTS

Savings banks are banks under the control of the government. They receive and invest deposits, chiefly savings, and pay interest thereon at stated intervals.

Interest on savings bank accounts is computed at from 2% to 4% annually, and is usually *payable the first of each January and July*. If not drawn when due, the interest is added to the deposit.

For example: If John deposited \$100 Jan. 1, in a bank paying 4% annually, his amount in the bank July 1 would be \$100 plus the interest on \$100 for 6 months at 4%, or \$102. The interest for the next 6 months would be computed on \$102.

NOTE. — As a rule, interest is reckoned only on deposits made at regular intervals; as, Jan. 1 to Jan. 10, April 1, July 1 to July 10, Oct. 1. Money deposited Jan. 2 to Jan. 10 draws interest from Jan. 1; money deposited Jan. 11 to March 31 draws interest from April 1, etc. Unless otherwise stated, the following examples are to be worked on this basis. Sometimes, however, interest is calculated from the 1st and 15th of each month succeeding the several deposits.

Written Work

1. On Jan. 1, 1918, John deposited \$10 in the savings bank at 4% interest. On March 25, he deposited \$5. What was John's amount in bank July 1, 1918?

\$ 10, prin. Jan. 1, 1918.	\$ 5, prin. Mar. 25, 1918.
.04, rate.	.04, rate.
2)\$.40, interest for 1 yr.	\$.20, interest for 1 yr.
\$.20, interest for 6 mo.	Interest from April 1 to July 1, —
	3 mo., or $\frac{1}{4}$ of a year.
\$10.00, first deposit.	4)\$.20, interest for 1 yr.
.20, interest on first deposit.	\$.05, interest for 3 mo.
5.00, second deposit.	
.05, interest on second deposit.	
\$15.25, total amount in bank July 1, 1918.	

2. Mary deposits \$5 in the savings bank Jan. 1, 1917. March 1, 1917, she deposits \$6. Find the total amount in bank Jan. 1, 1918, at 4%, payable January 1 and July 1.

NOTE. — Savings banks usually allow interest only on the *dollars* of the smallest balance on deposit during the entire term. In the interest, *fractional parts of a cent are omitted.*

3. Frank makes the following deposits: Jan. 1, 1917, \$10, Feb. 10, 1917, \$15. Find the total Jan. 1, 1918, at 4% interest, payable January 1 and July 1.

4. Find the interest July 1, 1916, on a deposit of \$100 made July 1, 1915, at 4%, payable January 1 and July 1.

5. The total deposits of the pupils in a certain school on Jan. 1, 1916, are \$395.75. Find the amount of these deposits on Jan. 1, 1918, at 4% interest, payable January 1 and July 1.

6. Henry's parents put \$100 in a savings bank Jan. 1, 1917, at 4% interest, payable January 1 and July 1. Find the amount of this deposit Jan. 1, 1918.

7. A newsboy put \$50 in a savings bank Jan. 1, 1917, \$50 Jan. 2, 1918, and \$50 July 1, 1918. Find the amount in bank Jan. 1, 1919, at 4% interest, payable January 1 and July 1.

8. The Jackson Street school has in the savings bank \$508.70 on Jan. 1, 1915, and makes a deposit of \$230.50 on April 1, 1916. Find the amount of these two deposits on Jan. 1, 1917, at 4% interest, payable January 1 and July 1.

9. A school teacher places \$30 in a savings bank the first of October, 1917, and the first of February, 1918. Find the amount in bank July 1, 1918, at 4% interest, payable January 1 and July 1.

Find the amount in bank from the following deposits :

	DEPOSIT	DATE	RATE	INT. PAYABLE	AMOUNT IN BANK
10.	\$ 200	Jan. 1, 1917	3 %	Jan. 1 and July 1	Jan. 1, 1918
11.	\$ 150	Mar. 16, 1917	4 %	Jan. 1	Jan. 1, 1918
12.	\$ 875	May 29, 1917	2½ %	Jan. 1	Jan. 1, 1918
13.	\$ 1200	Aug. 10, 1917	2 %	Jan. 1, Apr. 1 July 1, Oct. 1	Jan. 1, 1918

14. On July 1, 1916, Raymond Wilkinson makes a savings deposit of \$400 at 4 % interest, payable semiannually. If the interest at each period is added to the deposit, what is the total amount in bank January 1, 1918 ?

Deposit, July 1, 1916	\$400.00
Interest on \$400 at 4 % July 1, 1916, to Jan. 1, 1917	8.00
Amount in bank Jan. 1, 1917	408.00
Interest at 4 % on \$408 from Jan. 1, 1917, to July 1, 1917	8.16
Amount in bank July 1, 1917	416.16
Int. at 4 % on \$416 (why?) from July 1, 1917, to Jan. 1, 1918	8.32
Amount in bank Jan. 1, 1918	\$424.48

15. Find the difference between the simple interest on a note of \$200 dated July 1, 1918, due in two years at 4½ %, and the interest on \$200 deposited in a savings bank at 4 % compounded semiannually, for the same period.

16. A savings account of \$150 deposited April 1, 1916, at 3 % interest, payable January 1 and July 1, is withdrawn April 12, 1918. Find the amount withdrawn.


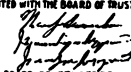
17. A savings bank pays 4 % interest, calculated from the 1st and the 15th of each month succeeding the several deposits. The deposits are Sept. 1, \$20 ; Oct. 10, \$15 ; Nov. 15, \$20 ; Dec. 10, \$25. Find the amount in bank the following January 1, if the interest periods are January 1 and July 1.

18. A school had on deposit in the Holmes Savings Bank Jan. 1, 1916, \$495.80. The deposits were: Feb. 1, \$76.90; March 1, \$105.05; April 1, \$114.29; May 1, \$129.70; June 1, \$98.75. Find the amount in the bank Jan. 1, 1917, at 4% interest, compounded the first of January and July, interest being calculated from the first of each month.

POSTAL SAVINGS SYSTEM

The postal savings system was established by the United States government to provide facilities for depositing small savings with the security of the government for repayment.

Accounts may be opened with \$1 or more by any person of the age of 10 or over. Smaller amounts than \$1 may be saved for deposit by the purchase of postal savings cards and stamps. No charge is made for the service by the United States government.

2	NOT TRANSFERABLE NOT NEGOTIABLE NEW YORK N.Y. Madison Square Station DEPOSITORY OFFICE	POSTAL SAVINGS SYSTEM UNITED STATES OF AMERICA 	ISSUE OF 1917 X 2507463	2
	CERTIFICATE OF DEPOSIT THIS CERTIFIES THAT THE SUM OF TWO DOLLARS HAS BEEN DEPOSITED WITH THE BOARD OF TRUSTEES OF THE POSTAL SAVINGS SYSTEM AND WILL BE PAYABLE TO THE OFFICE PER MATURATION OF THIS CERTIFICATE PROPERLY ENDORSED	NAME OF DEPOSITOR <i>Mary Brown</i> ACCOUNT NUMBER <i>47144</i>		
	DATE OF ISSUE APRIL 10 1917 INTEREST BEGINS ON THE FOLLOWING DATE MAY 1 - 1917	SAMPLE OF NO VALUE EXCEPT BY MATURATION OF THIS CERTIFICATE PROPERLY ENDORSED	SIGNATURE  BOARD OF TRUSTEES	

Deposits are evidenced by **postal savings certificates** issued in fixed denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$200, and \$500, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. The depositor signs a duplicate of each certificate, which is returned by the postmaster.

Interest is allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest is paid on money that remains on deposit for a fraction of a year only. Deposits bear interest from the first day of the month next following that in which the deposits are made.

Compound interest is not allowed; but a depositor may withdraw interest when due and include it as a new deposit, which will bear interest at the regular rate.

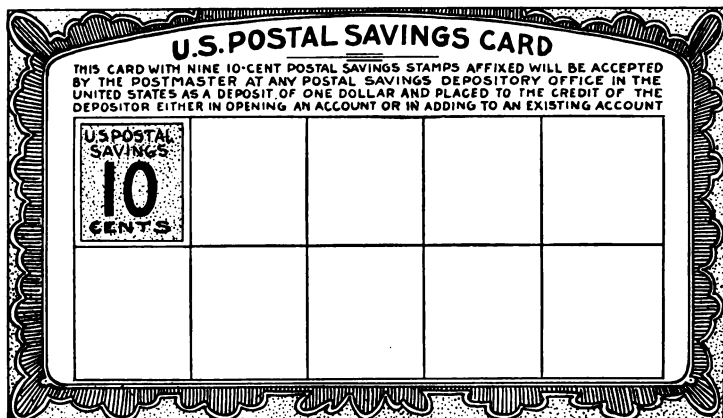
Deposits are **limited** to \$1000 exclusive of interest.

A depositor may exchange his deposits for registered or coupon United States postal savings bonds issued in denominations of \$20, \$100 and \$500, bearing interest at the rate of $2\frac{1}{2}$ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest being payable 20 years from such date in United States gold coin. Such exchange may be made Jan. 1 and July 1 of each year, provided such bonds are then available.

The **withdrawal** of a savings certificate together with any accrued interest may be made at any time on presentation of the certificate properly indorsed. When withdrawing merely the *interest*, the depositor simply signs a receipt for the amount of interest paid.

Oral and Written Work

1. Mary Collins makes a deposit on Saturday, and the postmaster gives her a card like the one on the following page. How many cents does she deposit? How many more ten-cent stamps must she affix to fill the card? How large a postal savings certificate can she get in exchange for the card when filled?



2. If I open a postal savings account April 3, 1917, when does interest begin on my first deposit?

3. A boy deposits \$5 a month in a postal savings bank. What amount has he to his credit at the end of a year?

4. If I open a postal savings account with \$10 April 3, 1917, and add \$10 to this account on May 1, 1917, how much interest is due me May 1, 1918?

5. Harry deposited \$2 May 1, 1917, and \$10 June 11, 1917. How much money did the postmaster give him for the certificate July 1, 1918?

6. A \$50 deposit made May 3, 1917, is worth how much at the post office June 1, 1918?

7. On July 1, 1916, John exchanges postal certificates amounting to \$500 for United States bonds. How much interest will be due on them Jan. 1, 1917, at $2\frac{1}{2}\%$ per annum?

8. James has \$120 on deposit in the postal savings bank and buys two $2\frac{1}{2}\%$ government bonds on January 1, — one for \$20 and one for \$100. How much interest will each of these bonds yield in five years?

9. John deposits \$5 June 2, 1917; \$5 June 9; \$5 June 16; \$5 June 23; and \$5 June 30. Explain when each deposit begins to bear interest and what will be the amount of the deposits July 1, 1918, including interest.

10. May deposits 10¢ each week day in the postal savings bank. How many \$1 certificates will she have in 52 weeks?

11. May deposited \$3 on June 5, 1917, and \$5 on July 2, 1917. What will be the amount of her deposits, including interest, if withdrawn August 1, 1918?

12. Frank Gross deposits \$10 January 5, 1917, and each month throughout the year on the same date the same amount. Explain when there is a year's interest due on each deposit.

13. My postal certificates June 11, 1917, show \$190. How much will be due me on July 1, 1918, including interest?

PROMISSORY NOTES

Mr. James H. Ames, a grocer, Salem, Oregon, has an account of \$52 against Robert Patterson for groceries, and Mr. Ames asks Mr. Patterson to give him a note at 6% interest for the amount of the bill.

The note reads as follows:

\$ 52.00

Salem, Oregon, Nov. 21, 1917.

Six months after date...I...promise to pay to the
order ofJames H. Ames.....

Fifty-two.....Dollars.

Value received, with interest at 6%.

Robert Patterson.

A **promissory note** is a *written* promise to *pay* to a *certain person* named in the note, or his *order*, a *specified sum* of money at a *specified time*.

The Essentials of a Promissory Note

1. It should state the **place** where and the **time** when given.
2. It should promise to pay to a **certain person** or to his **order**.
3. It should promise to pay a **certain sum of money**, expressed both in **figures** and in **writing**.
4. It should state **when** the money is to be paid.
5. It should state **by whom** the money is to be paid.
6. It should state for **value received**.
(Not absolutely necessary, but usually written in a note.)
7. It should state **with interest** and the **rate**, if it is an interest-bearing note.

The promissor is called the **maker** of the note.

The person who is to receive the money is called the **payee** of the note.

1. Who is the **maker** of the note on page 131?
2. Who is the **payee** of the note on page 131?
3. Find the **amount** to be paid when due.
4. The **face** of the note is the sum written in the note. What is the **face** of the note on page 131?
5. This note reads "pay to the *order of James H. Ames*," and means that Mr. Ames has the right to sell this note to any one by simply writing his name across the back of the note and delivering it to the purchaser. What words in the above note give Mr. Ames the right to sell it?

When the owner of a promissory note writes his name across the back of it, he is said to **indorse** the note.

If Mr. Ames indorses the note and then sells it to Mr. Bond, and Mr. Bond indorses it and sells it to Mr. Cox, to whom does the note belong?

A promissory note, therefore, like any other property may be bought and sold; hence it is called **negotiable paper**.

When a note is made payable to a definite person, it cannot be transferred, and is therefore *not* negotiable.

Promissory notes may be indorsed as follows :

(1) **In blank** : In this form the indorser simply writes his name across the back of the note, thus making the note payable to the holder.

INDORSEMENT IN BLANK

James Anderson.

(2) **In full** : In this form the indorser designates that the note is to be paid to the order of a definite person.

INDORSEMENT IN FULL

*Pay to the order of
John Burke.
James Anderson.*

(3) **In limited form** : In this form the indorser writes "without recourse" above his name. This means that the holder cannot compel him to pay if the maker fails to do so.

LIMITED INDORSEMENT

*Without recourse.
James Anderson.*

Every indorser in blank, or in full, makes himself *liable* for the amount of the note if the maker and the previous indorsers *fail* to pay. Banks are required to notify the indorsers in a manner prescribed by law, in case the note is not paid when due. This is called **protesting** the note. If the note is not protested, the indorsers are released from the liability of payment.

Forms of Promissory Notes

I. As to time :

1. If the words "on demand" are substituted for the words "six months" in the note of Mr. Ames, page 131, it will then be a "demand note"; that is, the maker may be called upon to pay it at any time after date.

2. The note of Mr. Ames is a "time note" because it is not to be paid until a certain time named in the note.

The *time* of payment in a note must be definite. A promise to pay "when able" is too indefinite, and not binding.

II. As to payees :

1. When a note is payable to the *order* of some particular person, he alone can collect it, or sell the note by indorsement.

2. When a note is payable to some particular person, or bearer, the holder can collect it, or sell it by indorsement.

III. As to the number of makers :

1. An individual note is a promise made by one person.

2. "A joint and several note" is a promise made by more than one person. It contains the words "we, or either of us," and is signed by the makers.

Maturity of Promissory Notes

A note is said to *mature* on the last day of the time named in the note. Some states allow three days, called "days of grace," from the time a note matures before the payee can proceed to collect the note. In this case three days are added to the time on which the interest is computed. Days of grace are now abolished in most states. The note on page 131 matures May 21, 1918.

If a note falls due on Saturday, Sunday, or a legal holiday, it is usually payable on the next *succeeding* business day. Some states require such notes to be paid on the *preceding* business day.

Interest on Promissory Notes

If either a time or a demand note contains the words "with interest," the note bears interest from *date* at the legal rate in that state.

If the words "with interest" are omitted from a time note, it bears interest from the date of *maturity* until paid.

If the words "with interest" are omitted from a demand note, it bears interest from the time payment is demanded until paid.

Oral and Written Work

1. Is the promissory note given by Mr. Patterson (p. 131) a *demand* or a time note? an individual or a joint and several note? payable to order or bearer?

2. Write a promissory note, in which you are the maker, for \$ 125 due in 6 months, payable to the order of Ellsworth Slater, with the legal rate of interest in your state.

3. Mr. Slater sells this note to Herman Gross, and indorses it in full. Write the indorsement on the note.

4. Write a joint and several note for \$250, dated Sept. 24, 1917, due on demand, with interest at 6%, payable to the order of James Harbison. Your teacher and yourself may sign this note as makers.

5. May the two names to the above note be written by the same person? Why not?

6. In case James Harbison sells this note to James Brown, but says to Mr. Brown, "I shall not be responsible for the collection of this note," write the indorsement and explain why you use that form.

7. Find the amount paid to the holder of Mr. Harbison's note (Ex. 4), if settled Jan. 4, 1918.

NOTE. — The time from Sept. 24, 1917, to Jan. 4, 1918, is 102 da. (Count the last date, but not the first date.)

8. Name the different kinds of negotiable notes:

(1) as to *time*; (2) as to *payee*; (3) as to *number of makers*.

9. Find the amount to be paid on the following note, if settled March 1, 1918. (Compute interest from maturity at 6%.)

\$ 300.00

Dayton, Ohio, March 1, 1917.

Three months after date I promise to pay to the order of James Bryce

Three hundred Dollars.

Value Received.

James Anderson.

SOLUTION. — Int. begins at maturity, June 1, 1917. Time = 273 da. Int. on \$300 for 1 yr. at 6% = \$18. Int. for $\frac{273}{360}$ yr. = \$13.65. Amount = \$313.65.

10. Find the amount to be paid on the following note, if settled Aug. 10, 1917, with interest at 6%:

\$ 175.50	Bangor, Maine, Jan. 20, 1917.
On demand, after date---I---promise to pay to the	
order of Theodore Noel-----	
One hundred seventy-five and $\frac{50}{100}$ -----Dollars.	
Value received, with interest.	
Arthur Mason.	

SOLUTION.—Time = 202 da. Int. for $\frac{202}{360}$ yr. at 6% = \$5.91.
 Amount = \$175.50 + \$5.91 = \$181.41.

11. If the words “with interest” are not mentioned in a time note, from what date does the note bear interest?

12. If a note reads “with interest,” and no rate is mentioned, what rate per cent is to be taken?

13. If the words “with interest” are not written in a demand note, does it bear interest?

14. If days of grace are allowed in your state, how do you find the time on which the interest is to be reckoned?

15. How do you compute time on a promissory note?

16. Which is the safer form:

“Pay to the order of Henry James,” or

“Pay to the order of Henry James, or bearer”? Why?

NOTE.—The note of Mr. Ames (p. 131) could have been payable to James H. Ames or bearer. Why is the form as written in the note better?

It is safest to indorse a note *in full*, for the reason that if it is lost in its delivery by mail or by messenger, it can be collected only by the party named on the back of it.

Find the amount due at date of settlement. On overdue, non-interest bearing notes, compute interest at 6 %.

	DATE	FACE	TIME	PAYEE	MAKER	INT. RATE	SETTLEMENT
17.	2/ 5/12	\$ 100	6 months	George Kimes	Yourself	6 %	Maturity
18.	4/21/12	250	On demand	A. J. Edwards	James Clyde	8 %	7/29/12
19.	6/10/12	500	1 year	John Dunn	John Grant	()	6/15/14
20.	8/16/13	350	6 months	N. J. Noel	James Palm	()	12/15/14
21.	7/12/12	125	On demand	James Bryce	Yourself	7 %	1/ 2/13
22.	5/15/13	1200	3 months	M. J. Boyce	B. J. Morrow	6½ %	Maturity

Solution to Ex. 17.—Time = 6 mo. Int. for 6 mo. on \$ 100 = \$ 3.
Amount = \$ 103.

PARTIAL PAYMENTS OF PROMISSORY NOTES

United States Rule

It is frequently inconvenient for the borrower to pay the face of the note all at one time. He is sometimes permitted, by special contract, to make payments at any time or at interest-bearing periods, until the note is paid. These amounts are called **partial payments** and are credited on the back of the note, together with the date of payment.

Find the amount of the principal to the time of the first payment, and from the amount subtract the first payment. Consider the remainder as a new principal and proceed as before until the time of final settlement. If any payment does not equal or exceed the interest, then find the interest to the time when two or more payments equal or exceed the interest.

This is the **United States rule** of partial payments, and is the legal one in most states.

The Supreme Court of the United States has decreed:

- (1) That the payment on a note must first be applied to cancel the interest then due, before the principal may be diminished.
- (2) That interest must not be charged upon interest.

Written Work

1. A borrower gives the following note:

<i>\$ 1000.</i>	<i>Portland, Oregon, Nov. 1, 1915.</i>
<i>On demand, for value received, I promise to pay</i>	
<i>James Jones.....or order,</i>	
<i>One thousand and $\frac{no}{100}$.....Dollars</i>	
<i>With interest at 6 %.</i>	<i>Henry Brown.</i>

The following payments are indorsed on this note:

What amount is due Nov. 1, 1917?

Nov. 1, 1916, \$160.00.
May 1, 1917, \$25.00.

SOLUTION

Principal	\$ 1000.00
Interest on \$1000, Nov. 1, 1915, to Nov. 1, 1916 (1 yr.)	60.00
Amount of \$1000, Nov. 1, 1916	\$ 1060.00
Payment Nov. 1, 1916	160.00
Balance = new principal due Nov. 1, 1916	\$ 900.00
Interest on \$900, Nov. 1, 1916, to May 1, 1917 (6 mo.), \$27.	
As the interest exceeds the payment of \$25 made May 1, 1917, a new principal is not formed.	
Amount of \$900, Nov. 1, 1916, to Nov. 1, 1917 (1 yr.)	\$ 954.00
Less payment \$25, May 1, 1917	25.00
Amt. due Nov. 1, 1917	\$ 929.00

2. On a claim of \$850, dated May 2, 1915, interest 5%, the following payments were made: May 2, 1916, \$200; May 2, 1917, \$200. How much was due May 2, 1918?

3. A note of \$1000, dated Aug. 1, 1915, bearing interest at 6%, had the following payments indorsed upon it: Feb. 1, 1916, \$200; Aug. 1, 1916, \$300. Find the amount due Aug. 1, 1917.

For Merchants' Rule see p. 343.

CHAPTER III

BANKS AND BANKING

1. Name several banks in your community. How many different kinds have you? In what kinds of banks can you deposit money and draw checks on your deposit? In what kinds can you only make deposits?

A **bank** is an institution that receives, lends, or exchanges money. Certain banks may also issue notes that circulate as money.

Among the various forms of banks in the United States may be mentioned **national banks** and **federal reserve banks**, which are under control of the federal government; **state banks**, which are under the control of the state; **private banks**; and **savings banks**.

A **trust company** is an institution empowered by its charter to accept and execute all kinds of trusts, to act as executor, administrator, assignee, and receiver. In most states it is empowered also to do a general banking business.

The chief business of banks is to receive deposits for safe keeping; to lend money on approved security; and to collect drafts and bills of exchange.

A **national bank**, besides its ordinary banking business, may issue notes, called **national bank notes**, that circulate as money.

Federal reserve banks are central banks in the national banking system. They were organized in order to extend our currency system and to lend money to other banks when needed.

Opening an account with a bank.

When a person opens an account with a bank, he first fills out a **deposit slip**, as here shown, and gives it, together with the deposit, to the "cashier" or "receiving teller."

DEPOSITED WITH		
First National Bank		
ELKHART, INDIANA		
By <u>James Anderson</u>		
Oct. 10, 1918		
	Dollars	Cents
<i>Bills</i> _____	50	
<i>Gold</i> _____	60	
<i>Silver</i> _____	20	
<i>Checks</i> _____		
ENTER CHECKS SEPARATELY		
<i>1st Nat. Bank</i>	65	80
<i>Union Trust Co.</i>	130	10
<i>Total</i>	325	90

Some banks require the name of the bank on which the check is issued to be written on the deposit slip.

The depositor then writes his name and address in a book kept by the bank, so that the bank may have his signature for identification.

He then receives a **bank book**, which should always be presented to the teller when a deposit is made, in order that the dates and amounts of all the deposits may be entered.

1. Get deposit slips from friends or from a bank.

2. Fill out these slips with various deposits of checks, cash, etc.

The depositor also receives a **check book**, each page of which has one or more blank checks and stubs. When he wishes to pay a bill by check, he fills out a form from the check book, similar to the following :

STUB

CHECK

No. 1401	Salem, Oregon, June 26, 1918. No. 1401
Date June 26, '18	Federal Trust Company
Payable to John R. Thompson	Pay to the order of John R. Thompson.....\$900. ²⁵ / ₁₀₀
For Rent to date	Nine hundred and ²⁵ / ₁₀₀ ~~~~~ Dollars.
Am't. \$900. ²⁵ / ₁₀₀	F. A. Smith.

A **check** is a written order by a depositor in a bank, directing the payment of money.

The **stubs** remaining in a check book, after the checks are torn out, give a complete record of the checks issued.

1. Who is the maker of this check?
2. To whose order is this check written?

For methods of indorsing notes or checks, see p. 133.

Checks, like promissory notes, may be written in different ways as follows:

- | | |
|--|--------------------------|
| 1. Pay to bearer, | } collectible by bearer. |
| 2. Pay to cash, | |
| 3. Pay to James Ogden, or bearer, | |
| 4. Pay to self (collectible by maker only). | |
| 5. Pay to the order of self (collectible by indorsement of the maker). | |
| 6. Pay to the order of James Ogden (collectible by indorsement of James Ogden only). | |

NOTE. — The last form of check is the one in general use.

Balancing Accounts ; Depositing : Checking on Accounts.

1. Your deposits in a bank for the month of October are as follows :

Oct. 1, currency, \$100; silver, \$50; check, \$45.

Oct. 5, currency, \$30; silver, \$80; check, \$100.

Oct. 9, currency, \$65; silver, \$6.75.

Oct. 15, currency, \$50; silver \$8.40; check, \$20.

Oct. 24, gold, \$50; check, \$50; silver, \$50.

Oct. 30, check, \$40; currency, \$30; silver, \$25.

Make out deposit slips and find amounts of deposits for October.

BORROWING FROM BANKS. BANK DISCOUNT

Banks usually lend money on promissory notes, which vary in form. A note may be payable to an indorser who signs his name across its back, or to the bank.

1. An *accommodation note* is one the maker or indorser of which lends his credit to another.

2. A *joint and several note* is one signed by two or more parties as makers.

3. A *collateral note* is one with which property is pledged, usually in the form of stocks, bonds, or mortgages.

\$ 200. ⁰⁰	Richmond, Va., Sept. 8, 1917.
<p><i>Three months after date.....I.....promise to pay to</i></p> <p><i>the order ofMyself.....at the</i></p> <p style="font-weight: bold; font-size: 1.2em;">first National Bank</p> <p><i>Two Hundred and $\frac{no}{100}$Dollars.</i></p> <p><i>Value received.</i></p> <p style="text-align: right; margin-top: 20px;"><i>J. B. Chandler.</i></p>	

This accommodation note **matures** three months after Sept. 8, or Dec. 8. If the time in the note were "ninety days" instead of "three months," the note would mature ninety days after Sept. 8, or Dec. 7.

If Mr. Chandler wishes to borrow money at the bank, he may make out a note as on p. 142 and indorse it. As an accommodation to Mr. Chandler, Mr. Watson further secures the note by adding his indorsement.

If both men are responsible from a financial point of view, the bank will *buy* the note and give Mr. Chandler the difference between the value of the note at its maturity and the interest on that value at the legal rate.

The **value** of Mr. Chandler's note is the amount the First National Bank will receive from Mr. Chandler at its maturity. If Mr. Chandler fails to pay, Mr. Watson will be held responsible.

The buying of notes by a bank is called **discounting notes**, and the interest deducted is called **bank discount**.

The **proceeds** of a note discounted by a banker or a broker is the value of the note at its maturity less the discount.

The **term of discount** is the exact number of days that the borrower has the use of the money.

There are two methods, however, of reckoning this *term*; the first method counts the day of *maturity* but not the day of *discount*; the second counts both. Thus, by the first method Mr. Chandler had the use of the money 22 days in Sept., 31 days in Oct., 30 days in Nov., and 8 days in Dec., or 91 days in all; by the second method he had the use of the money 23 days in Sept., 31 days in Oct., 30 days in Nov., and 8 days in Dec., or 92 days in all.

NOTE. — Answers in this book are given by the first method.

When *days of grace* are allowed they are included in the term of discount, but in this book days of grace are not reckoned.

1. How much does the bank pay to Mr. Chandler? How much does Mr. Chandler pay to the bank at maturity?

2. If Mr. Chandler had borrowed \$200 from Mr. Watson for 3 months at 6%, how much would he have paid Mr. Watson when the note became due?

3. How much would Mr. Chandler have received from Mr. Watson at the time he borrowed the money?

4. Find the difference between the discount paid to the bank and the interest he would have paid to Mr. Watson.

Comparative Study

Banks differ from individuals in lending money, as follows:

1. *Banks* require the interest on a note to be paid in advance; *individuals* demand interest when the note is due, or annually, if for a longer period than a year.

2. *Banks* compute interest for the exact number of days; *individuals* compute interest by months and years.

3. *Banks* lend money for short periods, usually not exceeding four months; *individuals* lend for longer periods, not exceeding five years in most states.

4. *Banks* require the maker to give additional security; *individuals* may or may not demand security.

5. *Interest* is computed on the face value of a note; *bank discount* is computed on the value of a note at its maturity.

Bank discount is the simple interest paid in advance on the value of a note at its maturity for the exact number of days the banker is without his money.

Written Work

Given the dates and time of notes, to find the date of maturity.

Find the date of *maturity* of the following:

DATE	TIME	DATE	TIME
1. June 1	2 months	5. Jan. 2	3 months
2. July 3	50 days	6. March 3	75 days
3. Aug. 5	100 days	7. April 1	70 days
4. Sept. 10	1 month	8. May 5	4 months

Find the date of *maturity* and the *term of discount*:

DATE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT	DATE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT
9. March 1	60 da.	April 1	14. June 3	90 da.	Aug. 1
10. April 10	3 mo.	June 15	15. March 23	4 mo.	June 2
11. July 10	4 mo.	Sept. 30	16. Oct. 8	60 da.	Nov. 1
12. May 24	30 da.	June 1	17. June 5	90 da.	July 10
13. August 5	70 da.	Sept. 1	18. Sept. 24	30 da.	Oct. 1

19. Write a promissory note for \$300 payable to John Jackson, dated Aug. 3, 1918, due in four months, without interest and signed by Glenn Campbell.

20. Mr. Jackson indorses the note in example 19, and Mr. Campbell borrows the money from the Park National Bank. Indorse the note in full and find the bank discount at 6% and the proceeds.

Write promissory notes from the following data: Find the date of maturity, the bank discount, and the proceeds, if the notes are discounted on the date when made.

DATE OF NOTE	TIME	FACE	RATE OF DISCOUNT
21. Aug. 10, 1917	90 da.	\$ 150	6%
22. June 12, 1917	2 mo.	515	6%
23. July 2, 1917	3 mo.	1000	7%
24. Jan. 8, 1917	4 mo.	625	7%
25. Mar. 5, 1917	60 da.	570	6%
26. May 8, 1917	60 da.	425	6%
27. Sept. 1, 1917	1 mo.	1200	8%
28. Dec. 1, 1917	20 da.	400	6%
29. Mar. 5, 1917	90 da.	1125	7%

30. Mr. Charlton sells a horse for \$174 and takes a promissory note from the buyer, Henry Austin, for ninety days

without interest. Thirty days after selling the horse, Mr. Austin discounts the note at his local bank at 5%. Find the proceeds.

31. Byers and Byers, merchants, take a note August 1, 1918, from Thomas Rowal for \$150 due in six months. Needing money, they get this note discounted at their local bank December 10, 1918, at 6% discount. Write the note. Then write the indorsement. Find the proceeds.

32. Mr. McAdoo is in business and takes a time note, without interest, for \$125 from Silas Hagerty, April 1, 1916, for one year. He discounts this note January 10, 1917, at his local bank at 6% interest. Write the note. Then indorse it and present to your teacher (as banker) for discount. Find the proceeds.

33. Mr. McAdoo takes Rudolph Austin's note on demand for \$200 May 1, 1917, at 6%. September 1, Mr. McAdoo makes demand for the money. How much will it take to pay the note?

INVESTING MONEY

It is quite as important to know how to invest money wisely as how to save it. When people hoard money in hidden places, they not only lose interest, but are likely to be robbed. When they are induced by an offer of a large interest to invest with dishonest men or in unsafe enterprises, they run the risk of losing their savings.

In many eastern states 5% to 6% interest and in western states 6% to 8%, on good security, is considered safe.

Safe investments. — The United States Government pays 2% interest on postal savings and $2\frac{1}{2}$ % to 3% on U. S. Government bonds. Cities and towns pay from $3\frac{1}{2}$ % to 5% on city

or village bonds. **Savings banks** pay 3% to 4% interest semiannually on the money deposited. Men who borrow money on **notes or mortgages on real estate** pay from 5% to 8% interest, depending on the locality.

Real estate loans in cities are generally made on a 50% valuation for property with buildings, or $33\frac{1}{3}\%$ for vacant property. In small towns and in the country the valuation is frequently higher.

1. Why is it not safe to loan as high a percentage on vacant property as on property with buildings?

2. Why is it usually safer to invest money at a low rate of interest at home rather than at a high rate of interest away from home?

3. Name the various ways of investing, either by purchase or by loan, in your community. Which is the safest? Which is the most common form of investment?

4. Compare the relative incomes in two years from the investment of \$500 in the United States postal bank; in a savings bank paying 4% interest; in a real estate loan on James Smith's house valued at \$1000, at 5% interest; in a loan to Miles Anderson, with security, at 6%.

5. Write the above loans in the order of their income; then in the order of their safety.

6. Why is the element of *safety* with people of small means of far greater importance than the amount of income?

The safety of a loan on real property is to be determined by considering whether the real property will sell for the amount lent in case the owner of the property fails to pay the loan when due. Care must be taken to see that all taxes and insurance are paid, and that the owner has the insurance policy to protect him in case the buildings burn.

Investing money in real estate.—Safety of investment in purchasing real estate is determined, first, by its *actual cash valuation*; second, by the *yearly returns* of the property; third, by the probability of *increase in valuation*.

Written Work

1. Mr. Arnold purchases a house and lot in a good neighborhood for \$3000. The taxes average \$45 per year, repairs \$50, and insurance \$5. What per cent does he realize yearly on his investment, if the monthly rental is \$30?

2. After two years Mr. Arnold sells the house for \$4200. What has been the average yearly income?

SOLUTION: $\$1200 \div 2 = \600 ; $\$600 + \$260 = \$860$

3. Mr. Cole buys a house for \$5000, which he rents for \$25 per month. The taxes average \$100; insurance \$5; repairs and loss of rent \$50. Find his yearly per cent of income on the investment.

4. Mr. Cole sells the property in 4 years for \$8000. Find the average yearly per cent of profit on the investment.

5. Mr. Reed buys a vacant lot for \$300. The taxes average \$7 per year. He keeps the lot 4 years and then sells it for \$325. Find his per cent of loss for the time, if the \$300 was previously invested at 5% interest.

6. Mr. Burt buys a house for \$3000 on which there is a mortgage of \$1500, with interest at 6%. The taxes, insurance, and repairs average \$80 per year. For how much must he rent the house to realize 6% on the investment?

7. The taxes on a house average \$40, insurance \$8, repairs \$50. It rents for \$25 a month. Which would yield the better income,—to keep the property and pay 5% for collecting rents, or sell it for \$3000 and loan the money on first mortgage at 6% interest?

8. In renting houses, an allowance must always be made for loss of rent when the house is vacant, or when the rent is uncollectible. Allowing \$50 per year for loss of rent and cost of collections, find the per cent of return on a house valued at \$3000 that rents for \$25 a month.

9. Charles Hoffman bought a house and lot in 1917 for \$4000 and kept it for one year. He spent \$54.75 for advertising, \$20 for insurance, \$15 for taxes, and 2% for selling the house at \$4250. Did he gain or lose, and how much, if money is worth 5% interest?

10. Mr. Ford decides to purchase a house and lot for \$3000. The rent is \$30 per month, and taxes, repairs, and other expenses are \$100 per year. After four years, he sells the property for \$4500. Counting interest only on the purchase price, how much better is this than a straight loan for four years at 6% interest?

11. Mr. Grant loans \$1000 at 6% interest for three years on a vacant lot valued at \$3000 secured by a bond and mortgage. At the end of three years the owner is compelled to sell the property for \$2000 to pay the loan. How much is due to Mr. Grant with interest?

12. Mr. Gray buys a one-half interest in a store for \$2000, April 1, 1916; April 1, 1918, he and his partner find that the money invested by both, namely \$4000, has yielded $26\frac{3}{4}\%$ profit. Find Mr. Gray's profit.

13. Mr. Buck buys a vacant lot for \$2000. The taxes for three years are \$66. Money being worth 6% interest, what is his profit or loss if he sells the lot after three years for \$2500?

14. James, Henry, and Susan have each \$2000, April 1, 1916: (a) Susan puts her money in a savings bank that day at 4% interest compounded April 1 and October 1 of

each year; (b) Henry loans his money at 6% interest with good security for three years; (c) James buys a house and lot that rents for \$15 per month. Examining the title, and putting the deed on record costs \$20; the amount paid for taxes, insurance, repairs, etc., for the three years is \$200. James sells the property at the end of the three years at \$3250. Which investment yields the best return at the end of the three years?

15. Mr. James buys a lot for \$600, May 1, 1917, paying \$500 cash, and giving a mortgage for \$100 payable in one year. May 1, 1918, he sells the lot for \$720 cash. He estimates the taxes at \$15 and other items of cost at \$15. Counting interest at 5% on the cash payment and on the mortgage, what rate per cent does his investment yield him?

16. Mrs. James buys a house and lot for \$3000, April 1, 1918. It rents for \$25 a month. The collector charges 5% for collecting the rent. The insurance, taxes, and repairs for the year are \$100. What per cent is realized on the investment?

17. John Gaunt buys a house and lot for \$3500, April 1, 1918. He estimates insurance, taxes, and repairs at \$125 a year. For how much per month must he rent the house, so that he may realize 6% net on his investment per year?

18. Mr. Gaunt sells this property for \$4250 after keeping it for two years, and paying an extra paving bill of \$185. How much profit does he make after deducting 6% interest on the cost of the property?

19. Mr. See has \$2500 to invest in a savings account at 4% interest compounded semiannually, or in a mortgage at 6% interest for three years, both being safe investments. Which yields the larger return for 3 yr. and how much?

20. Mr. Ames buys a vacant lot for \$1500, paying \$500 down and the balance on a mortgage due in 3 years at 6 % interest. The taxes average \$15 per year and a paving bill the second year cost \$128. Find the cost of the lot to Mr. Ames at the end of the 3 years, including mortgage, interest on mortgage, paving, and taxes.

21. Mr. Alden buys a house and lot in a residence section of the city for \$3500. The taxes average \$42.50, insurance \$7, and repairs \$50 annually. What will be the per cent of profit on the investment, if the house rents for \$30 per month?

22. Mr. Alden sells this house for \$3600 after owning it for 3 years. Find the average per cent of profit for each year.

23. Mr. Sands buys a house and lot for \$4000 in the business section of a city. The taxes average \$50, repairs \$20, insurance \$10 annually. A street improvement costs \$200. After renting the house for two years at \$40 per month, he sells the property for \$5600. Find his profit, counting interest at 5 % only on the purchase price.

24. In 1915 Mr. Crane buys a lot for \$400. The taxes average \$6 per year, and a street improvement costs \$80. Three years after buying the lot, he sells it for \$546. Counting interest at 6 % only on the cost of the property, find his per cent of loss.

25. Mr. Owen buys a house and lot for \$5000. It rents for \$40 per month. The taxes average \$60, repairs \$15, and insurance \$10. A paving bill amounts to \$175. After five years, he sells out for \$10,000. What per cent of gain on the investment of \$5000 does he realize from the

INVESTING IN LOANS

TO THE TEACHER.—Explain to the pupils that all forms of loans represent investments. Show a *mortgage deed* and explain that the real property is practically transferred to the lender, during the period of the loan.

Written Work

1. On Sept. 1, 1915, Paul Smith borrows from Miles Bryan \$200 for 3 years, with interest at 6%, reserving the privilege of making payments on the note at any time. Samuel Smith is the indorser. Write the note payable to Samuel Smith; then indorse it thus:

Samuel Smith

Paul Smith

Paul Smith gives the note to Miles Bryan and gets his money. How many people agree to pay the money?

2. Paul Smith makes a payment of \$75 on the note, Sept. 1, 1916, and a payment of \$100, Sept. 1, 1917. How much money will pay the note, Sept. 1, 1918? Show how the payments are indorsed on the back of the note.

3. John and James Welch give to John Davis a note for \$300 signed by both of them, due on demand at 6% interest, dated January 1, 1917. Write this note and find the amount due if paid January 25, 1918.

4. Charles Bronson borrows \$2000 from Thomas Corbet for three years from May 1, 1916, giving a bond and mortgage on his house and lot at 6% interest. It costs Mr. Bronson 2% commission to get the money, and \$30 for examination of title and recording fees. How much in all does it cost him for the use of the money for the three years?

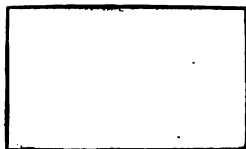
5. Mr. Grant gives Mr. Burt a demand note for \$100 May 1, 1918, without interest. Find the discount on this note for 90 days at 5%.

MEASUREMENTS

AREAS OF RECTANGLES

A **rectangle** is a figure having four straight sides and four right angles.

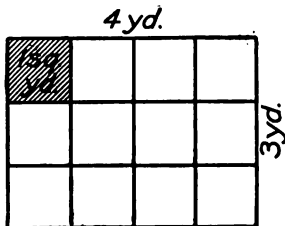
The **area** of any surface is the number of square units of surface that it contains.



RECTANGLE

Finding the area of a rectangle.

Find the area of a rectangle 4 yd. long and 3 yd. wide. How long is this rectangle? how wide? What is the *unit* of measure? How many such units are in the first row? in the second? in the entire surface?



If the length and the width of a rectangle are expressed in inches, the unit of measure is 1 *sq. in.*; if expressed in feet, the unit of measure is 1 *sq. ft.*; if expressed in rods, the unit of measure is 1 *sq. rd.* If the length and the width are expressed in related units, as feet and inches, or yards, feet, etc., the dimensions must be changed to like units before you find the area.

Written Work

1. Find the area of a flower bed 20 feet 8 inches in length by 10 feet 6 inches in width.

SOLUTION

$20\frac{2}{3}$ ft., length; $10\frac{1}{2}$ ft. width.

$20\frac{2}{3} \times 10\frac{1}{2} \times 1$ *sq. ft.*, or,

$4\frac{2}{3} \times 2\frac{1}{2} \times 1$ *sq. ft.* = 217 *sq. ft.*, area.

The area of a rectangle is found by multiplying its unit of measure by the product of its two dimensions when expressed in like units.

Find the areas of rectangles having the following dimensions :

2. 20.5 ft. by 12 ft.
3. 21 ft. by 6.9 ft.
4. 72 yd. by $40\frac{1}{2}$ yd.
5. 6 yd. 1 ft. by 3 yd. $2\frac{1}{2}$ ft.
6. 115 ft. by 54 in.
7. 45 yd. by 7 ft.
8. 108 in. by 3 ft.
9. 54 ft. by 108 in.
10. How many square yards are there in a lawn 45 feet long and 36 feet wide?
11. A square ball park 600 feet on a side is inclosed by a tight board fence 9 feet in height. Find the outside surface of the fence in square yards.

DRAWING TO SCALE

Draw to a suitable scale, and show graphically, the number of square inches in the surfaces of:

1. A rug $9' \times 12'$
2. A walk $4' \times 22'$
3. A blackboard $4' \times 22'$
4. A street $48' \times 180'$
5. A surface $36'' \times 48''$
6. A hall $10' \times 24'$
7. A wall $9' \times 18'$
8. A mirror $2' \times 6'$
9. A floor $12' \times 16'$
10. A window $3\frac{1}{2}' \times 6\frac{1}{2}'$
11. James sods a lawn for Mrs. Johnston at 25¢ a square yard. How much should he receive if the lawn is 15 ft. wide and 30 ft. long?
12. A room is $24' \times 60'$, and is covered with boxes. Each box is $1\frac{1}{2}'$ in width by 1' in height by 3' in length. Find the number of boxes that can be laid bottom side down to cover the floor.

13. Mr. Bain puts down a cement walk in front of the schoolhouse, and asks the pupils to find how much the School Board should pay him for the walk which is 5' in width by 200' in length. The price agreed upon is 17¢ per square foot.

14. Mr. Johns has a cement walk put around his house that is $103\frac{3}{4}'$ in length by $2\frac{1}{2}'$ in width. Find its cost at 18¢ per square foot.

15. Measure the cellars in your own houses and estimate the cost of cementing them at \$1.25 per square yard.

16. Measure the cement walks near your houses, and estimate their cost at 16¢ per square foot. If there are any steps, allow \$1 extra for each step.

SCHOOL SHOP PROBLEMS

1. John makes the key rack shown in Fig. 1. The back is 10" in length. How many hooks can he get in one row

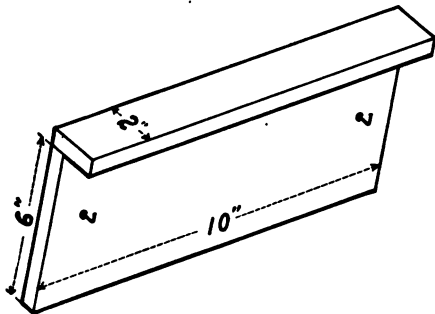


FIG. 1.

On the rack, if the hooks are $\frac{3}{4}"$ apart, and he starts them $1\frac{1}{4}"$ from the edge on each end?

HINT. — $10" - 2\frac{1}{4}" = 7\frac{3}{4}"$, space between two end hooks.

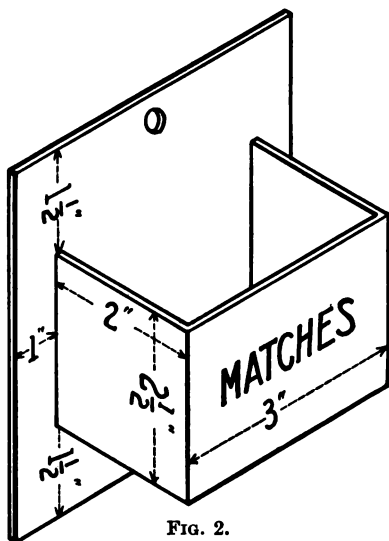


FIG. 2.

2. The shelf of the key rack on p. 155 is 2" in width. The back board is 6" in width. How many square inches of wood are there in each piece?

3. The match box shown in Fig. 2 is 3" \times 2", and 2 1/2" in depth. The back board projects beyond the box 1" on the sides and 1 1/2" on the top and bottom. The box is made of wood 1/4" in width. How many square inches of wood will it take to make the whole box?

4. What is the length of a board 6" in width necessary to make the whisk-broom holder, shown in Fig. 3, if the back is 8" \times 6", the front 5" \times 4" (4" at shorter end), and the side pieces each 1 1/2" in width?

NOTE. — For the front, a board 5" \times 6" is needed; and for the two side pieces, a board 3" \times 6".

5. What must be the number of square inches in the board necessary to make this whisk-broom holder?

6. Frank builds the footstool shown in Fig. 4 out of a board 34" in length by 10" in width. How many square inches of wood are wasted if the top is 14" \times 8", the legs are 8" \times 7", and the braces are 10" \times 2 3/4", not allowing for notches in legs and braces?

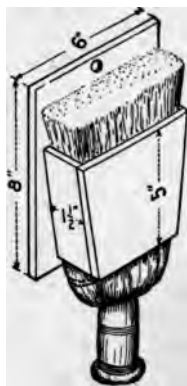


FIG. 3.

7. Show by a diagram the best way to cut the board with the least possible waste.

8. How high will the bookstand shown in Fig. 5 be, if the shelves are 12" apart, the bottom shelf being 6" from the floor, and the top of the stand 6" above the top shelf?

The upright boards are each 1" in thickness, and the shelves are $\frac{3}{4}$ " in thickness. The shelves are placed in grooves in the sides $\frac{1}{4}$ ".

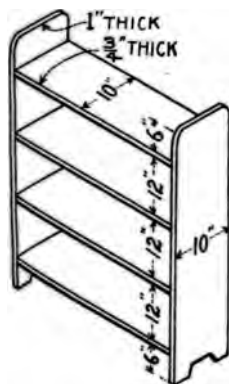


FIG. 5.

11. The plant stand shown in Fig. 6 is 18" in height. The board on the top is 14" square and $\frac{3}{4}$ " in thickness. How long a piece of wood will it take for the four legs?

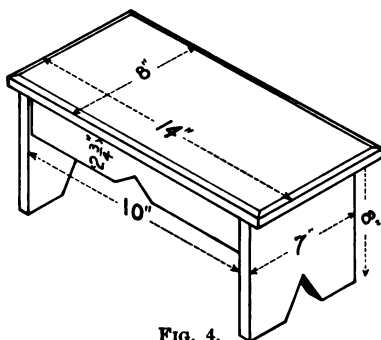


FIG. 4.

9. How many square feet of lumber are there in the stand if the shelves are $36\frac{1}{2}$ " in length and 10" in width, and the sides the same width as the shelves?

10. How many books will the stand hold if the average thickness of each book is $\frac{3}{4}$ "? How many books 1" thick?

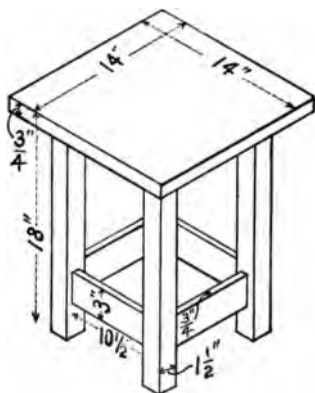


FIG. 6.

12. The legs are each $1\frac{1}{2}$ " square, and the braces between the legs at top and bottom are each $10\frac{1}{2}$ " \times 3". How many square feet of boards are there in the braces and the top if the braces are set into the legs $\frac{1}{2}$ ", making a mortise joint?

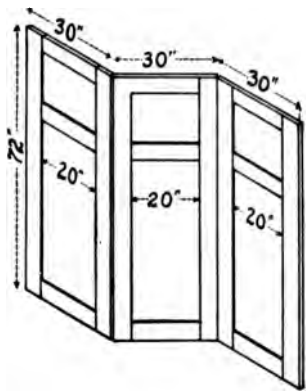


FIG. 7.

13. How far does the top extend beyond the legs?

14. John makes the three-fold screen shown in Fig. 7. Each screen is 72" by 30". The horizontal pieces, 30" in length, are fastened on the back of the upright pieces. Find the number of square feet in the entire screen

and the number of square feet of burlap necessary to cover the entire back of each section.

15. A boy has a board 3' long by $12\frac{1}{4}$ " wide. How many blocks 4" square could he saw out of the board, allowing $\frac{1}{8}$ " saw cut?

16. How many more blocks could be cut from a board 1" longer?

17. The school hall shown in Fig. 8 is to be covered with pieces of tile 10" square. How many pieces of tile will be required, no allowance being made for crevices?

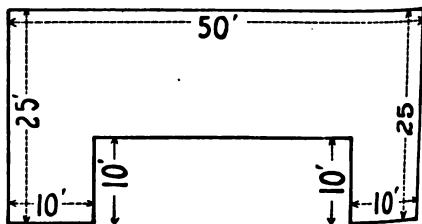


FIG. 8.

18. There is to be a border of red tile around the edges and the rest of white. How many of each will be required?

19. The door of the house in Fig. 9 is 4' wide and the windows are each 3' wide. If the house is 40' front, how

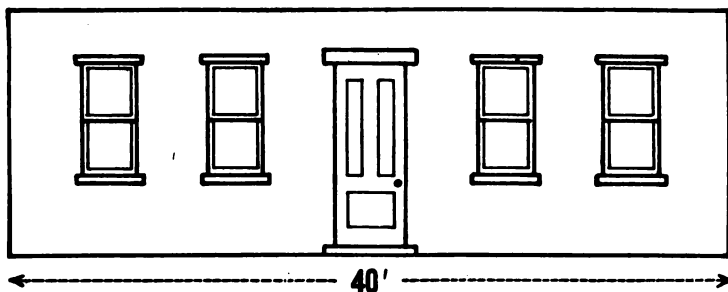


FIG. 9.

wide are the equal spaces between door and windows?

20. If this house is built on a lot 50' wide, and 160' deep, and the house is 40' by 40' with a front veranda 10' by 35' and a back porch 8' by 12', how many square feet will be left for walks and lawn?

21. A carpenter is building the cabinet shown in Fig. 10. The wood for the sides is $\frac{3}{4}$ " thick. The strips between the drawers are $\frac{1}{4}$ ". How high will the cabinet have to be if the large drawers are 8" high, and the small ones 4", and the lowest strip is 4" from the floor?

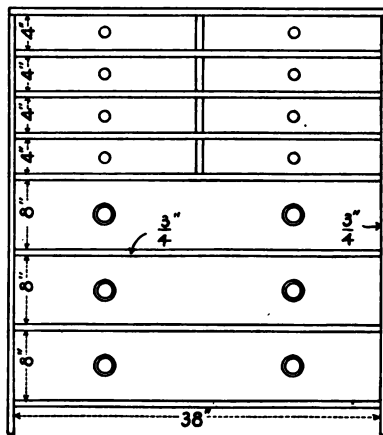


FIG. 10.

22. How many square feet of wood will he require to cover the back, which is 38" wide and 46" long?

23. Mr. Jones's lot (Fig. 11) slants so that one end is 10 ft. higher than the other end. How many cubic feet of earth must be removed to level the lot, if it is 40 ft. long on the level, and 20 ft. wide?

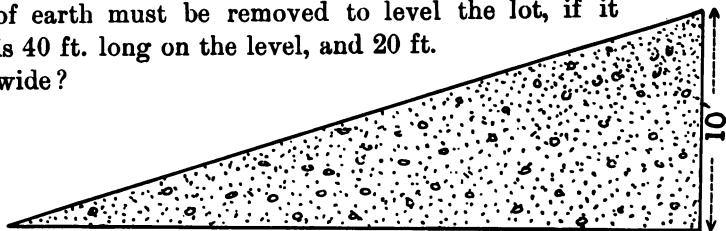


FIG. 11.

24. Every 10 sheets of a certain kind of drawing paper (Fig. 12) is $\frac{1}{8}$ " thick. How many sheets of paper are there in a package 1' 4" high?

25. How many square feet of paper are there in a package 1' 4" high if each sheet is 11" by 15"?

26. The diagram in Fig. 13 shows a lot around which Mr. Smith intends to build a fence. How many feet will it take?

27. What is the area of the lot?

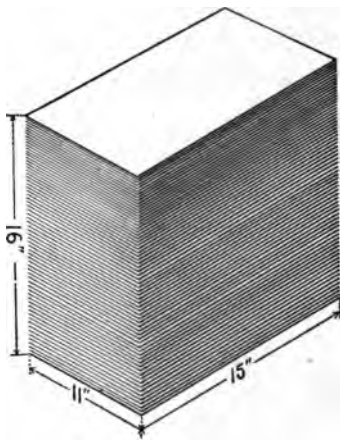


FIG. 12.

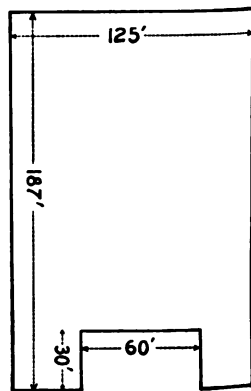
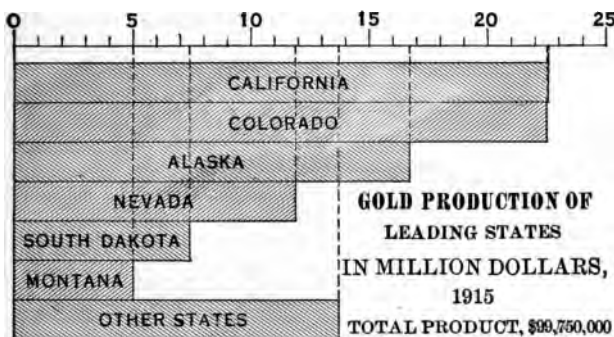


FIG. 13.

GRAPHICAL PROBLEMS



1. The scale shows that California produced over \$22,550,000, and Colorado, \$22,500,000. Find the production of gold for each of the other states named.

2. Represent graphically the area of each of the New England States; of the Middle Atlantic States; of the South Atlantic States; of the South Central States; of the North Central States; of the Mountain States; of the Pacific States.

3. Represent graphically the area of the United States from 1850 to 1916: 1850, — 2,997,119; 1880, — 3,088,519; 1900, — 3,088,519; 1916, — 3,088,519.

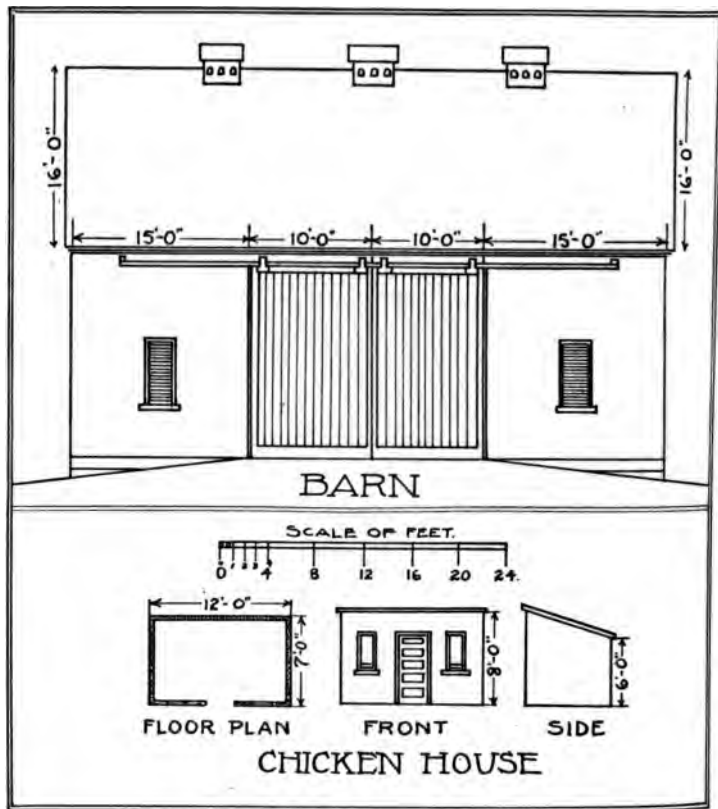
4. The imports of merchandise into the United States from 1800 to 1916 were: 1800, — \$91,252,768; 1850, — \$173,509,526; 1880, — \$667,954,756; 1900, — \$849,941,184; 1916, — \$2,391,654,835. Represent this graphically.

5. In 1880 the United States produced 498,549,868 bu. wheat; in 1900, — 522,229,505 bu.; and in 1915, — 1,011,505,000 bu. Represent this graphically.

6. Draw a graph to show that in 1850 there were 9021 mi. of railroad in the United States; in 1880, — 93,267 mi.; in 1900, — 194,262 mi.; and in 1914, — 252,230 mi.

DRAWING TO SCALE

1. Measure this drawing with your ruler, and find its scale in feet.



2. Test the floor plans of the chicken house to see if the scale is correct ; then the front ; then the end.

3. Make a copy of the plan on a scale of 1 in. to 8 ft.
4. According to the scale, what is the width of the barn floor? of the windows? the length of the barn? the height of the barn to the roof?
5. According to the scale, how high is the ridge of the roof above the eaves of the roof?

NOTE. — The line above the eaves does not show the slant of the roof, but the actual height of the ridge above the eaves.

6. If the barn is 40 ft. in width, how many square feet of siding does it take for the sides and ends, not allowing for any openings?

7. Show by a drawing the gable ends of this barn if there are two windows 4' \times 7' each 12 ft. from the side of the barn and 8 ft. from the bottom.

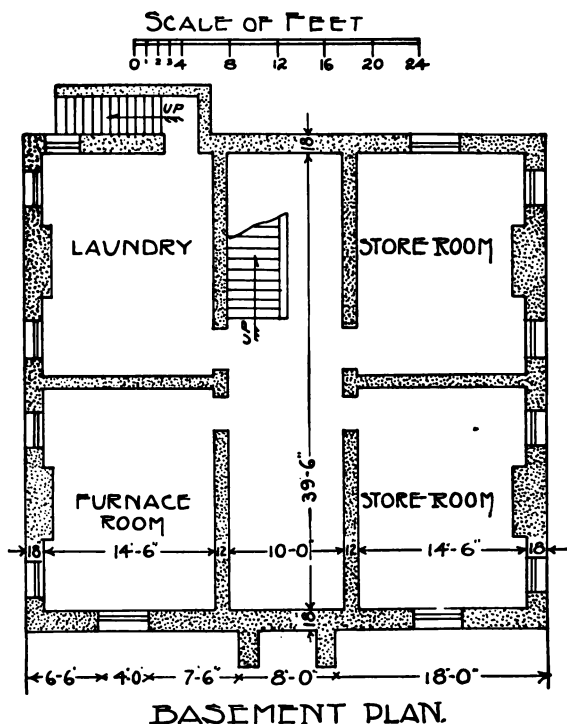
8. Make a drawing to show a plan of a wagon shed 24 ft. wide by 36 ft. long. The shed is to be 16 ft. to the roof, and the ridge of the roof is to be 10 ft. higher than the eaves of the roof.

9. Make a drawing for the plan of a chicken coop, three times the length of this one, and having three doors and six windows, of the size shown, and two partitions.

10. Find the cost of painting the sides and ends of this barn with three coats at 30¢ per square yard, not allowing for cornice and cupolas.

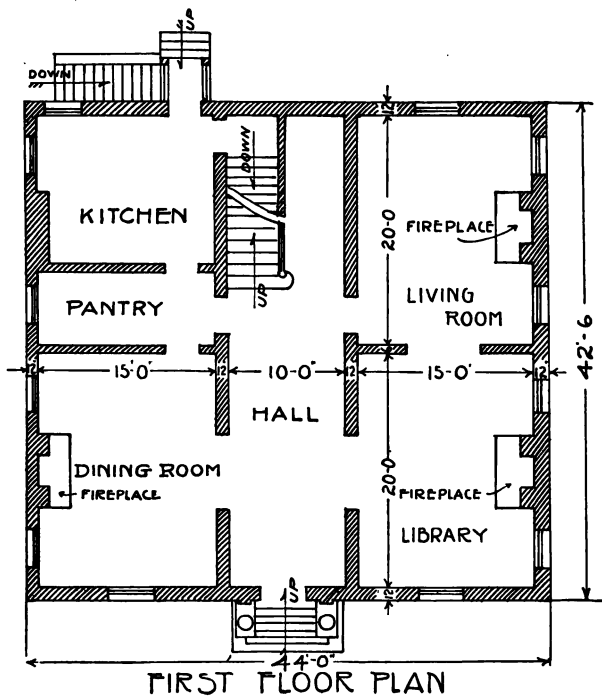
Mr. Smith gets from the architect the plans on pp. 164 and 165 of the cellar, first floor, and front elevation of his house. Find from the scale the dimensions of the rooms, windows, and doors.

11. Make a copy of the plan of this house on a scale of 1 in. to 8 ft.



12. Find the width and the length of the pantry, the fire-places, and the openings from the hall into each room; the height of the house to the eaves; the space between the windows; the space each window is from the end of the house; the height of the chimney tops from the eaves; and the height of the ridge of the roof from the eaves.

13. Make a drawing on a suitable scale to show the cellar plans, first floor, and elevation for a house 40' by 50', carrying out the design of the plans in this picture.



14. Make a drawing to show the front elevation of the house in which you live.

15. A city lot has a house with a gable fronting the street. The house is 24' wide and 42' long. Draw a plan, on a suitable scale, to show a first floor and hallway with stairway on side; a parlor with fireplace; a diningroom with fireplace; and a kitchen 10' by 16' back of the front hall. Also draw front elevation of gable end, showing a 4' door and a 5' window on first floor and two 4' windows on second floor, and the ridge of the roof 12' above the eaves.

16. Make drawings to show the plan and side elevation of your schoolroom, if in the country; or of some building of simple design, if in the city.

17. Make simple drawings to show (a) a barn 30' by 40' with a 16' barn floor and the ridge of the roof 12' above the eaves; (b) the cellar and first-floor plans and front elevation of a house of simple design; (c) a chicken house of simple design.

18. Make a plan of the front of some near-by store, showing the window and door spacing.

19. Make a plan of the first floor of a barn 30' by 36' with a row of 5 stalls 6' by 12', backed by a five-foot passageway on one side, the other side of the floor being occupied by a feed and harness room. Show a sliding door at one end, 10 feet in width.

20. A cellar wall is 40' by 24' with an extension on one side 16' by 12'. The walls are $1\frac{1}{2}$ ' thick. Make a drawing, on a scale of $\frac{1}{4}$ " to 1', showing an outside door 3' wide.

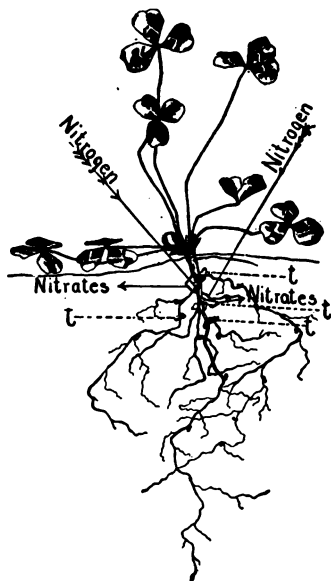
21. Make a drawing, on a scale of $\frac{1}{8}$ inch to 8 feet, to show the dimensions of your classroom, of each pupil's desk, and of each blackboard.

FARM PROBLEMS

Plant Foods.— Nitrogen, phosphoric acid, and potash must be in the soil in sufficient quantities to produce good crops. All the other constituents necessary are as a rule not wanting.

Nitrogen comprises 78 % of the air. It is difficult to get it into the soil so as to have it taken up by the plant. The most common method in the eastern states is by planting the ground in clover; in the southern states by planting cowpeas or soy beans; and in the western states by raising alfalfa. The picture shows how the nitrogen from the air is taken up by the roots and changed to nitrates and deposited in the soil.

In early times the *potash* was secured from wood ashes and both potash and phosphoric acid were obtained from farm manure. Now, since farming is done on a larger scale, the potash and phosphoric acid are usually purchased from dealers; but the nitrogen is produced as suggested above,



TUBERCLES ON CLOVER ROOTS.



SHOWING THE AMOUNTS OF THE THREE MOST IMPORTANT PLANT FOODS REMOVED FROM THE SOIL BY 1000 POUNDS EACH OF THE GRAIN OF INDIAN CORN, WHEAT, AND OATS.

CORN, WHEAT, OATS, AND COTTON

Principal Cereals Produced in 1916

STATES	OATS, BUSHEL	CORN, BUSHEL	WHEAT, BUSHEL
Indiana		182,618,000	
Illinois	172,210,000	339,947,000	
Wisconsin	81,141,000		
Minnesota	85,304,000		28,950,000
Iowa	186,813,000	377,235,000	
Missouri		152,599,000	
North Dakota			39,039,000
Nebraska	79,662,000	209,336,000	66,237,000
Kansas			89,742,000
Washington			35,404,000

1. Show by a graph the comparative corn crops in 1916 of Iowa and Missouri.

2. In 1916 the total oat crop of the United States was 1,229,000,000 bushels. What per cent (to the nearest tenth per cent) of the total crop was produced in Illinois? in Wisconsin? in Minnesota? in Iowa? in Nebraska?

3. In 1916 the United States produced 607,557,000 bu. wheat. What per cent of the wheat crop of the United States was produced in Minnesota? in North Dakota? in Nebraska? in Kansas? in Washington?



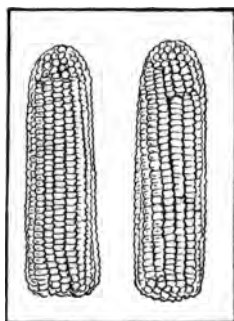
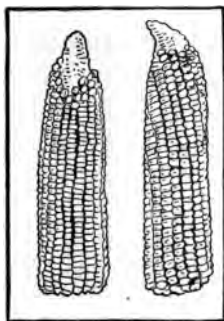
United States.

Rest of world.

THE WORLD'S CORN CROP.

4. The corn crop of the United States in 1916 was 2,718,000,000 bushels. Find out from the table what per cent of this crop Iowa produced; Illinois; Indiana; Missouri; Nebraska.

Before Mr. Adams began to improve his corn raising by selected seed, most of the yield was like the first two ears in the picture; but by proper selection of seed, the yield was like the last two ears in the picture.



IMPROVEMENT OF CORN BY SELECTION.

5. Mr. Adams's yield in 1916 was 80 bushels of ears per acre. In 1917, on the same land by proper selection of seed, it was increased 100 bushels of ears per acre. What was the per cent of increase by proper selection of seed?

6. In 1916 Texas produced about 3,725,000 bales of cotton; Georgia, 1,820,000 bales; Arkansas, 1,134,000 bales; South Carolina, 930,000 bales; Oklahoma, 823,000 bales.

The total crop was about 11,443,000 bales. Find the per cent of the total production of each of these states.

7. What per cent of the cotton crop of Texas was that of Georgia? What per cent of the crop of Texas was that of Arkansas? of South Carolina? of Oklahoma?

8. Mr. Jackson planted 50 acres in cotton, which yielded 3 bales of 256 lb. each per acre. How much did he receive for the cotton at $.9\phi$ a pound?

NOTE. — Cowpeas and soy beans are raised not only for food products, but also to enrich the soil by the formation of nitrates.

9. Mr. Jackson sowed the 50 acres in soy beans and the value of the increased fertility of the soil was 20% of the previous crop. How much was the increased fertility of the soil worth?

10. Mr. Jackson estimated that the nitrates necessary to fertilize an acre of land to produce cotton were worth \$6.75, and the cowpea stubble \$2.90, per acre. What per cent of the nitrates was the cowpea stubble worth?



United States.

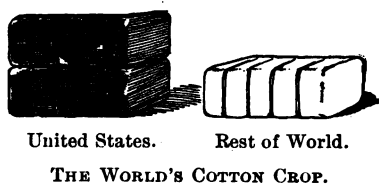
Rest of world.

THE WORLD'S WHEAT CROP.

11. What per cent of the world's wheat crop was produced by the United States in 1916?

12. Mr. Johnston plowed a clover sod of 20 acres and planted it in selected seed wheat. The yield on this land in 1916 was $17\frac{1}{2}$ bu. an acre; in 1918 the yield from the clover sod was 25 bu. an acre. What was the per cent of increase in 1916 over 1918? The weather conditions being the same, explain the effect the clover sod should have.

13. What per cent of the world's supply of cotton was produced by the United States in one year as represented graphically in this picture?



14. David Long, by proper selection of seed corn for five years, increased his yield of corn from 80 bushels of ears per acre in 1913 to 105 bushels in 1917. Find the per cent of increase.

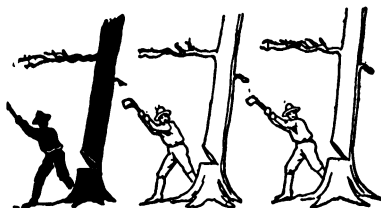
15. Frank Mays sowed 10 acres in wheat from the bin and 10 acres in carefully cleaned wheat. The two fields had had the same quality of soil and received the same care. The first field yielded 210 bu. and the second field yielded 254 bu. What was the per cent of increase by using carefully cleaned seed?

16. James's father gave him 10 trees on which to study scientific fruit raising. The trees in 1916, without care, yielded small fruit, salable at 30¢ a bushel. James pruned the trees and sprayed them for insects that injure the fruit. The next year the trees yielded the same amount of fruit valued at 60¢ a bushel. What was the per cent of increase per bushel by proper care?

17. On a certain plantation the average yield of cotton was 190 lb. an acre; but by seed selection, study of soil, and proper fertilizing, the yield in three years was increased to 392 lb. an acre. What was the per cent of increase by careful farming?

18. A farmer considers all clover stubble worth \$3.90 an acre in nitrates, and the clover hay crop worth \$27 an acre. What per cent of the clover crop is left in the soil?

FORESTRY AND TREE PLANTING



United States.

Rest of World.

LUMBERING OF THE WORLD.

1. What per cent of the world's lumber supply does the United States produce?

2. Mr. Williams had 50 acres of oak timber which he valued at \$100 an acre in 1912. If timber has increased $33\frac{1}{3}\%$ in value since that time, and there has

been no damage to the timber, how much per acre should it now be worth?

3. Mr. Long sold his timber tract at \$8 a thousand board feet on the stump. How much did he get from the sale of 150,500 feet of lumber?

4. Mr. Wayne purchased 160 acres of Washington timber for \$80 an acre in 1908, and in 1918 sold it for \$275 an acre. Counting interest for 10 years at 6%, how much profit did he realize?

5. Mr. Johnston had 20 acres of second-growth timber, consisting of oak, maple, chestnut, etc. The timber was valued at \$20 an acre in 1914. If the increase in valuation is 20% a year over each former year, how much should the timber be worth in 1917?

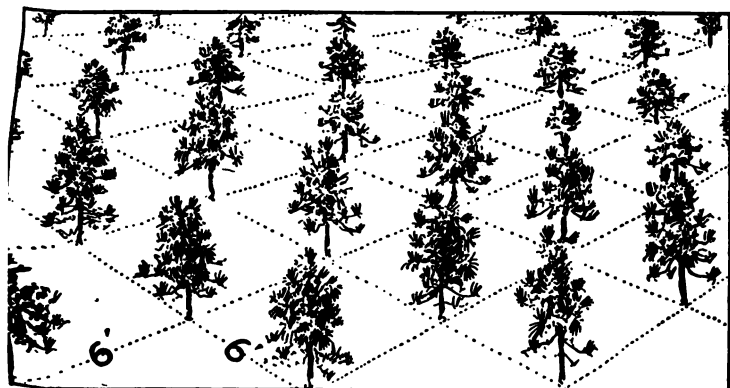
6. In 1896 Mr. Ford had 50 acres in timber land valued at \$60 an acre, including the timber. In 1918 he sold the

timber land at \$250 per acre, including the timber. What was the average annual increase per cent in the timber tract?

7. Counting money worth 6% simple interest, how much better investment was it to keep the timber tract, allowing \$1 per acre annually for taxes?

TREE PLANTING FOR TIMBER

NOTE. — The destruction of our forests is impressing on the people of our country the need of planting trees for timber. The principal trees planted for timber are: *white pine*, planted 6 ft. apart; *southern* or *hard pine*, 4 ft. apart; *black locust*, 6 ft. apart; and *catalpa speciosa* or *hardy catalpa*, 6 ft. apart. When spacing the trees 6 ft. apart, 1200 trees are estimated to the acre; and when spacing the trees 4 ft. apart, 2000 trees, to the acre.



This picture shows white pine trees, three years old, set out on soil that originally was covered with white pine forests.

1. How much will it cost to plant 20 acres in white pine trees, if the plants cost \$30 per thousand, and the planting \$12 per acre?

2. An Indiana farmer planted 30 acres in hardy catalpa. The trees cost \$27 per thousand, and the planting, \$9 per acre. Find the cost of the trees when set out.

3. The estimated value after 20 years, from the white pine trees set out in problem 1, is \$40 an acre. Every 5 years thereafter the increase is $33\frac{1}{3}\%$ over each former period. What is the value of the timber after 40 years' growth?

4. The hardy catalpa mentioned in problem 2 is estimated at \$40 an acre after 15 years' growth. If the increase every 5 years thereafter is 25%, what is the value of the catalpa after 30 years' growth?

5. A landowner in 1890 bought for \$750 fifty acres of land from which the timber had been cut. A white pine forest immediately started to grow. The value of the pine was estimated in 1918 at \$35 an acre. The owner estimated the taxes and care for the 28 years at \$400. Considering the land worth the same as in 1890, what per cent profit would the owner make if he sold the land and timber in 1918?



6. This picture shows hardy trees after 15 years' growth. The owner cut from the larger trees 50 railroad ties worth 75¢ each, and 500 fence posts worth 10¢ each. If his profit was 60% of the sale of the ties and posts, how much was his profit?

7. A Virginia farmer in 1900 cut the oak timber from 50 acres of land. A natural southern hard pine immediately grew up, which to-day is estimated to be worth \$100 an acre. If the land was worth \$10 an acre in 1900, what per cent of the value of the land in 1880 is the timber worth now?

8. A landowner planted 50 acres in black locust on mountain land valued at \$5 an acre. The plants cost \$20 a thousand and the planting \$15 an acre. He estimated the timber after 30 years' growth at \$80 an acre, for fence posts and cross ties. What was the average annual profit on the timber, after deducting for trees and planting, and \$10 an acre for taxes and care of the timber for the 30 years? (Allow 1200 trees per acre. See Note, page 173.)

9. In 1898 a man bought 160 acres of timber land, in the state of Washington, at \$35 an acre. In 1918 he sold it at \$245 an acre. If the taxes and care are estimated at \$3500, what average annual per cent of profit did he make?

10. Mr. Kirk in 1878 bought 50 acres of worn-out land in Virginia at \$8 an acre. A natural southern pine forest grew up on this land, which Mr. Kirk sold in 1918 at \$60 an acre, including land and timber. Considering the taxes and rental value of this land for 40 years at \$20 an acre, what per cent profit did Mr. Kirk make?

11. A railroad company is planting large areas in trees on each side of its tracks. The trees average 1200 to the acre. In 35 years it is estimated that each tree will produce 2 ties worth 75¢ each. What is the value of the ties on 160 acres at the end of 35 years? Estimating the cost of the land at \$30 an acre, the planting at \$5 an acre, the tax and care annually at \$3.70 an acre, what will be the company's profit at the end of 35 years?

12. Mr. Franklin bought 200 acres of land covered with second-growth timber at \$20 an acre. After 10 years he sold the land and timber for \$40 an acre. Allowing \$30 a year for taxes, and 6% interest on the investment, find the net profit on the investment.



HOUSEHOLD PROBLEMS

Oral Work

1. If $\frac{3}{4}$ of a cup of corn could be cut from 3 ears of green corn, how many ears would be required to get 4 cups?
2. Fine hominy requires 4 parts water to 1 part hominy. How much water should be used with $\frac{1}{3}$ cup of fine hominy?
3. A recipe called for 1 part coarse hominy and 5 parts cold water. How much coarse hominy should be used with $2\frac{1}{2}$ cups of water?
4. Mrs. Dunn had a small farm on which she saved \$50 the first year. The second year, by better methods of farming, she saved \$100, and the third year, by making and selling jellies, she saved \$300. What per cent of the first year's savings was the second year's? the third year's? What per cent of the second year's savings was the third year's?
5. The cost for a cooking laboratory equipment in a public elementary school was \$1200. In taking the inventory at the close of the school year, 5% was allowed for general depreciation and \$11 for the purchase of new napery, towels, etc. What was the amount of the depreciation and the additional purchase?
6. A housewife bought from her grocer potatoes at 45¢ a peck, but received only 15 lb. for each peck. She used in a given time 300 lb. A neighbor using the same quantity of potatoes bought them from a farmer, paying \$1.20 for 60 lb. How much did she save on 300 lb. of potatoes?

7. If $\frac{3}{4}$ of a pound of flour will make one pound loaf of bread, how many loaves can be made from 1 bbl. of flour?

8. A domestic science teacher explained how good beef fat could be rendered and used for cooking, in place of butter, in many cases. There were 60 families represented in these cooking classes. In giving their monthly reports for home work, it was found that the community had used 100 lb. of the beef drippings for cooking purposes, instead of an equal amount of butter. Compute the cost of these beef drippings selling at 15 cents a pound, and the total saving, when butter is 35 cents a pound.

9. Mary was told to use $\frac{1}{4}$ cup tapioca, $\frac{1}{2}$ cup water, and 1 cup milk for porridge. With the same proportions, how much tapioca would she have to use with $2\frac{1}{2}$ cups of milk?

10. Four cents per capita for each lesson was the amount allowed for cost of maintenance in an elementary school kitchen laboratory. Four classes reported for work. The classes contained 20, 25, 15, and 10 pupils, respectively. These classes reported once a week for 36 weeks. How much did it cost to maintain the running expenses of the cooking classes?

Written Work

11. Mr. Adams keeps a poultry farm. He buys 100 lb. shelled corn at 91¢ per bushel; 200 lb. bran at 80¢ per hundred pounds; 300 lb. mixed chicken feed at 75¢ per hundred pounds. How much does the feed cost?

12. John sells four chickens weighing $2\frac{1}{2}$ lb., $2\frac{3}{4}$ lb., $3\frac{5}{8}$ lb., and 3 lb., at 15¢ per pound. Find the amount of the sales.

13. Mrs. Rodman went shopping with her daughter. She bought 1 muff \$40, less 30%; 1 suit \$25, less 25%; 1 coat \$20, less 30%; 1 fur neckpiece \$35, less 40%. Find the total amount of the bill.

14. Mrs. Ford buys for the family $8\frac{1}{2}$ yd. gingham at 12¢ a yard; 3 pairs girls' shoes at \$2.50 a pair; 30 yd. muslin at 7¢ a yard; 2 boys' shirts at 59¢ each; 3 pairs boys' shoes at \$3.25 a pair; and 2 hats at \$3 each. Find the amount of these purchases.

15. Fred's father tested his ability in solving some practical problems by asking him to find the cost of: $3\frac{1}{2}$ lb. meat at 22¢ a pound; $3\frac{3}{4}$ lb. fish at 16¢ a pound; 3900 lb. soft coal at \$3.20 a ton; 5000 cu. ft. artificial gas at \$1.00 a thousand cubic feet, less 10% for cash; 22,000 cu. ft. of natural gas at 30¢ a thousand cubic feet, less 10% for cash; 1 dining room set for \$80, less 25%. What was the correct answer?

16. If the reading of your gas meter on Jan. 10 was 105,680 cu. ft., and on Feb. 10, 119,780 cu. ft., write the check for the gas bill payable to your gas company at \$1.00 a thousand cubic feet, less 10% by paying Feb. 19.

17. A butcher buys a dressed lamb that weighs 30 lb. at 14¢ a pound. He estimates that 20% of the lamb will sell at 28¢ a pound; 30% at 24¢ a pound; 40% at 18¢ a pound; and the remainder at 15¢ a pound. How much profit does the man make on the lamb, if 2% of the total sales are uncollectible?

18. Mrs. James Anderson buys from her grocer, Monday, 1 doz. oranges for 35¢; 2 bunches lettuce for 5¢; 1 lb. coffee for 32¢; 6 bars soap for 25¢; and 2 cans tomatoes for 25¢. How much change should she receive from a \$5 bill?

19. Frank Smith's soft coal sales for the day show 1875 lb.; 1750 lb.; 1500 lb.; 6000 lb.; 5420 lb.; 3900 lb. Make out bills to the customers if the coal is sold at \$3.60 a ton, delivered.

20. Mary went shopping for her mother and bought $7\frac{3}{4}$ yd. lawn @ $12\frac{1}{2}\phi$; $\frac{3}{4}$ yd. ribbon @ \$1; $8\frac{1}{4}$ yd. linen @ 20ϕ ; 15 yd. muslin @ 8ϕ ; and $16\frac{1}{4}$ yd. gingham @ 15ϕ . Find the amount of her bill.

21. Mrs. James buys $10\frac{1}{4}$ lb. of butter at 30ϕ a pound; 5 bu. 3 pk. apples at 15ϕ a peck; $2\frac{1}{2}$ bu. apples at 20ϕ a peck; $\frac{1}{2}$ bu. chestnuts at 6ϕ a quart; and 4 doz. tomatoes at 20ϕ a dozen. Make out and receipt her bill for these purchases.

22. Mrs. Bell's meat bill for a week was as follows: $2\frac{1}{4}$ lb. rump steak @ 20ϕ ; $3\frac{1}{2}$ lb. chuck roast @ 15ϕ ; $3\frac{3}{4}$ lb. sirloin steak @ 28ϕ ; $\frac{3}{4}$ lb. lamb chops @ 28ϕ ; $3\frac{1}{4}$ lb. sausage @ 18ϕ ; $1\frac{1}{2}$ lb. ham @ 20ϕ . Find the total amount of her bill.

23. Mrs. Clark buys a dining room table for \$30, 20% off; a dining room rug for \$50, $33\frac{1}{3}\%$ off; a set of dining room chairs for \$24. Make out and receipt the bill from her local dealer.

24. Mr. Smith checks up his heat and light bill for October. It shows 1600 kilowatt hours electric current at 10ϕ a kilowatt hour less 10% for cash if paid before November 10; 2000 cu. ft. of natural gas at 27ϕ a thousand cubic feet, less 10% for cash if paid before November 10; and 6 T. 15 cwt. hard coal at \$6.25 a ton. Write the receipt your local company would give you in payment of each bill.

25. Mrs. Grant keeps an account of her butcher's checks for each month. The first week they run as follows:

$1\frac{1}{2}$ lb. lard @ 18ϕ	$2\frac{3}{4}$ lb. steak @ 28ϕ
$2\frac{1}{2}$ lb. lamb chops @ 24ϕ	$3\frac{1}{4}$ lb. fish @ 15ϕ
$2\frac{3}{4}$ lb. fish @ 16ϕ	$5\frac{1}{2}$ lb. chuck @ 16ϕ

Find her bill for the week.

26. The present state of Mr. Ford's gas meter is 11,380 cubic feet. The previous state was 10,280 cubic feet. How many cubic feet did he use and what was the cost at 80¢ a thousand cubic feet?

27. Mrs. Flint's bill for painting includes \$87.50 for the outside of the house, \$25 for the dining room, and \$30 each for two bedrooms. How much does she pay all together, with 2% discount for cash?

28. Mrs. Ford buys 10 bu. oats @ 55¢; $1\frac{1}{4}$ bu. apples @ \$1; 25 lb. cornmeal @ 4¢. Make the change if a \$20 bill is given in payment.

29. Mrs. Lane paid for house furnishings: 1 bedroom suite \$35; 1 dining room set \$55; 1 set dishes \$15; pans, kettles, etc., \$8; 1 cook stove \$23; other articles \$40. Make out the bill at 10% discount for cash.

30. Nine boys bring to school the checks for hard coal delivered to their parents as follows: 1 ton 5 cwt., 3 tons 90 lb., 2 tons 19 cwt., 1500 lb., 25 cwt., 3400 lb., 1 ton 1 cwt., 2 tons 3 cwt., and 4000 lb. Make out the sale checks if purchased from your local coal man at \$6.40 a ton.

31. Write the receipts a tax collector would give for the following township taxes: (a) a township tax of \$16.80, with 5% added for collection; (b) a county tax of \$23.90, with 10% added for collection; (c) a city tax of \$90 paid in July, with 1% added.

32. Write the receipt the North Shore Insurance Company would send for \$25 insurance on a house, barn, and contents for the years 1916 to 1919.

33. Paul Grant's mother gives him the money to pay the gas bill \$3.80, less 10%; the rent \$30, less 5%; \$50 taxes on a piece of property owned by her, less 2% discount. Write the receipts he should get for his mother.

CURRENT PRICES

LAST WEEK	THIS WEEK	LAST WEEK	THIS WEEK
Eggs 20 ¢	24 ¢	Peaches per bu. 80 ¢	75 ¢
Creamery butter 32 ¢	35 ¢	Apples per bu. \$1.20	\$1
Sirloin steak 25 ¢	27 ¢	Apples per pk. 35 ¢	30 ¢
Lamb chops 24 ¢	27 ¢	Cheese per lb. 25 ¢	20 ¢
Ham 20 ¢	22 ¢	Prunes per lb. 12½ ¢	15 ¢

34. Find the per cent of increase or decrease in the price on each article for the week.

35. The school tax rate in a certain borough is 4.5 mills and the borough tax rate 6.3 mills. The assessed valuation of Mr. Bird's property is \$3250, and there is a poll tax of \$1 for school purposes. Find Mr. Bird's total tax, with 5 % discount for prompt payment.

36. Mr. Gray's property is assessed at \$2300 and the tax rate is 2.3 mills. Write the receipt the tax collector will give Mr. Gray for payment at a discount of 5 %.

37. Susan, John, and Harry take advantage of January bargain sales. Susan buys a \$15 coat for \$10; John, an overcoat at \$15 less 25 %; Harry an \$18 suit for \$13.75. Find the per cent of discount each one gets.

38. Mr. Burton buys 50 cwt. of hard coal and 25 cwt. of soft coal. The hard coal costs \$6.25 a ton and the soft coal costs \$3.20 a ton. Write the dealer's receipt for the bill if a discount of 2 % for cash is given.

39. Mrs. Smith gets the August sale discounts in furniture and carpets: 1 library rug \$75, 25 % off; 1 reading table \$32, 16⅔ % off; 4 library chairs \$24, less 20 %; and a Roman reading chair \$12, less 33⅓ %. Make out the bill.

40. Mrs. Bronson's bill for the week from a department store was as follows: $12\frac{1}{2}$ yd. gingham @ 15ϕ ; 3 chairs @ \$2.25; a coat \$12, less 10%; 3 lb. coffee @ 32ϕ and 1 pair of slippers at \$2.75. Find the total bill.

41. A delivery boy has in his wagon for Mrs. Arnold, 3 pk. apples @ 30ϕ ; $3\frac{3}{4}$ lb. butter @ 30ϕ ; 3 cans tomatoes @ 15ϕ ; 5 bunches beets @ 5ϕ ; 6 lb. cornmeal @ 5ϕ ; and 15 lb. of beans @ 3ϕ . Find the total amount of her bill.

42. Find the amount of the following deliveries of soft coal at \$2.80 a ton: 15 cwt.; 2950 lb.; 1800 lb.; 3900 lb.; 1 T. 9 cwt.; 6 cwt.; 950 lb.; 1400 lb.; 6000 lb.; 5450 lb.; 2950 lb.

43. The cost of clothing for Mr. Orr's family for three months is as follows: dressmaking 8 days at \$2.00 a day; 1 overcoat \$50, less 25%; 1 suit for \$30, less 30%; 15 yd. dress goods at $87\frac{1}{2}\phi$ a yard; miscellaneous items for dressmaker \$4.75; 1 hat \$12, less 30%. Find the total cost.

44. Mrs. Burke receives a bill of \$1500 for furnishing a house. She will be allowed a discount of 5% for cash, or $2\frac{1}{2}\%$ if she pays in one month. How much will she save by paying cash? by paying in one month?

45. Mrs. Clark who had been paying \$50 a month for her house bought it for \$6000. Taxes, insurance, and repairs cost her \$120 a year and interest 6% on the investment. How much a year did she save by purchasing the house?

46. Mr. Burt leases a house that cost \$4700 for \$40 a month. If he pays on an average \$198 a year for insurance, taxes, and repairs, what per cent of the investment does he receive on the property?

TESTS FOR ACCURACY AND SPEED

ADDITION — INTEGERS AND DECIMALS

Oral Work

Add these numbers, timing yourself. Observe the groups that make 5, 10, 15, or 20:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
4	5	8	5	7	7	7	5	5	8
3	5	7	4	3	3	3	6	5	2
3	4	9	1	5	8	9	4	4	7
4	3	6	9	8	2	1	9	6	3
6	3	6	8	2	9	6	6	9	6
3	7	9	2	6	1	4	7	1	4
7	3	8	7	4	5	9	2	8	5
5	8	5	3	6	9	1	8	2	5
4	2	2	6	3	1	3	7	6	9
1	6	7	4	1	5	7	3	4	1
8	4	7	5	9	8	8	9	8	7
2	5	1	3	1	7	2	1	2	3
7	4	4	2	7	9	7	7	3	5
3	1	9	7	7	6	3	8	7	5
8	9	2	2	1	5	4	6	5	8
1	1	6	1	9	6	6	5	6	1
1	5	7	6	4	8	7	4	4	1
6	4	2	9	7	1	8	8	1	9
—	—	—	—	—	—	—	—	—	—

Written Work

Add and test. Time yourself; then work the examples again, trying to better your record.

1.	2.	3.	4.	5.	6.	7.
45	48	56	458	\$56.01	\$50.01	\$109.76
34	36	42	19	89.88	48.09	56.99
16	42	48	549	23.00	6.10	7.08
15	14	65	119	10.01	5.98	.89
18	12	88	45	8.08	.87	5.90
29	10	19	98	7.89	1.05	403.98
23	11	11	400	.98	.67	5.01
56	18	78	19	.09	.45	6.00
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
8.	9.	10.	11.	12.	13.	14.
56	87	16	546	\$51.83	\$45.10	\$190.01
78	98	27	890	9.10	8.10	490.00
91	19	83	78	14.09	4.19	28.98
10	12	26	980	.98	.98	.76
98	64	78	45	.48	.92	4.67
70	76	89	10	5.89	.87	.78
45	44	65	780	45.09	34.01	211.23
68	32	45	98	.87	9.11	.45
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
15.	16.	17.	18.	19.	20.	21.
56	18	56	450	\$10.00	\$50.71	\$178.10
54	28	78	890	19.90	45.09	45.70
32	38	19	98	87.90	5.90	5.01
21	89	12	560	6.00	.75	251.89
19	91	32	78	.98	.09	6.78
18	10	67	671	14.50	6.90	15.79
56	11	14	679	.76	16.90	.10
56	15	70	87	7.80	6.90	2.50
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

Instead of decimal points, bookkeepers often use lines to separate dollars and cents. Add and test, timing yourself:

22.	23.	24.	25.
\$6,567 54	\$5,876 00	\$5,987 00	\$4,867 99
456 76	4,876 01	569 01	695 10
45 90	265 09	45 89	56 90
9 76	7 09	76	248 90
98	98	2 50	6 90
610 01	534 28	4,567 00	87
6 90	34 89	98	4,589 01
78	4 00	24 56	5 09
4,567 90	87	10 09	578 00
26.	27.	28.	29.
\$2,004.56	\$5,678.90	.05	.005
.95	428.01	50.5	40.5
6.25	.91	.003	6.645
568.01	4.20	4.65	.008
.09	598.00	7.10	.9
24.91	24.25	.009	.19
423.29	.98	.9	40.5
.45	1.25	.41	.6
30.	31.	32.	33.
\$5,678.00	\$9,800.00	.008	98.567
265.21	90.00	4.5	1.09
24.50	.09	40.789	.87
.98	1.98	.002	24.9
1.50	.78	4.10	1.003
.78	24.78	.98	45.9
275.25	324.67	46.06	.005
25.90	56.09	1.98	1.780

Write the sum of each column separately on a slip of paper; then add the separate sums as illustrated in example 34.

34.	35.	36.	37.	38.	39.
881	248	467	814	591	737
769	217	178	419	672	642
435	846	679	897	723	229
821	215	146	926	880	312
989	724	715	463	993	414
706	373	474	417	726	514
<u>31</u>	212	833	318	628	819
22	219	302	502	584	719
43	318	768	831	427	991
<u>4551</u>					

Add these problems and test the work:

40.	41.	42.	43.	44.	45.
8709	7209	9545	7889	5156	9685
6293	6048	2615	4274	7367	5044
4375	2597	8505	8916	978	7956
5885	7536	4066	307	8789	4747
9746	7585	8344	7933	8395	7588
3968	3264	4989	4065	543	4717
2276	7343	9653	8725	8724	7507
8888	5832	4398	4584	3462	5794
7783	5861	8235	2954	8578	6687
6977	7342	9845	3768	4602	5798
5206	7253	308	7947	5402	8687
2873	7504	9847	3848	8313	4323
7848	4385	7494	5896	9411	2808
<u>5674</u>	<u>65</u>	<u>3086</u>	<u>4059</u>	<u>8109</u>	<u>6832</u>

SUBTRACTION — INTEGERS AND DECIMALS**Oral Work**

Business men make change by the **adding method**. Thus, if a purchase is made for \$1.69, and \$2.00 is given in payment, the clerk will probably say: "One dollar sixty-nine cents, seventy cents, seventy-five cents, two dollars," each time laying down the piece of money that makes the sum named.

Acting as clerk when the following purchases and payments are made, give the exact language you might use, if the purchaser were actually present to receive the change:

COST OF PURCHASE	AMOUNT GIVEN	COST OF PURCHASE	AMOUNT GIVEN
1. \$1.85	\$2.00	16. \$1.85	\$5.00
2. 2.96	3.00	17. 7.24	10.00
3. 2.15	5.00	18. 3.77	5.00
4. 3.18	5.00	19. 2.86	4.00
5. 1.14	1.50	20. 5.09	6.00
6. 4.15	5.00	21. .95	2.00
7. 2.25	5.00	22. .74	2.00
8. 3.56	3.75	23. 6.05	10.00
9. 1.18	2.00	24. 7.08	8.00
10. 4.07	5.00	25. 3.98	10.00
11. 2.89	4.00	26. 2.09	5.00
12. 2.46	3.00	27. 6.69	10.00
13. 3.09	4.00	28. 7.08	7.50
14. 2.61	4.00	29. 8.34	10.00
15. 3.08	4.00	30. 2.99	5.00

Written Work

Subtract five problems in 1 minute. Test your work.
Practice in order to beat your own record:

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
1.	400566 <u>298578</u>	500632 <u>298783</u>	464898 <u>398927</u>	\$3009.85 <u>2783.72</u>	39355.07 <u>590.075</u>
2.	203001 <u>19785</u>	300102 <u>198456</u>	100102 <u>98986</u>	\$1010.31 <u>983.67</u>	40327.03 <u>5998.07</u>
3.	100003 <u>99998</u>	109203 <u>56789</u>	301422 <u>298563</u>	\$9097.60 <u>7628.73</u>	67800.7 <u>59723.08</u>
4.	720023 <u>545637</u>	430023 <u>287645</u>	500103 <u>298346</u>	\$1023.02 <u>984.57</u>	2100.02 <u>1768.075</u>
5.	500678 <u>375989</u>	200102 <u>145678</u>	100903 <u>94536</u>	\$6000.07 <u>4875.69</u>	20035.8 <u>17648.08</u>
6.	304201 <u>246879</u>	506102 <u>398469</u>	456720 <u>258937</u>	\$4501.03 <u>2789.36</u>	2190.075 <u>1567.086</u>
7.	246702 <u>176496</u>	450021 <u>387346</u>	560071 <u>356895</u>	\$3001.08 <u>2453.89</u>	23742.07 <u>19867.047</u>
8.	310432 <u>287467</u>	451002 <u>367894</u>	450600 <u>359618</u>	\$1000.02 <u>986.56</u>	5678.02 <u>4872.76</u>
9.	450612 <u>275239</u>	416701 <u>98878</u>	400210 <u>376897</u>	\$2003.45 <u>1463.84</u>	4123.24 <u>2754.074</u>
10.	450061 <u>378972</u>	142867 <u>137958</u>	340013 <u>278945</u>	\$4005.34 <u>2784.67</u>	3210.1 <u>2986.61</u>

MULTIPLICATION — INTEGERS AND DECIMALS**Oral Work**

Multiply each of the following numbers first by 4 ; then by 3 ; by 5 ; by 6 ; by 7 ; by 8 ; by 9 ; by 10 ; by 11 ; by 12 ; by 20 ; by 50 ; by 100.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
1.	15	10	44	34	54	49
2.	49	21	60	29	34	90
3.	38	30	80	64	73	70

Written Work

Solve the following problems, selecting the most economical method of solution :

- | | | |
|--------------------------|-------------------------------------|---|
| 1. $21 \times \$25.75$ | 16. $.16\frac{2}{3} \times \$600$ | 31. $11 \times \$480$ |
| 2. $62 \times \$5.92$ | 17. $.25 \times \$800$ | 32. $29 \times \$300$ |
| 3. $99 \times \$2400$ | 18. $101 \times \$6.50$ | 33. $.002 \times \$850.50$ |
| 4. $101 \times \$1500$ | 19. $1.75 \times \$280$ | 34. $2.025 \times \$550$ |
| 5. $51 \times \$500$ | 20. $1002 \times \$7.50$ | 35. $49 \times \$684.75$ |
| 6. $26 \times \$300$ | 21. $25 \times \$75$ | 36. $.25 \times \$400.20$ |
| 7. $54 \times \$5000$ | 22. $105 \times \$3.25$ | 37. 1.02×425.5 |
| 8. $109 \times \$200$ | 23. $90 \times \$350$ | 38. $37\frac{1}{2} \times \$240$ |
| 9. $26 \times \$50.25$ | 24. $1.01 \times \$10.90$ | 39. $.62\frac{1}{2} \times 96.60$ |
| 10. $25 \times \$25.50$ | 25. $2.5 \times \$60.50$ | 40. $.12\frac{1}{2} \times 12\frac{1}{2}$ |
| 11. $11 \times \$4000$ | 26. $.10 \times \$43.75$ | 41. $.16\frac{2}{3} \times 180$ |
| 12. $14 \times \$1000$ | 27. $.75 \times \$30$ | 42. $.16\frac{2}{3} \times 16\frac{2}{3}$ |
| 13. $53 \times \$75.50$ | 28. $.33\frac{1}{3} \times \$25.50$ | 43. $.66\frac{2}{3} \times 66$ |
| 14. $21 \times \$50$ | 29. $5.5 \times \$5.75$ | 44. $.11\frac{1}{3} \times 87$ |
| 15. $99 \times \$581.50$ | 30. $1.25 \times \$560$ | 45. $.37\frac{1}{2} \times 40.72$ |

- | | | |
|------------------------------------|---------------------------|------------------------------------|
| 46. $125 \times \$640$ | 56. $12.5 \times \$750$ | 66. $.08\frac{1}{2} \times \$9600$ |
| 47. $175 \times \$600$ | 57. $.125 \times \$240$ | 67. $.8\frac{1}{2} \times 9000$ |
| 48. $1.8 \times \$2000$ | 58. $.125 \times \$2800$ | 68. $.06\frac{1}{4} \times 2400$ |
| 49. $99 \times \$75.50$ | 59. $124 \times \$35.75$ | 69. $.6\frac{1}{4} \times 2400$ |
| 50. $1001 \times \$50$ | 60. $1.8 \times \$50$ | 70. $.12\frac{1}{2} \times 7200$ |
| 51. $.12\frac{1}{2} \times \$2400$ | 61. $21.25 \times \$3.25$ | 71. $.03\frac{1}{2} \times 3300$ |
| 52. $.33\frac{1}{3} \times \$60$ | 62. $109.5 \times \$5.50$ | 72. $.3\frac{1}{2} \times 3300$ |
| 53. $33\frac{1}{3} \times \$48$ | 63. $125 \times \$360$ | 73. $10.5 \times \$62.30$ |
| 54. $1007 \times \$7.50$ | 64. $107 \times \$500$ | 74. 9.5×9500 |
| 55. $99 \times \$325.50$ | 65. $9 \times \$680$ | 75. 7.5×124 |

Write only the products and find the total of each bill, timing your work:

76. Aaron Ward and Sons sold G. H. Graham:

June 1, 1917, 3 doz. bananas	@	\$.15
June 4, 1917, 5 doz. eggs	@	.25
June 18, 1917, 8 bu. apples	@	.95
June 20, 1917, 25 lb. sugar	@	.07
June 25, 1917, 5 gal. molasses	@	.50
June 28, 1917, 15 lb. butter	@	.40
June 30, 1917, 6 bu. apples	@	1.00

77. L. S. Platt and Company sold to J. T. Franklin:

April 3, 1917, 10 yd. muslin	@	\$.10
April 7, 1917, 15 yd. velvet	@	1.50
April 9, 1917, 7 yd. linen suiting	@	.50
April 11, 1917, 2 ladies' suits	@	25.00
April 16, 1917, 5 yd. silk	@	1.00
April 18, 1917, 8 yd. dress trimming	@	.75
April 21, 1917, 10 yd. satin ribbon	@	.35

78. Packer, Bender, and Willard Company sold to F. T. Phillips:

Jan. 3, 1918, 3 bbl. apples	@	\$ 2.50
Jan. 7, 1918, 3 doz. lemons	@	.35
Jan. 9, 1918, 10 lb. butter	@	.45
Jan. 11, 1918, 8 lb. cheese	@	.20
Jan. 25, 1918, 3 boxes soap	@	3.50

79. Simon Judson and Sons sold to H. E. Morton:

Feb. 11, 1918, 3 bu. peaches	@	\$ 1.50
Feb. 12, 1918, 2 bu. blackberries	@	2.00
Feb. 20, 1918, 15 qt. strawberries	@	.10
Feb. 23, 1918, 5 doz. oranges	@	.25
Feb. 25, 1918, 8 lb. tea	@	.60
Feb. 26, 1918, 5 lb. prunes	@	.12

80. S. P. Durham and Co. sold to John T. Robinson:

Aug. 1, 1917, 20 yd. gingham	@	\$.15
Aug. 3, 1917, 25 yd. voile	@	1.25
Aug. 6, 1917, 10 yd. cambric	@	.25
Aug. 9, 1917, 8 yd. lawn	@	.35
Aug. 11, 1917, 7 yd. lace	@	.20
Aug. 14, 1917, 3 pieces ribbon	@	.18
Aug. 18, 1917, 6 yd. lawn	@	.18

81. The Cutler Company sold to J. H. Brown:

Nov. 23, 1917, 3 lb. lamb	@	\$.25
Nov. 26, 1917, 1 ham	@	2.25
Nov. 27, 1917, 4 lb. veal roast	@	.25
Nov. 29, 1917, 8 lb. fish	@	.18
Nov. 30, 1917, 5 lb. boiled ham	@	.35

82. J. L. Simon & Co. sold to R. J. Rhodes :

Sept. 11, 1917, 5 lb. best creamery butter @	\$.40
Sept. 15, 1917, 10 lb. rice @	.10
Sept. 18, 1917, 12 doz. ears corn @	.35
Sept. 21, 1917, 3 bunches carrots @	.07
Sept. 23, 1917, 5 cans peaches @	.20

83. T. L. Thomas sold to R. J. Campbell :

March 2, 1918, 3 lb. coffee @	\$.25
March 4, 1918, 5 heads cabbage @	.08
March 9, 1918, 7 cans peaches @	.15
March 11, 1918, 4 pk. pears @	.20
March 15, 1918, 4 cans corn @	.10
March 18, 1918, 6 boxes strawberries @	.10
March 23, 1918, 2 pk. apples @	.20
March 29, 1918, 4 lb. tea @	.60

DIVISION—INTEGERS AND DECIMALS**Oral Work**

Divide the following numbers, first by 3; then by 4; then by 5; 6; 7; 8; 9; 10; 11; 12; 20.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
1.	61	45	72	3.42	50	\$8.75
2.	45	30	84	6.08	40	\$3.21
3.	32	53	92	7.00	30	\$1.00

Written Work

Divide and test:

- | | | |
|-----------------|-----------------|-----------------|
| 1. 93460 by 809 | 4. 68890 by 508 | 7. 46594 by 757 |
| 2. 88578 by 835 | 5. 77089 by 247 | 8. 37483 by 907 |
| 3. 72809 by 405 | 6. 37089 by 360 | 9. 37498 by 445 |

- | | | |
|---------------------|---|----------------------|
| 10. 47084 by 607 | 20. 74837 by 987 | 30. 12730 by 458 |
| 11. 47083 by 734 | 21. 74895 by 786 | 31. 26301 by 657 |
| 12. 37809 by 640 | 22. 84958 by 234 | 32. 29801 by 478 |
| 13. 74809 by 490 | 23. 99363 by 945 | 33. 30009 by 246 |
| 14. 74633 by 487 | 24. 46487 by 467 | 34. 27003 by 756 |
| 15. 64839 by 357 | 25. 74849 by 945 | 35. 37483 by 882 |
| 16. 74783 by 678 | 26. 80032 by 587 | 36. 36733 by 956 |
| 17. 74893 by 359 | 27. 21389 by 903 | 37. 28476 by 767 |
| 18. 74843 by 598 | 28. 12730 by 846 | 38. 37003 by 420 |
| 19. 74837 by 456 | 29. 35301 by 649 | 39. 37003 by 407 |
| 40. 464341 | <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">by</div> <div style="border-left: 1px solid black; padding-left: 10px;"> <p>a. 169 Form 100 problems by di-</p> <p>b. 254 viding each dividend by each</p> <p>c. 376 of the divisors, thus:</p> <p>d. 458 40 a. $464341 \div 169 = ?$</p> <p>e. 365 40 b. $464341 \div 254 = ?$</p> <p>f. 484 45 c. $784100 \div 376 = ?$</p> <p>g. 989 Write, solve, and test each</p> <p>h. 397 problem in 2 minutes.</p> <p>i. 209</p> <p>j. 190</p> </div> </div> | |
| 41. 846760 | | |
| 42. 864548 | | |
| 43. 645341 | | |
| 44. 624872 | | |
| 45. 784100 | | |
| 46. 810404 | | |
| 47. 904025 | | |
| 48. 867045 | | |
| 49. 234567 | | |
| 50. $12 \div .12$ | 58. $.008 \div 800$ | 66. $.125 \div .025$ |
| 51. $10 \div .10$ | 59. $25 \div 250$ | 67. $90 \div .045$ |
| 52. $.8 \div .08$ | 60. $4.5 \div 1500$ | 68. $.84 \div 21$ |
| 53. $8 \div .008$ | 61. $125 \div .25$ | 69. $264 \div 8.4$ |
| 54. $.025 \div .5$ | 62. $7500 \div .125$ | 70. $84 \div .21$ |
| 55. $70 \div .7 =$ | 63. $.75 \div 250$ | 71. $840 \div .21$ |
| 56. $900 \div .003$ | 64. $950 \div 5000$ | 72. $840 \div 2.1$ |
| 57. $200 \div 2000$ | 65. $7500 \div .0125$ | 73. $.84 \div .021$ |

Find the quotients:

- | | | |
|---------------------|---------------------|-----------------------|
| 74. $84 \div .021$ | 85. $.84 \div .042$ | 96. $1.8 \div 30$ |
| 75. $8.40 \div 210$ | 86. $84 \div 420$ | 97. $45 \div 150$ |
| 76. $840 \div .004$ | 87. $18 \div .6$ | 98. $450 \div .15$ |
| 77. $24 \div .12$ | 88. $180 \div .06$ | 99. $.45 \div 150$ |
| 78. $240 \div .12$ | 89. $180 \div 600$ | 100. $4500 \div 1500$ |
| 79. $24 \div .012$ | 90. $180 \div .60$ | 101. $.045 \div 1.5$ |
| 80. $2.4 \div .12$ | 91. $1800 \div .06$ | 102. $.0045 \div 150$ |
| 81. $240 \div .012$ | 92. $18 \div 600$ | 103. $.045 \div 150$ |
| 82. $240 \div 1.20$ | 93. $18 \div .06$ | 104. $.0045 \div 1.5$ |
| 83. $24 \div 120$ | 94. $18 \div .006$ | 105. $.045 \div .015$ |
| 84. $24 \div .024$ | 95. $18 \div .003$ | 106. $.045 \div 1.5$ |

Find the quotients:

- | | | |
|---------------------|---------------------|----------------------|
| 107. $1 \div .1$ | 115. $3 \div .03$ | 123. $.18 \div 72$ |
| 108. $.1 \div 10$ | 116. $30 \div .3$ | 124. $.04 \div 50$ |
| 109. $25 \div 50$ | 117. $6 \div .006$ | 125. $2 \div .025$ |
| 110. $2.5 \div .5$ | 118. $16 \div .04$ | 126. $20 \div .002$ |
| 111. $25 \div 2.5$ | 119. $60 \div 300$ | 127. $200 \div 12.5$ |
| 112. $.15 \div .15$ | 120. $.6 \div 30$ | 128. $64 \div .016$ |
| 113. $1.5 \div 15$ | 121. $.66 \div 1.1$ | 129. $64 \div 160$ |
| 114. $.15 \div 2.5$ | 122. $.9 \div .009$ | 130. $.4 \div 400$ |

COMMON FRACTIONS

Oral and Written Work

Find the sum of:

- | | |
|---|--|
| 1. $5 + \frac{3}{4} + \frac{1}{2} + 1 + \frac{4}{5}$ | 4. $\frac{5}{6} + \frac{1}{2} + \frac{1}{4} + \frac{5}{12} + 1\frac{1}{2}$ |
| 2. $\frac{1}{3} + \frac{1}{4} + 1 + \frac{3}{4} + \frac{7}{12}$ | 5. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{9}{16} + \frac{1}{2}$ |
| 3. $\frac{1}{2} + .2 + .25 + \frac{3}{4} + \frac{9}{10}$ | 6. $\frac{2}{3} + \frac{3}{4} + \frac{7}{8} + \frac{1}{2} + 1\frac{3}{4}$ |

Add :

- | | | |
|---|--|---|
| 7. $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}$ | 11. $\frac{2}{3}, \frac{5}{9}, \frac{5}{18}$ | 15. $\frac{2}{3}, \frac{5}{8}, \frac{11}{24}$ |
| 8. $\frac{5}{12}, \frac{1}{2}, \frac{5}{8}$ | 12. $\frac{5}{12}, \frac{2}{24}, \frac{7}{24}$ | 16. $\frac{1}{2}, \frac{11}{16}, \frac{5}{8}$ |
| 9. $\frac{5}{16}, \frac{3}{4}, \frac{7}{8}$ | 13. $\frac{3}{4}, \frac{7}{12}, \frac{5}{12}$ | 17. $\frac{2}{3}, \frac{5}{9}, \frac{11}{18}$ |
| 10. $\frac{2}{5}, \frac{2}{10}, \frac{7}{10}$ | 14. $\frac{6}{7}, \frac{2}{7}, \frac{11}{21}$ | 18. $\frac{1}{3}, \frac{5}{8}, \frac{23}{24}$ |

Give sums orally :

- | | | | |
|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 19. $\frac{1}{3} + \frac{1}{7}$ | 22. $\frac{2}{3} + \frac{4}{5}$ | 25. $\frac{4}{9} + \frac{7}{12}$ | 28. $\frac{5}{9} + \frac{1}{8}$ |
| 20. $\frac{2}{5} + \frac{1}{9}$ | 23. $\frac{1}{18} + \frac{2}{8}$ | 26. $\frac{1}{3} + \frac{3}{4}$ | 29. $\frac{5}{24} + \frac{2}{3}$ |
| 21. $\frac{1}{6} + \frac{1}{8}$ | 24. $\frac{3}{4} + \frac{4}{5}$ | 27. $\frac{1}{2} + \frac{7}{12}$ | 30. $\frac{3}{4} + \frac{7}{12}$ |

Find differences orally :

- | | | | |
|-----------------------------------|----------------------------------|---------------------------------|----------------------------------|
| 31. $\frac{1}{3} - \frac{1}{8}$ | 34. $\frac{1}{2} - \frac{1}{12}$ | 37. $\frac{2}{5} - \frac{2}{8}$ | 40. $\frac{5}{6} - \frac{5}{8}$ |
| 32. $\frac{1}{5} - \frac{1}{6}$ | 35. $\frac{5}{9} - \frac{1}{3}$ | 38. $\frac{3}{4} - \frac{3}{5}$ | 41. $\frac{5}{9} - \frac{1}{4}$ |
| 33. $1\frac{1}{5} - \frac{1}{10}$ | 36. $\frac{7}{8} - \frac{2}{4}$ | 39. $\frac{5}{6} - \frac{4}{5}$ | 42. $1\frac{5}{6} - \frac{2}{4}$ |

Find the value of :

- | | |
|---|---|
| 43. $6\frac{1}{2}$ yd. @ $5\frac{1}{2}$ ¢ | 47. $18\frac{1}{2}$ yd. @ $16\frac{3}{4}$ ¢ |
| 44. $11\frac{3}{4}$ lb. @ $11\frac{3}{4}$ ¢ | 48. $12\frac{1}{2}$ oz. @ $3\frac{1}{2}$ ¢ |
| 45. $12\frac{1}{2}$ lb. @ $8\frac{1}{2}$ ¢ | 49. $20\frac{1}{2}$ lb. @ $9\frac{1}{2}$ ¢ |
| 46. $20\frac{3}{4}$ yd. @ $12\frac{1}{2}$ ¢ | 50. $19\frac{3}{4}$ lb. @ $6\frac{1}{2}$ ¢ |

Find products :

- | | | |
|--|--|--|
| 51. $124\frac{3}{4} \times 3\frac{1}{2}$ | 55. $75.9 \times 50\frac{9}{10}$ | 59. $24\frac{3}{4} \times 27\frac{1}{2}$ |
| 52. $150\frac{3}{4} \times 27\frac{3}{4}$ | 56. $200\frac{1}{2} \times 30\frac{1}{2}$ | 60. $38\frac{3}{8} \times 22\frac{1}{2}$ |
| 53. $19\frac{1}{2} \times 20\frac{3}{8}$ | 57. $29\frac{7}{12} \times 9\frac{1}{2}$ | 61. $25\frac{3}{4} \times 27\frac{1}{2}$ |
| 54. $19\frac{3}{8} \times 120\frac{4}{5}$ | 58. $95\frac{1}{2} \times 15\frac{3}{4}$ | 62. $16\frac{1}{4} \times 23\frac{1}{2}$ |
| 63. $6\frac{2}{3} \times 2\frac{3}{11} \times 7\frac{8}{9}$ | 69. $35\frac{1}{16} \times 22\frac{1}{32} \times 12$ | |
| 64. $8\frac{7}{10} \times 2\frac{7}{5} \times 2\frac{3}{8}$ | 70. $51\frac{2}{7} \times 19\frac{9}{15} \times 12\frac{1}{8}$ | |
| 65. $10\frac{1}{2} \times 1\frac{1}{3} \times 7\frac{3}{4}$ | 71. $6\frac{2}{7} \times 5\frac{4}{5} \times \frac{7}{8} \times 2\frac{3}{10}$ | |
| 66. $14\frac{1}{7} \times 17\frac{8}{9} \times 8\frac{2}{5}$ | 72. $20\frac{2}{3} \times 20\frac{4}{7} \times 20\frac{11}{16} \times 10$ | |
| 67. $27\frac{15}{16} \times 42\frac{3}{8} \times 6\frac{1}{4}$ | 73. $7\frac{3}{2} \times 15 \times 2\frac{19}{56} \times 12\frac{9}{17}$ | |
| 68. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{5}{6} \times 5\frac{5}{8}$ | 74. $172\frac{1}{2} \times 2\frac{7}{5} \times 3\frac{7}{8}$ | |

Find quotients:

- | | | | |
|--|------------------------------------|--------------------------------------|--------------------------------------|
| 75. $3 \div \frac{1}{6}$ | 84. $9 \div \frac{3}{4}$ | 93. $\frac{3}{4} \div \frac{3}{8}$ | 102. $\frac{7}{8} \div \frac{7}{10}$ |
| 76. $4 \div \frac{1}{4}$ | 85. $12 \div \frac{5}{6}$ | 94. $\frac{7}{8} \div \frac{1}{6}$ | 103. $\frac{3}{4} \div \frac{1}{20}$ |
| 77. $5 \div \frac{1}{7}$ | 86. $12 \div \frac{5}{9}$ | 95. $\frac{5}{8} \div \frac{3}{5}$ | 104. $\frac{7}{8} \div \frac{3}{8}$ |
| 78. $6 \div \frac{1}{8}$ | 87. $12 \div \frac{3}{5}$ | 96. $\frac{1}{12} \div \frac{1}{6}$ | 105. $\frac{1}{16} \div \frac{1}{4}$ |
| 79. $6 \div \frac{1}{3}$ | 88. $15 \div \frac{3}{4}$ | 97. $\frac{5}{6} \div \frac{7}{10}$ | 106. $\frac{3}{4} \div \frac{1}{4}$ |
| 80. $6 \div \frac{2}{3}$ | 89. $16 \div \frac{7}{8}$ | 98. $\frac{2}{3} \div \frac{5}{21}$ | 107. $\frac{7}{8} \div \frac{2}{3}$ |
| 81. $8 \div \frac{1}{4}$ | 90. $20 \div \frac{1}{6}$ | 99. $\frac{3}{8} \div \frac{9}{16}$ | 108. $\frac{7}{8} \div \frac{3}{8}$ |
| 82. $8 \div \frac{1}{5}$ | 91. $\frac{2}{3} \div \frac{1}{5}$ | 100. $\frac{3}{4} \div \frac{5}{14}$ | 109. $\frac{8}{9} \div \frac{3}{8}$ |
| 83. $9 \div \frac{2}{3}$ | 92. $\frac{1}{8} \div \frac{3}{4}$ | 101. $\frac{9}{10} \div \frac{5}{8}$ | 110. $\frac{1}{18} \div \frac{1}{4}$ |
| 111. $8 \div 2\frac{3}{8}$ | 128. $135\frac{3}{8} \div 8$ | 145. $5619 \div 35\frac{1}{2}$ | |
| 112. $12 \div 2\frac{1}{6}$ | 129. $145\frac{1}{2} \div 8$ | 146. $3648 \div 40\frac{3}{8}$ | |
| 113. $15 \div 3\frac{3}{4}$ | 130. $160\frac{3}{8} \div 6$ | 147. $3648 \div 34\frac{5}{8}$ | |
| 114. $20 \div 4\frac{3}{8}$ | 131. $250\frac{3}{8} \div 4$ | 148. $6475 \div 45\frac{3}{4}$ | |
| 115. $8\frac{1}{2} \div \frac{1}{2}$ | 132. $450\frac{3}{8} \div 15$ | 149. $4648 \div 56\frac{5}{8}$ | |
| 116. $15\frac{1}{2} \div 2\frac{3}{8}$ | 133. $235\frac{1}{2} \div 20$ | 150. $7289 \div 45\frac{5}{8}$ | |
| 117. $20\frac{3}{4} \div 5\frac{1}{2}$ | 134. $320\frac{3}{8} \div 18$ | 151. $7485 \div 78\frac{3}{8}$ | |
| 118. $10\frac{3}{4} \div 2\frac{3}{8}$ | 135. $425\frac{3}{8} \div 24$ | 152. $5648 \div 72\frac{3}{8}$ | |
| 119. $15\frac{1}{2} \div 2\frac{1}{4}$ | 136. $345\frac{1}{2} \div 30$ | 153. $4649 \div 58\frac{5}{8}$ | |
| 120. $35\frac{3}{8} \div 4\frac{3}{8}$ | 137. $280\frac{3}{8} \div 35$ | 154. $3739 \div 65\frac{7}{12}$ | |
| 121. $27\frac{3}{8} \div 5\frac{3}{8}$ | 138. $3140\frac{3}{8} \div 45$ | 155. $7845 \div 23\frac{5}{8}$ | |
| 122. $20\frac{1}{10} \div 1\frac{1}{10}$ | 139. $2345\frac{1}{2} \div 50$ | 156. $3648 \div 50\frac{3}{8}$ | |
| 123. $17\frac{3}{8} \div 4$ | 140. $4580\frac{3}{8} \div 56$ | 157. $4679 \div 48\frac{3}{16}$ | |
| 124. $18\frac{3}{8} \div 4$ | 141. $3245 \div 26\frac{3}{8}$ | 158. $4648 \div 12\frac{5}{16}$ | |
| 125. $15\frac{3}{8} \div 5$ | 142. $2568 \div 32\frac{1}{2}$ | 159. $7894 \div 10\frac{3}{10}$ | |
| 126. $180\frac{3}{8} \div 7$ | 143. $4531 \div 40\frac{3}{8}$ | 160. $3748 \div 15\frac{3}{8}$ | |
| 127. $250\frac{3}{8} \div 9$ | 144. $4651 \div 27\frac{3}{4}$ | 161. $8976 \div 48\frac{5}{12}$ | |

Change to decimals and test:

162. $\frac{5}{8}$	164. $\frac{5}{12}$	166. $\frac{7}{8}$	168. $\frac{9}{16}$	170. $\frac{7}{16}$
163. $\frac{2}{5}$	165. $\frac{5}{16}$	167. $\frac{11}{25}$	169. $\frac{19}{32}$	171. $\frac{17}{25}$

Change to decimals of not more than four places:

172. $\frac{7}{11}$	176. $\frac{17}{27}$	180. $\frac{35}{27}$	184. $\frac{1}{60}$
173. $\frac{9}{31}$	177. $\frac{9}{16}$	181. $\frac{37}{49}$	185. $\frac{6}{18}$
174. $\frac{45}{78}$	178. $\frac{11}{7}$	182. $\frac{7}{18}$	186. $\frac{27}{17}$
175. $\frac{29}{58}$	179. $\frac{25}{17}$	183. $\frac{5}{11}$	187. $\frac{13}{24}$

Change to common fractions:

188. .75	197. .375	206. .55	215. .45
189. $.87\frac{1}{2}$	198. .875	207. .28	216. .38
190. .64	199. .625	208. .68	217. .54
191. $.33\frac{1}{3}$	200. .800	209. .44	218. .65
192. .95	201. .500	210. .32	219. .85
193. $.62\frac{1}{2}$	202. .400	211. .58	220. .96
194. $.83\frac{1}{3}$	203. .750	212. .72	221. .24
195. $.66\frac{2}{3}$	204. .250	213. .92	222. .48
196. .84	205. .125	214. .36	223. .62

Write as per cents:

224. .039	230. .012	236. .705	242. $1.01\frac{1}{2}$
225. $.16\frac{2}{3}$	231. $.001\frac{1}{4}$	237. .905	243. 1.025
226. .20	232. .5	238. $2\frac{7}{8}$	244. 1.012
227. 1.	233. $.08\frac{1}{3}$	239. .0025	245. 1.0058
228. 2.5	234. .30	240. 1.09	246. 1.75
229. .002	235. .025	241. 1.25	247. 2.045

SCHOOL SHOP WORK AND HOUSEHOLD PROBLEMS

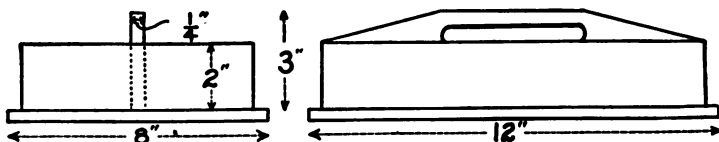
TO THE TEACHER.—The problems under these topics are simply suggestive. In every school in which any of these activities forms a part of the course, the problems should be secured anew each term, the actual work in the subjects furnishing the material for the problems.

1. A bookcase has three shelves 18" long and 8" wide. The two sides are 25" long and 9" wide. The wood used is chestnut, $\frac{5}{8}$ " thick, costing 7¢ a board foot. How much will the material for the case cost, allowing 1 foot for waste?

NOTE.—A board foot is a square foot 1 in. or less thick. Thus, a board 10 ft. long, 10 in. wide, and 3 in. thick contains $10 \times \frac{10}{12} \times 3$ board feet, or 25 board feet (see p. 248).

2. How many board feet are there in a board 10" wide, 14' long, and $\frac{1}{2}$ " thick?

3. If the material is worth 6¢ a board foot, what will be the value of a box 14" long, 4" wide, and 5" high, not taking into account the labor?



WORKING SKETCH OF KNIFE AND FORK BOX (Material $\frac{1}{4}$ " white wood).

4. What are the dimensions and area of the bottom of this box?

5. What is the length of each side, if the bottom board projects $\frac{1}{2}$ " all around?

6. The sides are nailed over the ends. What is the length of each end?

7. How long a strip will be required to make the sides and ends, allowing $\frac{1}{2}$ " on each piece for squaring?

8. The handle is made from a piece $\frac{1}{4}$ " \times 3" \times 1'. How many board feet will a class of 24 need for this problem, adding 1 ft. to every 10 ft. for waste?

9. If the lumber costs \$40 per M., what is the cost of the material for each box?

10. If a recipe for Chili sauce calls for 1 bu. of tomatoes, 5 qt. of onions, 12 green peppers, 1 cup of salt, 1 cup of sugar, $\frac{1}{2}$ cup of cinnamon, and 3 qt. of vinegar, what proportions should you use for 8 qt. of tomatoes?

11. If 2 cups of flour, 4 tablespoonfuls of baking powder, 1 teaspoonful of salt, 1 tablespoonful of lard, $\frac{3}{4}$ cup of milk, and 1 tablespoonful of butter will make 12 biscuits, how much should you use to make 30 biscuits?

12. If a housekeeper used 2 cups of flour for biscuits, $2\frac{1}{2}$ cups for a cake, $1\frac{1}{2}$ cups for a pie, and 3 qt. for bread, what part of a 25-pound bag did she have left? (See p. 117.)

13. Given the following recipe for ice cream: 1 qt. of milk, 1 cup of sugar, 1 cup of cream. If milk is 9¢ a quart, cream 60¢ a quart, and sugar 5¢ a pound, what would be the cost of the ice cream, allowing 7¢ for ice and salt for freezing?

14. The above recipe will serve 10 people. How much of each ingredient must I use to serve 35 people?

15. It required $1\frac{3}{4}$ yd. of material to cut a child's dress. How many dresses could be cut from $14\frac{1}{2}$ yd.? The material cost 10¢ per yard. What was the cost of each dress?

16. Towels have $\frac{3}{4}$ -inch hems at the top and bottom and are 30 in. long when finished. How many towels could be made from 7 yd.?

17. How much toweling must I buy for a class of 20 so that each pupil will have a towel to hem like those described in Ex. 16?

18. How many yards of cheesecloth are required to make dust cloths $18'' \times 18''$, for a class of 16, if the material is 1 yd. wide?

19. The running expenses of the house for a family of four for a month were as follows: Groceries \$15.65, meat \$10.50, rent \$20, milk \$3, gas \$2.57. The average cost of coal per month was \$7. How much were the expenses per day, counting 30 days to the month? the expenses for each person per day?

20. A cold frame 40 ft. \times 6 ft. was planted in lettuce. The plants were set 6 in. from the border line on all sides. Rows were 1 ft. apart running lengthwise; plants 6 in. apart. How many plants were there in the frame?

21. If $33\frac{1}{3}\%$ of the 474 matured heads were sold for 80¢ a dozen, and the remainder for 50¢ a dozen, what amount of money did the cold frame yield?

22. Deducting from \$23.70, \$1.25 for fertilizers and \$4.75 for labor, find what per cent was made on the cost of the frame, which was \$40.

23. Allowing 10% per year for depreciation of the frame, what would be the yearly profit? What would be the net per cent profit? (See Ex. 22.)

24. At the rate of \$23.70 for 240 sq. ft., what would be the total income from one half an acre inclosed in frames?

25. What would be the net profit on $\frac{1}{2}$ an acre at \$13.70 for 240 square feet.

INDUSTRIAL PROBLEMS

FARM PROBLEMS

TO THE TEACHER. — Problems relating to the industries of a state afford practice in the application of arithmetical principles already studied, and, if based upon recent statistics, are of much value in giving information. Pupils should be encouraged to make problems illustrating the industries of the locality in which the school is situated.

POUNDS OF FERTILIZING CONSTITUENTS IN ONE TON

MATERIALS	NITROGEN	PHOSPHORIC ACID	POTASH
Timothy hay .	19	16	47
Clover hay . .	39	13	44
Alfalfa hay . .	46	13	35
Corn, grain . .	34	13	9
Corn, stover . .	12	9	39
Bran	51	62	35
Oat straw . . .	13	7	38
Milk	12	4	3
Butter	1.6	1	1
Farm animals .	53	37	3
Farm manure .	10	5	10

This table shows the number of pounds of plant food, such as nitrogen, phosphoric acid, and potash, in one ton of the article named. All other constituents necessary to plant food can be obtained from the soil, if the water supply is sufficient and the heat conditions are proper.

The composition of farm produce varies greatly, so that the same kind of crop may be almost twice as

rich in certain constituents in some cases as it is in other cases. The table given here is intended to show the *average* composition of each.

1. How many pounds of nitrogen are there in each of the following: 6 tons clover hay? 3600 lb. grain corn? 5500 lb. timothy hay? 4 tons alfalfa?

2. A farmer cut on an average 3 tons per acre of clover hay from a 5-acre field. How many tons of manure were required for the field to supply the phosphoric acid in the hay?

3. A farmer sold 50 tons of timothy hay in one season. How many tons of farm manure balanced the loss in potash?

4. When nitrogen is worth 16 ¢ per pound, phosphoric acid 5 ¢, and potash 6 ¢, find the fertilizing value of 5 tons of farm manure.

5. A commercial fertilizer when analyzed was found to contain by weight 3.4 % nitrogen, 6.6 % phosphoric acid, and 9 % potash. Find its value per ton, when nitrogen is worth 15 ¢ per pound, phosphoric acid 3 ¢ per pound, and potash 5 ¢ per pound.

6. The analysis of a certain brand of bone meal showed the following: 1.6 % nitrogen and 27.9 % phosphoric acid. What was its value per ton, at 15 ¢ per pound for nitrogen and $3\frac{1}{2}$ ¢ per pound for phosphoric acid?

• The **Bordeaux mixture** used for killing fungus growths contains 4 pounds of copper sulphate (blue vitriol) and 4 pounds of stone lime to the barrel (50 gallons).

7. For a vineyard of 25 acres of grapes suffering from black rot of the fruit, how many pounds of sulphate of copper and lime are necessary, if 100 gallons of Bordeaux mixture will spray $\frac{2}{3}$ of an acre?

8. If sulphate of copper can be bought at 6 ¢ per pound, and stone lime at $\frac{1}{2}$ ¢ per pound, what is the cost of materials for 500 gallons of Bordeaux mixture?

9. 1000 bushels of apples from 60 trees, which were not sprayed, were sold as follows: 200 bu. of perfect fruit at \$1.25 a bushel, and 800 bu. of scabby and wormy fruit at \$.40

a bushel. If spraying the 60 trees at \$.20 a tree caused them to yield 800 bu. of perfect fruit @ \$1.25 and 200 bu. of wormy fruit @ \$.40, how much was gained by the spraying?

10. The estimated value of a crop of grapes is \$400 an acre. Thorough spraying costs \$6 an acre. If, without spraying, fungus diseases destroy 30% of the crop, what is the net value of the spraying of 10 acres?

11. The average yield from 4 acres of sprayed grapes was 4770 lb. an acre, and the yield from unsprayed grapes was 3108 lb. The grapes were sold for $2\frac{1}{2}\phi$ a pound. The cost of spraying was \$8.60 an acre. What was the net gain on the 4 acres, as a result of spraying?

A Paris green solution used for killing chewing insects that destroy the plants by eating the leaves, and for destroying the codling moth to prevent wormy fruit is made by mixing 4 ounces of Paris green with 50 gallons of water.

12. An apple orchard of 250 trees is sprayed for the apple worm, with Paris green at 32ϕ a pound. Find the cost of the material if 50 gal. of the solution are used to 20 trees.

13. Mr. Wagner sprayed $8\frac{1}{4}$ acres of potatoes 4 times with Bordeaux mixture containing Paris green, to control blight, rot, and insects. His expense account was as follows: 183 lb. copper sulphate @ 8ϕ ; 204 lb. lime at \$1.50 per hundred; 10 lb. Paris green @ 35ϕ ; 48 hr. labor for a man @ 20ϕ ; 40 hr. labor for a team @ 15ϕ ; wear of sprayer, \$1.50. What was the cost of spraying per application per acre?

14. The $8\frac{1}{4}$ acres mentioned above yielded 1567.5 bu. potatoes. A portion of the same field that was left unsprayed yielded 156 bu. an acre. The crop was sold for \$1.25 a bushel. What was the net profit an acre as a result of spraying, allowing \$4.65 an acre for spraying?

15. Thirty-four apple trees sprayed to control scab and codling moth yielded 90 bu. merchantable fruit and 32 bu. culls and windfalls. In the same orchard 21 unsprayed trees yielded 11 bu. merchantable fruit and 40 bu. culls and windfalls. The merchantable apples were sold for \$1.00 a bushel and the culls and windfalls for \$.45 a bushel. The cost of spraying was 35¢ a tree. What was the net gain from spraying per tree?

16. A farmer planted 12 acres with potatoes. His expenses were as follows: Preparing the soil and planting, \$8.50 an acre; gathering and marketing the crop \$6.50 an acre; seed \$12 an acre. If his crop averaged 150 bushels to the acre, worth \$1.25 a bushel, what was his profit?

17. What per cent of the total expense was the cost of the seed?

18. A field 80 rd. in length by 40 rd. in width is planted with sweet potatoes. How much will it cost to plow it at \$3.75 an acre?

19. How much will the crop be worth if it averages 125 bu. to the acre, worth \$1.29 a bushel?

20. A farmer bought a rectangular ten-acre lot, 20 rd. on one side. He put up a fence at \$1.64 a rod. How much did the fence cost?

21. The expense for cultivation, fertilizer, and seed was \$175. His crop was 460 bu. of corn worth 80¢ a bushel. How much, if any, profit did he realize from the first year's use of the field?

22. A poultry raiser had a flock of 250 hens. During the year he raised and sold 240 lb. of poultry at an average price of 21½¢ a pound. He sold 50 doz. eggs a week at an average price of 32¢. His expense for labor, feed, etc., was \$504. What was his profit?

23. If his whole investment amounted to \$775, what per cent did he realize on his investment?

24. If 200 hens lay on an average 150 eggs a day, what will be the profit for the month of March from 500 hens, eggs being worth 43¢ a dozen and the cost per week for feeding each hen being 8¢?

25. A farmer had a herd of 12 cows averaging 8 qt. of milk a day each. He received 3¢ a quart for milk delivered at the railroad station. What was his income for the month of June?

26. If 500 qt. of this milk made 55 lb. of butter worth 45¢ a pound, what would be the difference in profit if the farmer referred to in Ex. 25 had converted his milk into butter, allowing \$32 for labor?

27. The quantity of milk consumed by factories in the United States in the manufacture of dairy products was 9,888,727,000 lb. in 1909 and 14.7% less in 1914. What was the quantity consumed in 1914?

28. The quantity of cream increased from 1,406,144,000 lb. in 1909 to 69.5% more in 1914. What was the number of pounds for 1914?

29. The total value of dairy products increased from \$275,277,000 in 1909 to \$370,819,000 in 1914. What was the per cent of increase to the nearest tenth per cent?

30. The production of condensed and evaporated milk increased from 495,198,000 lb. valued at \$33,587,000, in 1909, to 884,647,000 lb. valued at \$59,375,000 in 1914. Find the Per cent of increase in quantity and in value.

31. The production of butter in 1914 amounted to 786,013,000 lb. valued at \$223,179,000 as compared with 627,146,000 lb. valued at \$180,175,000 in 1909. Find the Per cent of increase in quantity and in value.

MANUFACTURING PROBLEMS

The following table gives a comparative summary of manufacturing statistics in 1909 and 1914 for the geographic divisions of the United States :

DIVISION	CENSUS YEAR	NUMBER OF ESTABLISHMENTS	WAGE EARNERS	CAPITAL	WAGES	VALUE OF PRODUCTS
New England	1914	25,193	1,140,233	\$ 2,948,040,000	\$ 628,409,000	\$2,926,676,000
	1909	25,351	1,101,290	2,503,854,000	557,631,000	2,670,065,000
Middle Atlantic	1914	85,466	2,355,940	7,836,071,000	1,370,131,000	8,053,644,000
	1909	81,315	2,207,747	6,505,675,000	1,182,568,000	7,141,761,000
East North Central	1914	59,896	1,680,281	5,913,681,000	1,073,538,000	6,542,261,000
	1909	60,013	1,513,764	4,547,225,000	827,152,000	5,211,702,000
West North Central	1914	27,199	381,595	1,424,181,000	235,471,000	2,032,192,000
	1909	27,171	374,337	1,171,572,000	204,792,000	1,803,899,000
South Atlantic	1914	28,925	685,342	1,644,539,000	293,063,000	1,682,999,000
	1909	28,088	663,015	1,368,475,000	244,378,000	1,381,186,000
East South Central	1914	14,410	264,378	713,357,000	117,987,000	700,668,000
	1909	15,381	261,772	586,276,000	102,191,000	630,488,000
West South Central	1914	12,417	211,940	687,819,000	116,128,000	802,538,000
	1909	12,339	204,520	547,739,000	97,646,000	625,443,000
Mountain or Plateau	1914	6,079	81,113	469,971,000	66,358,000	437,568,000
	1909	5,254	75,435	348,977,000	56,870,000	363,996,000
Pacific	1914	16,206	235,515	1,153,321,000	178,247,000	1,067,889,000
	1909	13,879	213,166	848,477,000	153,810,000	843,512,000

1. Find the total number of wage earners in manufacturing industries in the United States in 1914 ; the total wages in 1914 ; the average wage for each person.

2. Find the total number of establishments in the United States in 1909 ; in 1914 ; the increase in number.

3. Find the per cent of increase in number of establishments from 1909 to 1914.

4. What was the difference in the amount of capital invested in the United States in 1909 and 1914?

5. What was the per cent of increase in the amount of capital from 1909 to 1914?

6. Find the total value of the products in the United States in 1909; in 1914.

7. Find the per cent of increase in total value of production in the United States from 1909 to 1914.

Find the per cent of increase in the value of products from 1909 to 1914 in :

8. New England.

9. Middle Atlantic States.

10. East North Central States.

11. West North Central States.

12. South Atlantic States.

13. East South Central States.

14. West South Central States.

15. Mountain States.

16. Pacific States.

17. Draw a graph showing the relative rank of the divisions for value of products in 1914.

18. Draw another graph showing the relative amount of capital invested in each division in 1909 and in 1914.

The following table shows the condition of the woolen industry in 1909 and 1914:

INDUSTRY	NUMBER OF ESTABLISHMENTS		VALUE OF PRODUCTS	
	1914	1909	1914	1909
Worsted (combed wool) goods	294	324	\$275,668,474	\$312,624,863
Woolen (carded wool) goods	561	587	103,815,905	107,118,868
Carpets and rugs	97	139	69,128,185	71,188,162
Felt goods	53	43	13,692,765	11,852,626
Wool-felt hats	30	31	1,944,484	4,382,411
Total				

19. Find the totals for each item.
20. What was the total decrease in value in the five years?
21. Find (to the nearest tenth per cent) the per cent of decrease or increase in the value of each product from 1909 to 1914.

MERCANTILE PROBLEMS

W. V. Garland Company, of Omaha, Nebr., order from the Model Furniture Company, of Grand Rapids, Mich., 200 chairs at \$1.50 each; 150 chairs at \$2 each, and 12 sideboards at \$28.50 each, discount 20 %, 10 %. Terms 30 days net, 2 % off in 10 days.

1. Write the order for the goods.
2. Make out a receipted bill if paid in 10 days.
3. The freight and cartage bills are \$30.32. If W. V. Garland Company sell the goods at a profit of 26 %, how much do they receive for them?

4. What would be the selling price if the goods were sold at an advance of 16 %, with 5 % off for cash ?

5. If 40 % of the profit is used in paying general expenses, such as light, heat, salaries, etc., what is the net profit ?

6. What per cent is the net profit of the original outlay ?

7. On June 17, 1918, The Miller Company, of Baltimore, Md., bought of the Rogers Silk Company, Allentown, Pa., 5 pieces of silk containing 40, 43, 41, 44, 40 yards, respectively, at \$1.75 a yard, discount 5 %, terms 5 % off in 10 days. Make out the receipted bill if paid June 21, 1918.

8. If the transportation cost \$4.49, what selling price per yard would be an advance of 20 % on the whole outlay ?

9. The total cost of 208 yd. of silk was \$333. If 168 yd. were sold at \$1.93 and the remainder at \$1.60 less a discount of 25 %, what was the profit ?

10. If a part of the profit equal to 5 % of the original outlay was applied to general expenses, what was the net profit ?

11. What per cent of the original outlay was the net profit ?

Write and receipt bills for the following :

12. A school board bought of Wyckoff and Jones, New York, N. Y., 53 gross white crayons @ 23¢ ; 45 gross pens @ 24¢ ; 65 gross lead pencils @ \$1.60 ; discount 10 %.

13. Robert Brown bought of the Holt Company 4 10'' saws @ 93¢ ; 1 set bits, $\frac{1}{4}$ '' to 1'' @ \$4.90 ; 3 1'' chisels @ 34¢ ; 2 block planes @ 76¢ ; discount 10 %.

14. William Evans bought of the Scientific Company, 5 magnets @ 50¢ ; 3 glass tubes with stopcock @ \$1.50 ; 10 coils for induction @ 55¢ ; 1 set resistance spools @ \$3 ; discount 10 % ; 5 lb. mercury @ \$1 net.

15. W. A. Wessler bought of the General Book Company 5 "Principles of Economics" @ \$1, less $\frac{1}{8}$; 8 "Elements of Chemistry" @ \$1.25, less $\frac{1}{8}$; 9 "American History" @ \$1.05, less $\frac{1}{8}$; 28 "Choice Readings" @ \$1.50, less $\frac{1}{8}$. W. A. Wessler has returned to the General Book Company, 4 "Elementary Algebra" billed at \$1.12, less $\frac{1}{8}$.

16. H. P. Curtis bought of the Carter Company 12 electric fans @ \$13; 10 magnetos @ \$9.60; 20 jump-spark coils @ \$3.30; 18 belt-drive magnetos @ \$9.90; discount 10%, 5%. What was the entire outlay for the goods?

TRANSPORTATION PROBLEMS

1. If the tank of a tank car contains 1000 cu. ft., how many gallons of oil will a train of 10 tank cars transport and what is its value at $9\frac{1}{2}\text{¢}$ a gallon?

NOTE. — $7\frac{1}{2}$ gallons = 1 cubic foot.

2. How many such cars will be required to carry the contents of a tank whose capacity is 30,000 bbl. of 42 gal. each?

3. The cost of a freight locomotive is \$20,000 and of a steel coal car 6% of this sum. What will be the value of a train consisting of a locomotive and twenty-five cars, each containing 50 tons of coal worth \$2.50 a ton?

4. Allowing \$1.75 a ton for handling and 90¢ a ton for freight, for what must this coal be sold in New York to yield a profit of 25%?

5. A train consists of 3 steel day coaches and 2 steel sleeping cars. The steel day coaches weigh 112,000 lb. each and the steel sleeping cars 152,000 lb. each. What per cent of the whole weight is the weight of the sleeping cars?

6. A box car 40 ft. long, 8 ft. 6 in. wide, and 8 ft. high inside measurements was received in Boston, Mass., loaded with wheat to a depth of 4 ft. 6 in. At 60 lb. to the bushel what was the weight of the wheat? What was the freight charge at 16¢ a hundred?

NOTE. — 2150.42 cubic inches = 1 bushel.

7. A quantity of wheat which cost \$1.62 a bushel was sold at an advance of $11\frac{1}{3}\%$ on the cost price. What was the net profit on 1230 bu., allowing \$118 for freight?

8. Find the freight charge on 73,767 lb. of wheat shipped by water at 6¢ a hundred pounds.

9. A shipment of 600 bales of cotton weighing 256,950 lb. was made from Galveston to New York. The freight charges were \$1233.36. What was the average weight of a bale of cotton and what was the freight rate per hundred?

10. What would be the freight charges on 75,600 lb. of machinery shipped from an eastern to a western city, at $40\frac{1}{4}$ ¢ a hundred?

11. If a steel rail weighs 80 lb. to the yard, what will be the cost of rails necessary to relay a double track 12 miles long, if steel rails are worth \$38 a ton?

12. My agent bought 100 barrels of flour at \$7.50 a barrel. He paid \$50 freight and \$6 storage. I sent him \$872. What was his commission?

13. A collector succeeded in collecting 90% of the amount of a consignment of cotton sold for \$6700. He remitted \$5874, retaining the balance to pay collector's fees and \$6 freight charges. What amount did he charge for collecting?



PAYING AND COLLECTING MONEY

Among uncivilized peoples who have not learned the use of money, articles are directly exchanged one for another. The Indian trades his furs for powder, trinkets, blankets, or other articles. In communities on the frontier, the same practice prevails, the settler trading his products with his neighbor for what he needs. In gold-mining countries, the miner frequently pays for what he buys with gold dust.

As trade increases in amount and variety, this method of barter becomes inadequate and better methods must be devised. Pieces of metal, generally of gold or silver, are given or received in more advanced communities, in exchange for articles of use or value. These pieces of metal, stamped by the government and made in convenient shapes and sizes, are called **money**.

For further convenience, governments issue bills, which are commonly called paper money. These, being so much lighter and less bulky than metallic money, greatly facilitate the transaction of business on a large scale. These bills are not money, but simply promises to pay the face value of the bills in gold or silver at any time ; yet they are accepted as money because the governments that issue them are well able to keep their promises.

Even these methods of exchange, however, are entirely insufficient for the world's enormous trade, the more so as localities far remote from one another are engaged in trade. Other methods of paying debts, without the transfer of large sums of money from place to place, have therefore been devised. The various methods of **exchange** make it possible to transact business on a large scale, with little trouble, between places widely separated.

EXCHANGE

Exchange is a method of paying or collecting bills at a distance, without the actual transfer of money.

There are several different ways in which bills may be paid at a distance without the transmission of money :

- (1) By a **personal check**.
- (2) By a **bank draft** (banker's check).
- (3) By a **postal money order**.
- (4) By an **express money order**.
- (5) By a **telegraphic money order**.

CHECKS

The most commonly used of the above methods is that by check. We have under "Banking" studied some of the uses of checks. Their most important use is for paying bills without the actual transfer of money.

Checks may be made payable to *bearer*, to *cash*, to *John Smith or bearer*, to *the order of John Smith*, to *self*, etc. (See p. 141.)

When the payee writes only his name across the back of the check, it is called a *blank indorsement* of the check, as it does not state to whom the check is made payable. If

the payee writes across the back of the check the following:

Pay to the order of
WARD, PEET, AND COMPANY,
New York

and signs his name, this is known as a *full indorsement*, for no one but Ward, Peet, and Company can collect or indorse the check.

1. Suppose Mr. J. R. Thompson, Toledo, Ohio, wishes to send a check for \$500 to Mr. Sage Allen of Hartford, Conn., in payment of an account. First, write the check as indorsed by Mr. Thompson in blank; second, write the check as indorsed by Mr. Thompson in full.

2. Give reasons why it will be better for Mr. Thompson to indorse the check in full.

3. When a check is indorsed and sent by mail, what form of indorsement should always be used? Why?

Balancing Accounts ; Depositing ; Checking on Accounts.

1. Your deposits in a bank for the month of September are as follows :

Sept. 1, currency, \$ 50; silver, \$10; check, \$15.

Sept. 6, currency, \$20; silver, \$10; check, \$100; gold, \$20.

Sept. 10, currency, \$45; silver, \$4.75.

Sept. 16, currency, \$20; silver, \$3.40; check, \$80.

Sept. 25, gold, \$40; check, \$40; silver, \$10.

Sept. 29, check, \$80; currency, \$80; silver, \$35.

Make out deposit slips and find amount of deposits for September.

2. Your check book shows the following:

Balance in bank Sept. 1, \$847.10.

No. 1, Sept. 4, Keller Bros., for coal, \$15.50.

No. 2, Sept. 4, the George K. Stevenson Company, for groceries for August, \$49.50.

No. 3, Sept. 4, Dr. S. N. Pool, for services to date, \$90.

No. 4, Sept. 5, cash, \$55.

No. 5, Sept. 7, Jos. Horne, for merchandise, \$65.30.

No. 6, Sept. 11, Midland Lumber Co., for lumber, \$93.75.

No. 7, Sept. 15, the Johnson Company, for repairs on automobile, \$29.35.

No. 8, Sept. 19, cash, \$25.

No. 9, Sept. 24, J. H. McFarland, for interest due on note, \$24.

Write the checks for the bills paid for September, and find balance in bank.

3. Arriving at Chicago, I find in my mail a check from Quinn and Company, Phoenix, Ariz., for \$415.40, in payment of my salary and expenses for September. I wish to deposit the check to my account in the Federal Trust Company, Phoenix, Ariz. How should I indorse the check before sending it through the mail?

Sometimes the seller does not know the financial standing of the purchaser, and therefore requires the check accompanying the order to be *certified*; that is, the cashier of the bank on which the check is drawn stamps the word "certified," with the date and his signature, across the face of the check. The check is thereafter the check of the bank, and is good as long as the bank is solvent.

A **certified check** is a notice to the payee of the check that the amount named on the face has been taken from the maker's deposit and placed with the bank's funds for the payment of the check when presented.

Certified checks are frequently demanded in payment of notes and collections at banks, and in payments where the payee does not wish to make a personal check. Like other checks, they are mailed daily in payment of bills in all parts of the country.

1. What is a check? What are the essentials of a check?
2. In buying a lot from James Carothers for \$ 800, you are asked to give your certified check for the amount. Write your check on your local bank.
3. The Moore Company, Madison, Wis., purchased \$825 worth of furniture at 20 % and 10 % off from James Boyds, Detroit, Mich., 3 % off for cash in 10 days. The Moore Company sent a certified check within ten days on the Lincoln Trust Company, of which James Patterson is secretary and treasurer. Write the certified check.

NOTE.— The *secretary and treasurer* of a trust company corresponds to the cashier of a bank.

4. William Anderson, 7531 Hermitage Ave., Chicago, Ill., receives a check on the People's Bank of Cheyenne, Wyo., from Freeman Lewis for \$730.80 in settlement of an estate. The Business Bank of Chicago charges Mr. Anderson \$1.50 for collecting the check. This fee is called the **exchange** for collecting.

A person who cashes a check at a bank in which he is not a depositor is frequently charged an *exchange* of 10 ¢ and upward, according to the amount of the check.

BANK DRAFTS

A **draft** is a check drawn by one bank on another.

As New York and Chicago banks collect exchange on outside checks, nearly all banks keep deposits there, as well as in most of the other large commercial centers, to accommodate their depositors and others who have occasion to remit payment for bills in any part of the country.

Banks usually charge exchange on drafts to cover the cost of keeping funds on deposit at these commercial centers. This fee varies from $\frac{1}{16}$ % to $\frac{1}{4}$ % of the face. When the draft is less than \$ 100, a fixed charge is frequently made, varying from 10 ¢ to 50 ¢.

The custom of banks is not to charge depositors for drafts.

In issuing or collecting a draft, the **exchange** is either a fee or a certain per cent of the face of the draft.

New York and Chicago drafts are usually cashed without exchange at any point in the United States. Drafts on other large cities are cashed without exchange in the territory contiguous to those cities.

Ball and Company, Dover, Del., buy \$ 2500 worth of merchandise from the T. W. Trent Company, New York, and \$ 650 worth of machines from the Mason Company, Bridgeport, Conn. The business method of paying these bills is either by a check or by a bank draft. The draft is made payable to the order of the purchaser, who indorses it in full to the payee. For example :

First National Bank	
<i>Dover, Del., June 2, 1917, No. 1040</i>	
<i>Pay to the order of Ball and Company ----- \$2500 ⁰⁰/₁₀₀</i>	
<i>Twenty-five Hundred and ^{no}/₁₀₀ ----- Dollars.</i>	
<i>To The Mercantile National Bank,</i>	
<i>New York, N.Y.</i>	
<i>A. M. Holmer,</i>	
<i>Cashier.</i>	

This draft means that Ball and Company purchased at the bank where they kept their deposit a draft (banker's check) for the above amount. The First National Bank had money on deposit at the Mercantile National Bank, and simply checked on its deposit. If Ball and Company had not been depositors in the First National Bank, they would probably have been charged $\frac{1}{10}\%$ exchange. The draft would then have cost them \$ 2502.50.

The party who signs a draft is called the **drawer** of a draft.

The party to whose order the draft is drawn is called the **payee** of the draft.

The party who is to pay the money is called the **drawee** of the draft.

Thus, in the draft on p. 217 the cashier of the First National Bank is the **drawer**; Ball and Company the **payee**; and The Mercantile National Bank the **drawee**.

Written Work

1. Find the cost of a New York draft for \$550.25 at $\frac{1}{10}\%$ exchange.

2. Mr. Amidon buys \$2500 worth of farm implements at 30 % and 10 % off and pays $\frac{2}{3}$ of the debt by a Chicago draft at $\frac{1}{8}\%$ exchange. Find the face and the cost of the draft.

3. A. Madison of Des Moines, Iowa, bought of Simpson Brothers, Chicago, 2500 bu. of corn at 90 ¢ and paid with a Chicago draft at $\frac{1}{10}\%$ exchange. How much did the draft cost ?

4. At $\frac{1}{4}\%$ exchange what will be the cost of a New York draft to pay for \$1600 worth of merchandise bought at a discount of 20 % and 10 % ?

5. What will be the cost of a draft on New York at $\frac{1}{10}\%$ exchange to pay the following bill ?

20 doz. Kirk's "Standard Geography" at \$1.00 apiece.

8 " Flash's "History of U. S." at .80 "

30 " James's "School Arithmetic" at .60 "

Discount $\frac{1}{8}$

Express prepaid 3.25

Cartage .50

6. Write a draft for \$2600, making one of your local banks the drawer, the First National Bank of Buffalo, N. Y., the drawee, and the American Power Company, the payee. Indorse the draft in full to James Osborne, Syracuse, N. Y.

7. Mr. Robinson had a note for \$1000 discounted for 60 days at 6%. He bought a draft on Cleveland for \$410, at $\frac{1}{10}\%$ exchange and deposited the balance of the proceeds in the bank. What was the amount of the deposit?

8. My settlement of an account in New Orleans gave me \$26,785. I invested \$13,750 of this amount in land on which I paid an agent's commission of 2%. I also paid by draft, exchange $\frac{1}{10}\%$, a debt of \$8200. The remainder I deposited in the bank. What was the amount of the deposit?

9. James Sands and Son, Helena, Mont., order \$790 worth of goods from a Buffalo, N. Y., firm and send in payment a New York draft at $\frac{1}{4}\%$ exchange. Find the cost of the draft.

10. A dealer in San Francisco buys \$2000 worth of goods at 30% and 10% off, and sells them at an advance of 25% on the cash price. He pays for these goods with a Chicago draft at $\frac{1}{4}\%$ exchange. Find his profit.

POSTAL AND EXPRESS MONEY ORDERS

If you wish to order from H. K. Brooks, Chicago, \$46.25 worth of merchandise, unless you have credit with him, you will probably send him (1) either a *postal money order*, or (2) an *express money order*. The first will direct the postmaster at Chicago, the second some express agent at Chicago, to pay to the order of H. K. Brooks \$46.25.

WHEN COUNTERSIGNED
BY AGENT AT POINT OF ISSUE

EXPRESS MONEY ORDER

10-

American Express Company

NOTES AND RECEIPTS

PAY TO THE ORDER OF H. K. Brooks

THE SUM OF Forty six and 25/100 DOLLARS

PAID TO THE ORDER OF THE PAYEE BY THE ORDER OF THE ISSUING OFFICE

ISSUED AT New York DATE May 17 1918

AGENT My TREASURER C. Smith

DO NOT SIGN, ALTER, INTERPUN, OR INFLUENCE THIS ORDER OR ELSE IT WILL BE VOID

10-

AMERICAN EXPRESS CO.
MONEY ORDER

REMITTANCE RECEIPT

KEEP IT.

AMOUNT OF ORDER

DATE

NAME OF PAYEE

NAME OF ISSUING OFFICE

NAME OF AGENT

The cost of either of the above orders is the same; the only difference being that a postal money order is payable to the order of the

party or firm upon identification at the place named in the order, while an express money order is payable to the party or firm upon identification at *any* office of the same company where orders are sold.

Money orders may be purchased for any amount up to \$100.

The rates charged in the United States are as follows:

For orders from	\$.01 to	\$ 2.50	3 ¢
	\$ 2.51 to	\$ 5.00	5 ¢
	\$ 5.01 to	\$ 10.00	8 ¢
	\$ 10.01 to	\$ 20.00	10 ¢
	\$ 20.01 to	\$ 30.00	12 ¢
	\$ 30.01 to	\$ 40.00	15 ¢
	\$ 40.01 to	\$ 50.00	18 ¢
	\$ 50.01 to	\$ 60.00	20 ¢
	\$ 60.01 to	\$ 75.00	25 ¢
	\$ 75.01 to	\$ 100.00	30 ¢

The rates to *foreign* countries are from 10 ¢ to \$1 for the same amounts as domestic orders. This fee of from 3 ¢ to 30 ¢ for domestic orders, and from 10 ¢ to \$1 for foreign orders, to cover the cost of paying the bills at a distance, is called the **exchange** for issuing the orders.

1. I sent \$75.80 to New Orleans, La., by express money order. How much could I have saved by purchasing a bank draft at 15 cents exchange?

TELEGRAPHIC MONEY ORDERS

Such orders are drawn by agents of the telegraph company, and direct the agent at some designated office to pay to the person named in the telegraphic message, upon identification, the sum specified.

The present rates for sending money by telegraphic order are in addition to the regular charge for a 15-word message between the two places:

For \$25 or less	25 ¢
Above \$25 and not above \$50	35 ¢
Above \$50 and not above \$75	60 ¢
Above \$75 and not above \$100	85 ¢

After the first \$100, up to and including \$3000, add 20 cents for each \$100 or part thereof. (In addition, there is a war tax of 5 ¢ regardless of amount. This is to be disregarded in these problems.)

2. If the rate for a 15-word message is \$1, how much will it cost to send \$25 by telegraphic money order?

3. How much will it cost to send \$475 by telegraphic money order between two cities, the rate for 15 words being 90¢?

4. Find the cost of sending \$80 between two cities by telegraphic money order, the rate being 45¢ for fifteen words.

SIGHT COMMERCIAL DRAFTS

Bills are collected at a distance in *two* ways:

- (1) By a sight commercial draft of a creditor on a debtor.
- (2) By a time commercial draft of a creditor on a debtor.

If Klein Brothers, produce dealers, Portland, Me., order from Harris Brothers, Chicago, Ill., 1 carload of No. 1 oats, Harris Brothers will ship the carload of oats to Portland to the order of themselves and draw a sight draft on Klein Brothers, payable to the order of some Chicago bank, and deposit it, with the bill of lading, for collection. The Chicago bank will then mail the draft, together with the bill of lading, to some bank in Portland. The Portland bank will notify Klein Brothers. If the carload of oats is accepted by Klein Brothers, they will pay the draft and receive the bill of lading, which will entitle them to the oats.

This form of draft is commonly known as a **commercial sight draft** and reads as follows:

\$450 ⁰⁰ / ₁₀₀	Chicago, Ill., June 27, 1918
~~~~~At sight~~~~~Pay to	
the order of-----1st National Bank, Chicago,-----	
Four Hundred Fifty ^{no} / ₁₀₀ ~~~~~Dollars.	
Value received, and charge to account of	
To Klein Brothers No. 14. Portland, Maine	} Harris Brothers, <div style="text-align: right;">Chicago, Ill.</div>

A **bill of lading** is a receipt given by the carrier to the shipper. The goods shipped and their value are described on its face, and on the back of the receipt is stated the contract of shipment.

In case Klein Brothers refused to accept the oats, the draft would be returned to the Chicago bank, which in turn would notify Harris Bros.

Creditors use sight drafts in the collection of debts due or past due.

### TIME COMMERCIAL DRAFTS

Often a draft reads "30 to 90 days after sight." Such a draft is called a **time commercial draft**.

The method of collecting by a **time commercial draft** is as follows:

<p>$\\$1200 \frac{00}{100}$.</p>	<p>Ogden, Utah, June 27, 1918</p>
<p>Sixty days after sight.....</p>	
<p>Pay to</p>	
<p>the order of.....First National Bank.....</p>	
<p>Twelve Hundred and $\frac{no}{100}$.....Dollars.</p>	
<p>Value received, and charge to account of</p>	
<p>To Acme Buggy Company No. 132. Cincinnati, Ohio</p>	<p>} Bell Company..... Ogden, Utah</p>

The Cincinnati bank to which this draft is mailed immediately notifies the Acme Buggy Company and if the Company agrees to pay the draft, when due, the following is written across the face of it:

"Accepted  
Date .....  
Acme Buggy Co."

If the Company refuses to accept the draft, the Cincinnati bank will return it to the First National Bank, and the Bell Company will be notified that the goods are at Cincinnati at their risk.

The term of discount in a draft payable "after sight" begins to run from the date of acceptance; in a draft payable "after date," from the date of the draft.

In collecting by draft, the exchange is always collected on the face, not on the proceeds, of the draft.

### Written Work

1. If the draft of the Bell Company was accepted June 29, 1918, and discounted July 1, 1918, find the term of discount.

2. Find the proceeds remitted to the Bell Company, if this draft was discounted July 1 at 7%, with  $\frac{1}{4}\%$  exchange for collecting.

3. The Fiske Company, Sandusky, Ohio, draw at sight on James Howard, Canonsburg, Pa., for \$159.70 through the Erie National Bank, Sandusky, Ohio. Write the draft.

4. Freeman Brothers, Fargo, N. Dak., Jan. 2, 1918, sell on 60 days' time to the Milling Company, Minneapolis, Minn., 12,000 bu. corn at  $92\frac{1}{2}\%$  a bushel delivered at Minneapolis, and draw a time draft which is accepted by the Milling Company. The Merchants National Bank of Minneapolis buys this draft Feb. 1, 1918, at 7% discount. If exchange is  $\frac{1}{8}\%$ , find the proceeds from the sale of the corn.

5. Charles Boyd, Fremont, Ohio, owed Samuel Johnson, Jacksonville, Ill., \$600 due June 10, 1916. Mr. Johnson desired the money immediately, and drew on Mr. Boyd March 24, 1916, through the Illinois National Bank, Jacksonville, a time draft due June 10, which Mr. Boyd accepted March 30. The Fremont bank discounted the draft at 6% on the day of acceptance. If the exchange was  $\frac{1}{8}\%$ , how much was remitted to Mr. Johnson?

6. What will be the proceeds of a 60-day draft for \$1900, discounted for the full term at 7%, the bank charging \$1.20 for collection?



### LETTERS OF CREDIT AND TRAVELERS' CHECKS

People intending to travel abroad generally secure a letter of credit or a foreign express check.

A **letter of credit** is a circular letter issued by an international banking house, authorizing the person securing the letter to draw money on demand, at specified banking houses, or other financial institutions, for any sums not exceeding the face value of the letter.

The purchaser is required to sign several blanks, one of which is sent to each correspondent bank. When he wishes to draw funds, he presents his letter at any one of these banks, and signs a draft or a check for the amount he desires to draw. After comparison of the signature with that on the signature blank, the money is paid and charged on the letter, which is then returned to the owner. A commission of 1 % is usually charged by the bank issuing the letter of credit.

**Travelers' checks** are checks issued in denominations of \$10, \$20, \$50, \$100, and \$200, by banks and express companies, at a fixed rate of  $\frac{1}{2}$  % of the face value, with a minimum charge of 30 cents. The purchaser signs each check on the face when purchased, and repeats the signature when he cashes the check. These checks are accepted at all hotels, railroads, shops, etc.

The following values were printed on a \$20 check :

£	s.	d.	FRANCS	MARKS	LIRE	CROWNS	FLORINS
4	1	2	102.50	82.50	102.50	73.39	49.02

1. Find how much English, French, and German money would be received in return for a \$10 check; for a \$50 check.

# CHAPTER IV

## MENSURATION

### TRIANGLES

A **triangle** is a surface bounded by three straight lines.  
(*Tri* means *three*.)

A **vertex** of a triangle is a point where two sides meet.

The **base** of a triangle is the side on which it is supposed to rest.

The **altitude** of a triangle is the perpendicular distance from the vertex opposite the base to the base, or the base extended.

Triangles are named in *two* ways :

I. From their angles :

- (1) **Right-angled** triangles. (One right angle.)
- (2) **Acute-angled** triangles. (All angles less than a right angle.)
- (3) **Obtuse-angled** triangles. (One angle greater than a right angle.)



RIGHT-ANGLED.



ACUTE-ANGLED.



OBTUSE-ANGLED.

II. From their sides :

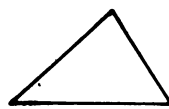
- (1) **Equilateral**. (Having three sides equal.)
- (2) **Isosceles**. (Having two sides equal.)
- (3) **Scalene**. (Having no two sides equal.)



EQUILATERAL.



ISOSCELES.



SCALENE.

By geometry it is shown that the sum of the angles in any triangle is equal to  $180^\circ$ . This can also be shown by measuring the angles with a protractor.

*The sum of all the angles of any triangle is equal to two right angles, or  $180^\circ$ .*

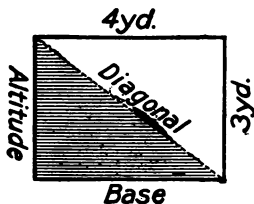
### Written Work

The following numbers in each case represent the size of two angles of a triangle. Find the size of the third angle:

- |                                                     |                                      |
|-----------------------------------------------------|--------------------------------------|
| 1. $90^\circ$ and $45^\circ$                        | 5. $60^\circ$ and $40^\circ$         |
| 2. $90^\circ$ and $60^\circ$                        | 6. $100^\circ 45'$ and $37^\circ$    |
| 3. $120^\circ$ and $30^\circ$                       | 7. $75^\circ 10'$ and $95^\circ 30'$ |
| 4. $120\frac{1}{2}^\circ$ and $40\frac{1}{2}^\circ$ | 8. $100^\circ$ and $45^\circ 40'$    |

### Finding the area of a right triangle.

- Find the area of a right triangle whose base is 4 yards and whose altitude is 3 yards.



Observe: 1. That the diagonal divides the rectangle into two equal *right* triangles.

2. That the *unit* of measure is 1 *sq. yd.*

3. That the area of one of the right triangles is  $\frac{1}{2}$  of the area of the rectangle; that is,  $\frac{1}{2}$  of  $4 \times 3 \times 1$  *sq. yd.*, or 6 *sq. yd.*

*The area of a right triangle is found by multiplying the unit of measure by half the product of the base and the altitude.*

Name the unit of measure, and find the area of each of the following right triangles:

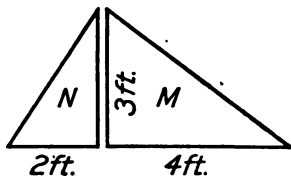
- |                                |                                 |
|--------------------------------|---------------------------------|
| 2. Base 10 in., altitude 6 in. | 5. Base 10 ft., altitude 7 ft.  |
| 3. Base 12 yd., altitude 8 yd. | 6. Base 14 ft., altitude 10 ft. |
| 4. Base 9 ft., altitude 6 ft.  | 7. Base 6 ft., altitude 20 ft.  |

# Finding the area of any triangle.

## Written Work

1. Find the area of two right triangles, the base of one being 2 ft. and of the other 4 ft. and the altitude of each 3 ft.

Draw the triangles as shown in the figure.

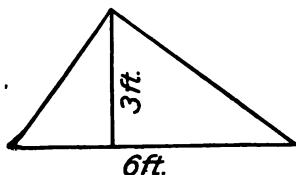


Observe: 1. That the *unit* of measure is 1 sq. ft.

2. That the area of the right triangle *N* is equal to  $\frac{1}{2}$  of  $2 \times 3 \times 1$  sq. ft., or 3 sq. ft.

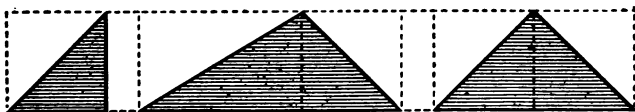
3. That the area of the right triangle *M* is equal to  $\frac{1}{2}$  of  $4 \times 3 \times 1$  sq. ft., or 6 sq. ft. Therefore, the area of *N* plus the area of *M* is equal to  $\frac{1}{2}$  of  $6 \times 3 \times 1$  sq. ft., or 9 sq. ft.

2. Find the area of a triangle whose base is 6 ft. and whose altitude is 3 ft.



Observe that the area of the triangle in example 2 is equal to the area of the two right triangles in example 1, and is, therefore, equal to  $\frac{1}{2}$  of  $6 \times 3 \times 1$  sq. ft., or 9 sq. ft.

Show by cutting and folding paper, as indicated in the following figures, that the area of each triangle is equal to one half the area of a rectangle, having the same base and altitude.

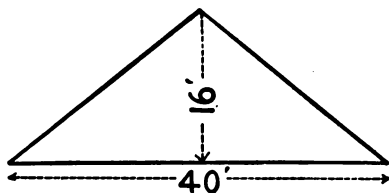


*The area of any triangle is found by multiplying the unit of measure by one half the product of the base and the altitude.*

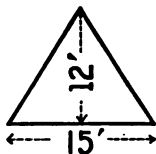
Find the area of the following triangles:

3. Base 20 ft., altitude 14 ft.    4. Base 15 ft., altitude 8 ft.

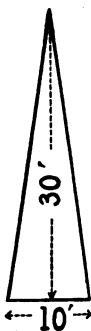
5. Base 10 ft., altitude 30 ft.
6. Base 18 ft., altitude 50 ft.
7. Find the areas of the following gables:



GABLE OF A BARN OR A HOUSE.

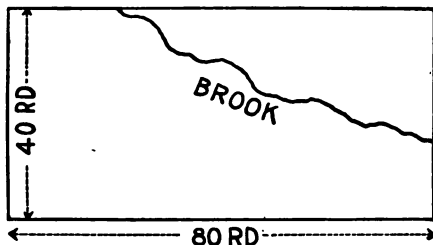


GABLE OF A TENT.

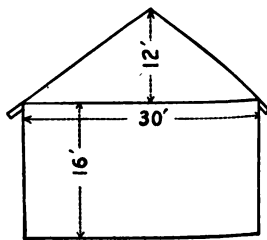
SIDE OF A  
CHURCH SPIRE.

NOTE. — The gable is only the surface included between the side of the roof and a straight line connecting the eaves.

8. Mr. Smith's farm is in the form of a rectangle and has a running stream across one corner. If the corner cut off by the stream is 58 rd. long and 24 rd. wide, find the approximate area of each part.



9. Find the number of square yards of tent cloth in a sleeping-room tent, the sides of which are  $15' \times 10'$  and the triangular ends  $12'$  long and  $8'$  high; in a cooking tent with sides  $22' \times 15'$  and triangular ends  $18' \times 12'$ .



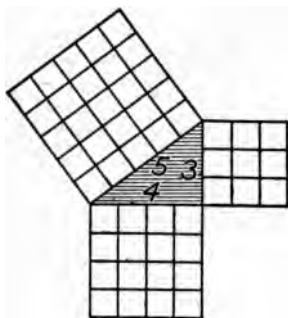
10. Find the area of the gable; then the area of the gable end of this house.

### Finding the hypotenuse of a right triangle.

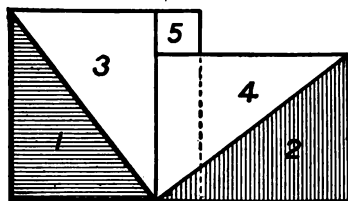
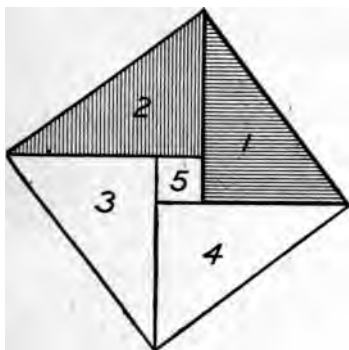
The **hypotenuse** of a right triangle is the side opposite the right angle.

1. How many square units are there in the square described upon the hypotenuse? in the square described upon the perpendicular? in the square described upon the base?

2. How do the number of square units described upon the hypotenuse compare with the *sum* of the square units described upon the other two sides?



That this is universally true is shown by the following diagram :



Take any right triangle, as 1; lay it off on a piece of cardboard and draw the square on its hypotenuse. Cut this square into the four equal triangles 1, 2, 3, and 4, and the small square 5, as here shown.

By changing the position of the triangles 1 and 2 as indicated, we change the first diagram into the second. But the first is the square on the hypotenuse, and the second is the sum of the squares on the other two sides. Since they are equal, the truth of the proposition is evident.

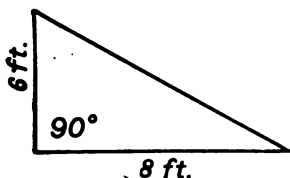
*The square on the hypotenuse of a right triangle equals the sum of the squares described on the other two sides.*

The **square root** of a number is one of its two equal factors. Thus, the square root of 25 is 5, written  $\sqrt{25} = 5$ .

Memorize this table :

Square of 13 = 169	Square of 19 = 361
Square of 14 = 196	Square of 20 = 400
Square of 15 = 225	Square of 21 = 441
Square of 16 = 256	Square of 22 = 484
Square of 17 = 289	Square of 23 = 529
Square of 18 = 324	Square of 24 = 576
Square of 25 = 625	

### Written Work



1. Find the hypotenuse of this right triangle.

$$6^2 + 8^2 = 100, \text{ hypotenuse}^2$$

$$\sqrt{100} = 10, \text{ no. of ft., hypotenuse}$$

Draw figures to a convenient scale and find the unknown side.

	BASE	PERPENDICULAR	HYPOTENUSE
2.	3 in.	4 in.	?
3.	?	3 ft.	5 ft.
4.	5 ft.	12 ft.	?
5.	15 yd.	20 yd.	?
6.	?	6 in.	10 in.

7. Find the length of the longest straight line that can be drawn on a table 12 ft. by 9 ft.

8. A has a field 40 rd. long and 30 rd. wide. B has a square field whose side equals the diagonal of A's field. What is the difference in the areas of the two fields?

### QUADRILATERALS

A **quadrilateral** is a surface having four straight sides. (Quadrilateral means having *four sides*.)

1. A **square** is a quadrilateral having four equal sides and four right angles.

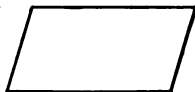
2. A **rectangle** is a quadrilateral having four straight sides and four right angles.

3. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

4. A **rhombus** is a quadrilateral whose sides are *equal*, and whose angles are *not* right angles.



RHOMBUS.



RHOMBOID.

5. A **rhomboid** is a quadrilateral whose *opposite* sides are equal and whose angles are *not* right angles.



TRAPEZOID.



TRAPEZIUM.

6. A **trapezoid** is a quadrilateral having but *two* sides parallel.

7. A **trapezium** is a quadrilateral having *no* two sides parallel.

8. Describe each of the six quadrilaterals named above with reference to its sides and angles. How many of these quadrilaterals are parallelograms? Give reasons.

### AREAS OF RECTANGLES

NOTE. — Review areas of rectangles, pp. 153, 154.



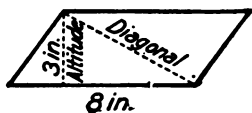
## Written Work

Find the areas of rectangles having the following dimensions :

1. 41 ft. by 6 ft.
2. 5.25 ft. by 27.6 ft.
3. 144 yd. by  $20\frac{1}{4}$  yd.
4. 4.5 yd. by 7 yd. 2 ft.
5. 57.5 ft. by 9 ft.
6.  $22\frac{1}{2}$  yd. by 14 ft.
7. 54 in. by 6 ft.
8. 108 ft. by 54 in.

## Finding the area of a parallelogram.

Find the area of a parallelogram whose base is 8 in. and altitude 3 in.



Observe: 1. That the diagonal of the parallelogram divides it into two equal triangles.

2. That the area of each triangle is equal to  $\frac{1}{2}$  of  $8 \times 3 \times 1$  sq. in., and the area of the parallelogram is equal to  $\frac{1}{2}$ , or once, the product of the base and altitude; that is,  $8 \times 3 \times 1$  sq. in., or 24 sq. in.

*The area of a parallelogram is found by multiplying the unit of measure by the product of the base and the altitude.*

## Written Work

Find the area in acres of :

1. A parallelogram whose base is 140 rd. and altitude 60 rd.

2. A rhomboid whose base is 90 rd. and altitude  $50\frac{1}{2}$  rd.

3. A rhombus whose base is 120 rd. and altitude 100 rd.

Find the altitude of :

4. A rhomboid whose area is 7.5 A. and base 48 rd.

5. A rhomboid whose area is 6.125 A. and base 140 rd.

6. Find the base of a parallelogram whose altitude is  $60\frac{1}{2}$  rd. and whose area is 30.25 A.

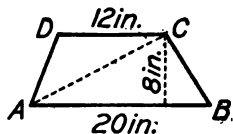
### Finding the area of a trapezoid.

#### Written Work

1. Find the area of a trapezoid whose parallel sides are 20 in. and 12 in., and whose altitude is 8 in.

Examine the trapezoid  $ABCD$ . Draw the diagonal  $AC$ , dividing it into two triangles.

Observe: 1. That the area of the trapezoid is equal to the area of its two triangles  $ABC$  and  $ACD$ .



2. That the area of the triangle  $ABC$  equals  $\frac{1}{2}$  of  $20 \times 8 \times 1$  sq. in.
3. That the area of the triangle  $ACD$  equals  $\frac{1}{2}$  of  $12 \times 8 \times 1$  sq. in.
4. That the area of the trapezoid equals  $\frac{1}{2}$  of  $(20 + 12) \times 8 \times 1$  sq. in., 128 square inches.

*The area of a trapezoid is found by multiplying the unit of measure by the product of the altitude and one half the sum of the parallel sides.*

2. The parallel sides of a trapezoid are 38 in. and 62 in., and its altitude is 21 in. Find its area.
3. The area of a trapezoid is 2.5 A. The sum of its parallel sides is 80 rd. Find its altitude.

### Finding the area of a trapezium.

#### Written Work

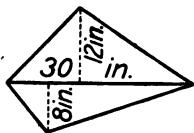
1. Find the area of a trapezium whose diagonal is 30 in. and whose altitudes are 12 in. and 8 in.

Observe: 1. That the area of the trapezium equals the area of its two triangles.

2. That the area of one triangle equals  $\frac{1}{2}$  of  $30 \times 8 \times 1$  sq. in.

3. That the area of the other triangle equals  $\frac{1}{2}$  of  $30 \times 12 \times 1$  sq. in.

4. That the area of the trapezium equals  $\frac{1}{2}$  of  $30 \times 20 \times 1$  sq. in., or 300 sq. in.

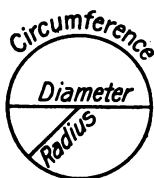


*The area of a trapezium is found by dividing it into triangles and finding the sum of their areas.*

2. The base line dividing a trapezium into two triangles is 40 ft. The altitude of one triangle is 10 ft., of the other is 12 ft. Find the area of the trapezium.

3. A trapezium is divided into two triangles by a line 28 ft. long. Find the area of the trapezium, if the altitude of one triangle is 8 ft. and of the other triangle, 14 ft.

## CIRCLES



Observe the figure. What is its shape? Observe that its boundary line changes its direction at every point.

A circle is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center*.

The **circumference** of a circle is its bounding line.

A **diameter** is a straight line passing through the center, with both ends terminating in the circumference.

A **radius** is a straight line extending from the center to the circumference.

Measure carefully with a cord the distance around a circle 1 foot in diameter, and you will find that it is about 3.1416 ft. in circumference. This relation of diameter to circumference is true of all circles.

*The circumference of a circle is found by multiplying the diameter by 3.1416. This ratio is represented by the symbol  $\pi$  ( $\pi$ ).*

*The diameter of a circle is found by dividing the circumference by 3.1416.*

## Written Work

Find the circumference if the diameter is:

- |                                      |                        |                      |
|--------------------------------------|------------------------|----------------------|
| 1. 15 ft.                            | 4. $10\frac{3}{4}$ ft. | 7. 40 ft. 4 in.      |
| 2. 25 ft.                            | 5. 12 ft. 6 in.        | 8. 30 in.            |
| 3. 60 ft.                            | 6. 28 yd.              | 9. 8 yd. 2 ft. 4 in. |
| 10. Circumference equals 25.1328 ft. | Radius = ?             |                      |
| 11. Circumference equals 125.664 yd. | Diameter = ?           |                      |

**Finding the area of a circle.**

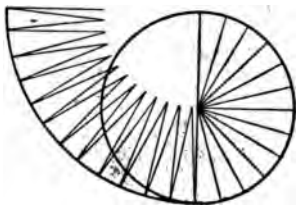
## 1ST METHOD

Examine the figure:

Observe: 1. That the circle may be considered as made up of triangles whose bases form the circumference.

2. That the radius of the circle is equal to the altitudes of the triangles.

3. That the area of the circle is equal to the areas of all the triangles, or  $\frac{1}{2}$  of the sum of their bases (circumference) by their altitude (radius).



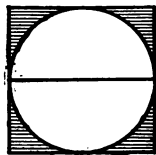
*The area of a circle is found by multiplying the circumference by one half the radius.*

## 2D METHOD

Examine the circle inscribed in the square.

Observe: 1. That the diameter of the circle is just equal to the side of the circumscribed square.

2. That *much* of the surface of the square, but *not all of it*, lies within the circumference. Careful measurement shows that about .7854 of the surface of any square lies within the circumference of the inscribed circle.



*The area of a circle equals .7854 of the surface of the circumscribed square.*

**Written Work**

Circumference =  $C$ . Diameter =  $D$ . Radius =  $R$ . Area =  $A$ .

Find the area if :

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| 1. $D = 10$ rd. | 4. $R = 18$ ft. | 7. $R = 40$ rd. |
| 2. $R = 10$ rd. | 5. $D = 20$ in. | 8. $D = 25$ yd. |
| 3. $D = 18$ ft. | 6. $R = 20$ in. | 9. $R = 60$ ft. |

Find the area to two decimal places if :

- |                      |                  |                   |
|----------------------|------------------|-------------------|
| 10. $C = 3.1416$ ft. | 13. $D = 35$ yd. | 16. $R = 125$ ft. |
| 11. $C = 6.2832$ rd. | 14. $R = 10$ ft. | 17. $D = 120$ yd. |
| 12. $C = 94.248$ in. | 15. $D = 10$ yd. | 18. $R = 19$ yd.  |
19. A circular fountain 20 ft. in diameter is surrounded by a cement walk 4 ft. wide. How much will the walk cost at \$1.50 per square yard ?

NOTE. — Find the difference between the areas of the two circles, the first bounded by the circumference of the fountain, and the second, by the circumference of the walk, the diameter of which is 28 ft.

**RECTANGULAR SOLIDS**

A **solid** is anything that has length, breadth, and thickness.

The **faces** of a solid are the surfaces that bound it.

The **lateral** or **convex surface** of a solid is the area of its sides, or faces.

The **volume** of a solid is the number of cubic units it contains.

Learn this table of solid or cubic measure :

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.)	= 1 cubic yard (cu. yd.)
128 cubic feet (cu. ft.)	= 1 cord of wood or tanbark
128 cubic feet (cu. ft.)	= 1 cord of stone

1 cubic yard of earth equals 1 load.

A **cube** is a solid bounded by six equal square faces.

The unit of cubic measure is a cube whose edge is one of the linear units. Thus, a cube each edge of which is one inch in length is a **cubic inch**.

A **rectangular solid** is a solid bounded by six rectangular surfaces.



### Written Work

Find the entire surface of :

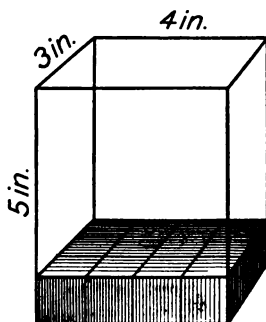
#### RECTANGULAR SOLIDS

1. 12 ft. by 8 ft. by 6 ft.
2. 20 in. by 10 in. by 10 in.
3. 16 ft. by 2 ft. by  $1\frac{1}{2}$  ft.
4. 10 ft. by 8 ft. by 7 ft.
5. 6 ft. by 5 ft. by 5 ft.
6. 13 ft. by 8 ft. by 3 ft.
7. 20 ft. by 9 ft. by 7 ft.

#### CUBES

8. 4 inches on an edge.
9. 12 inches on an edge.
10. 2 feet on an edge.
11.  $12\frac{1}{2}$  inches on an edge.
12. 14 inches on an edge.
13.  $11\frac{1}{2}$  feet on an edge.
14.  $14\frac{1}{2}$  inches on an edge.

Each cube in the solid represents **one cubic inch**. How many cubic inches are there in the first layer? How many such layers does this solid contain? How many cubic inches does the solid contain? Observe that the product of the three dimensions expresses the number of *cubic units*.



Scale:  $\frac{1}{4}$  inch = 1 in.

If the dimensions are expressed in inches, the unit of measure is 1 *cubic inch*; if expressed in feet, the unit of measure is 1 *cubic foot*; if expressed in yards, the unit of measure is 1 *cubic yard*. If the dimensions are expressed in related units, as feet and inches, or yards and feet, they must first be changed to like units.

*The volume of a rectangular solid is found by multiplying the unit of measure by the product of its three dimensions when expressed in like units.*

### Written Work

1. Find the volume of a rectangular solid 8 ft. 6 in. square and 12 ft. 4 in. in length.

SOLUTION. —  $8.5 \times 8.5 \times 12\frac{1}{3} \times 1 \text{ cu. ft.} = 891.08\frac{1}{3} \text{ cu. ft., volume.}$

Find the contents or volume of the following solids:

2. 10 ft. by 6 ft. 3 in. by 4 ft.
3. 12 ft. by 9 ft. 6 in. by 6 ft.
4. 10 ft. square and 8 ft. high.
5. 1 yd. by 2 ft. by 18 in.
6. 68 in. by 1 ft. by 10 in.
7. 5 yd. by  $1\frac{1}{2}$  yd. by 2 ft.

8. How many loads of earth must be removed in excavating for a cellar 30 ft. by 24 ft. and 8 ft. in depth?

9. Estimate the number of cakes of soap 3 in. by 2 in. by 2 in. that can be packed in a box 3 ft. by 2 ft. by 2 ft.

10. A schoolroom is 40 ft. by 28 ft. by 16 ft. How many cubic feet of air space are there for each of 39 pupils and their teacher?

11. How many cords of 4-foot wood are there in a pile 40 ft. long and 4 ft. high?

12. Estimate the number of cords of 18-inch wood in 3 piles each 60 ft. long and 4 ft. high.

NOTE. — The number of cords of short wood in a pile is found by dividing the number of square feet in one side by 32. See page 366.

13. How many cubical boxes 3 ft. 6 in. on an edge can be placed in a storage room 14 ft. in length, width, and height?

In a certain township, the piles of 20-inch wood in the yards of four schools were as follows :

	NO. OF PILES	LENGTH OF PILES	HEIGHT OF PILES
14. Sykes,	{ 1	60 ft.	4 ft.
	{ 2	40 ft.	6 ft.
15. Graham,	{ 2	50 ft.	5 ft.
	{ 1	60 ft.	4 ft.
16. Wilson,	{ 1	72 ft.	5 ft.
	{ 2	45 ft.	6 ft.
17. Clark,	{ 2	36 ft.	5 ft.
	{ 1	60 ft.	4 ft.

Estimate the number of cords at each school and the value of the wood at \$1.85 per cord.

18. Find the number of loads of earth removed in excavating for a cellar 16 ft. wide, 30 ft. long, and 6 ft. in depth.

19. A pile of tanbark is 8 ft. wide, 9 ft. high, and 100 ft. long. Find the number of cords.

20. A cubical block of granite 2 ft. on an edge is what part of a cubical block of granite 6 ft. on an edge?

When possible, use cancellation in the following problems:

21. Cape Cod cranberries are shipped in a crate whose inside dimensions are 20 in.  $\times$   $10\frac{1}{2}$  in.  $\times$   $6\frac{3}{4}$  in. How many cubic inches are there in a crate?

22. Sweet potatoes are sometimes sold in a box  $19\frac{1}{4}$  in.  $\times$   $11\frac{1}{4}$  in.  $\times$  10 in. How much does this differ from a bushel (2150.4 cu. in.)?

23. Colorado apples are sometimes shipped in a box 18 in.  $\times$   $11\frac{1}{2}$  in.  $\times$  11 in. How many cubic inches more or less than a bushel does such a box contain?

24. Colorado apples are sometimes shipped in a box 16 in.  $\times$   $11\frac{3}{4}$  in.  $\times$   $8\frac{5}{8}$  in. How many cubic inches does such a box contain?

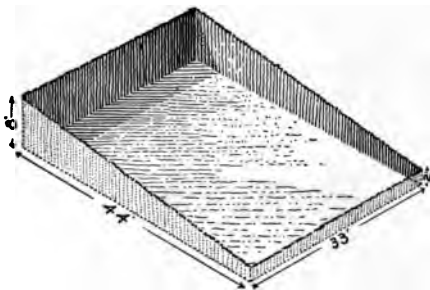


25. California celery is shipped in crates  $24\frac{1}{4}$  in.  $\times$  22 in.  $\times$   $20\frac{1}{2}$  in. How many cubic inches are there in such a crate?

26. California dates are sold in boxes  $17\frac{1}{4}$  in.  $\times$  10 in.  $\times$   $9\frac{1}{4}$  in. How many cubic inches are there in such a box?

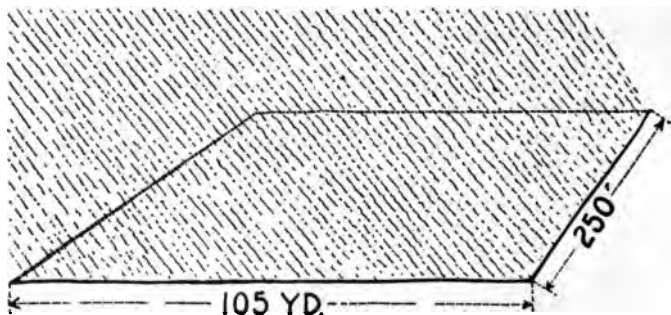
27. The standard California orange box is now 24 in.  $\times$   $11\frac{1}{2}$  in.  $\times$  12 in. Tangerines are shipped in boxes 24 in.  $\times$  12 in.  $\times$   $6\frac{1}{2}$  in. Which is the larger, and how many cubic inches larger is it?

28. How many cubic inches are there in a box  $5\frac{1}{2}$  in.  $\times$   $8\frac{1}{4}$  in.  $\times$   $2\frac{3}{4}$  in.? Find the number of cubic inches in a box  $8\frac{1}{2}$  in.  $\times$   $12\frac{1}{2}$  in.  $\times$   $4\frac{1}{4}$  in.



29. A man had a cellar to dig  $44' \times 33'$ . The rear end was to be dug 8 ft. deep, and the front end 2 ft. deep. Find the number of loads to be removed.

30. Mr. Brown had a lot on which  $\frac{3}{4}$ " of rain fell in two days. How many barrels of water fell, if the lot was 105 yd. long and 250 ft. wide?



**31.** Charles Franklin dug a cellar 25 ft. by 50 ft. The depths at the corners after excavation were 3 ft., 4 ft., 5 ft., and 6 ft., respectively. How many loads of earth were taken out ?

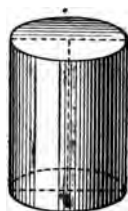
**NOTE.**— The average of the four corners gives the average depth of the cellar excavated.

## CYLINDERS

Examine this solid.

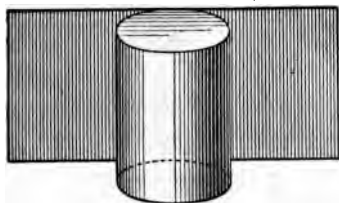
How many ends or bases has it ? What is the shape of each ? Are the bases equal and parallel ? Describe the shape of the body.

A **cylinder** is a solid whose two bases are equal and parallel circles and whose diameter is uniform.



The **convex surface** of a cylinder is the lateral or curved surface. The **altitude** is the perpendicular distance between its two bases.

Examine this cylinder.



Observe: 1. That if a piece of paper is fitted to cover its convex surface and then unrolled, its form will be that of a rectangle.

2. That the circumference of the base is the *length* of the rectangle, and the altitude of the cylinder is the *width* of the rectangle.

*The convex surface of a cylinder is found by multiplying the unit of measure by the product of the circumference and the altitude.*

*The entire surface of a cylinder is found by adding the area of the bases to the convex surface.*

**Written Work**

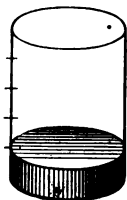
Find the convex surface of a cylinder:

- |                                   |                                  |
|-----------------------------------|----------------------------------|
| 1. <i>D</i> 10 in., height 24 in. | 4. <i>D</i> 20 in., height 4 ft. |
| 2. <i>D</i> 15 in., height 30 in. | 5. <i>D</i> 8 in., height 4 ft.  |
| 3. <i>D</i> 2 ft., height 10 ft.  | 6. <i>D</i> 6 ft., height 15 ft. |

Find the entire surface of:

7. A water tank that is 12 ft. in diameter and 12 ft. in height.
8. A steam boiler that is 15 ft. long and 3 ft. in diameter.

9. Find the volume of a cylinder 3 ft. in diameter and 5 ft. high.



Observe: 1. That the area of the base is  $3^2 \times .7854$   $\times 1$  sq. ft., or 7.0686 sq. ft.

2. That the first row of cubic units contains 7.0686 cu. ft.

3. That the cylinder contains 5 times 7.0686 cu. ft., or 35.343 cu. ft.

*The volume of a cylinder is found by multiplying the unit of measure by the area of the base, and this product by the height of the cylinder.*

Find the volume of a gas tank, silo, cistern, etc. :

- |                                    |                                   |
|------------------------------------|-----------------------------------|
| 10. <i>D</i> 15 ft., height 18 ft. | 14. <i>R</i> 2 ft., depth 8 ft.   |
| 11. <i>D</i> 25 ft., height 30 ft. | 15. <i>R</i> 8 ft., height 30 ft. |
| 12. <i>D</i> 16 ft., height 20 ft. | 16. <i>D</i> 1 ft., length 16 ft. |
| 13. <i>D</i> 20 ft., depth 15 ft.  | 17. <i>D</i> 5 ft., length 12 ft. |

## BINS AND TANKS

Wheat and other grains are generally sold by weight, but the capacity of bins is often estimated in bushels. The capacity of tanks and cisterns is estimated in gallons or in barrels.

NOTE. — The *standard bushel* in the United States contains 2150.42 cubic inches, stricken measure, and 2747.71 cubic inches heaped measure.

231 cu. in. = 1 gal.    31½ gal. = 1 bbl. when estimating contents.

## Written Work

Find to hundredths the number of bushels in :

1. A bin 20' by 10' by 5'.      3. A bin 12' by 15' by 8'.
2. A box 12' by 9' by 6'.      4. A bin 10' by 18' by 6'.
5. A metal trough for watering cattle is 12 ft. long, 3 ft. wide, and 20 in. deep. Find the number of gallons it holds.
6. The rainfall on a certain day was  $1\frac{1}{4}$  inches. Find the number of barrels of water that fell on Mr. Anderson's flower plot, which is 20 ft. long and 10 ft. wide.
7. Mr. James's house has a roof almost flat, 42' by 27'. The rainfall on a certain day was .3 of an inch. If the spouting carries away all the water, how many barrels fell on the roof ?
8. The swimming pool at the school averages 3 ft. and 8 in. of water in a pool 40 ft. by 100 ft. How many gallons of water are there ? how many barrels ?
9. A man has 40 head of cattle that drink on an average, in July, 10 gallons of water per day. How many barrels is that ? If the windmill averages 10 barrels per hour, how many hours per day must it run to pump sufficient water for the cattle ?

**APPROXIMATE MEASUREMENTS**

Approximate equivalents of the following measures are:

1 bu. shelled grains	= $1\frac{1}{4}$ cu. ft.
1 bu. apples	= $1\frac{5}{9}$ cu. ft.
1 bbl. in estimating contents	= $4\frac{1}{6}$ cu. ft.
1 cu. ft. of water	= $62\frac{1}{2}$ lb.
1 gal. of water	= $8\frac{1}{8}$ lb.
1 cu. ft of any liquid	= $7\frac{1}{2}$ gal.
1 ton of clover hay	= 550 cu. ft.
1 ton of soft (bituminous) coal	= 42 cu. ft.
1 ton of hard (anthracite) coal	= 35 cu. ft.

**Written Work**

1. A water meter registered 900 gallons of water consumed in a month. Estimate the weight of the water used and its volume in cubic feet.

2. The inside measurement of a wagon box is 12 ft. 4 in. by 3 ft. 6 in. by 16 in. Estimate the number of tons, etc., of hard coal it would contain; of soft coal.

3. The rainfall on a flat roof 20 ft. by 30 ft. during April and May was 9.5 in. Find the weight of the water that fell on the roof during that time.

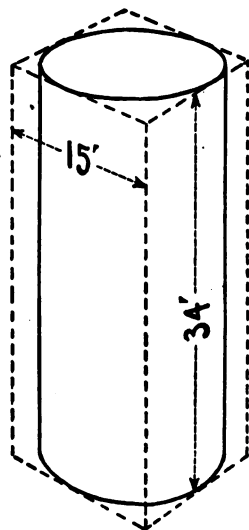
4. A swimming pool is 80 ft. long, 60 ft. wide, and 5 ft. deep. Estimate approximately the number of barrels of water in the pool.

5. Estimate the number of bushels in a bin for oats 14 ft. long and 10 ft. wide, if the bin is filled with oats to a depth of 6 ft.

6. How many tons of hard coal are there in a bin 16 ft.  $\times$  12 ft., when the pile is 1 ft. high?
7. There are 5 ft. of water in a cistern 4 ft. square. How many gallons of water are there in the cistern?
8. Find the weight of the water in a railroad tank 12 ft. square and 16 ft. in depth, if the tank has 12 ft. of water in it.
9. How many bushels of wheat can be shipped in a car whose inside measurements are 36 ft. by  $8\frac{1}{2}$  ft. by 8 ft.?
10. Estimate the number of tons of clover hay in a mow 60 ft. by 18 ft. by 16 ft.

### CIRCULAR SILOS, CISTERNS, ETC.

We learn in geometry that the cubical contents of a cylindrical vessel is approximately .80, or  $\frac{4}{5}$ , as much as that of a square vessel, whose side is equal to the diameter of the cylindrical vessel and whose height is the same. Observe the picture. What are the dimensions of each vessel?



#### Written Work

1. Mr. Ford builds a silo 20 ft. in diameter, inside measurement, and 25 ft. in height. How many cubic feet, approximately, of silage (green corn fodder) will it hold? It is estimated that this silo when full will hold 143 tons of silage. Find from this the weight of a cubic foot of silage.

2. On the tops of three buildings there are round tanks to hold water for use in case of fire. They are, respectively, 7, 8, and 9 feet in diameter, and 10 feet in height. About how many gallons of water will each hold?

3. How much water will a water main hold that is 20 ft. in length, with an inside diameter of 15 in.?

4. A small town has a cylindrical water tank to supply the water. It is 30 ft. in diameter and 30 ft. in height. How many gallons, approximately, will it hold?

5. William measures the depth of the water in his father's well and finds it to be 8 ft. The well is 5 ft. in diameter. Estimate the number of barrels of water in the well.

6. An oil tank of a railroad car is 30 ft. in length and  $4\frac{1}{2}$  ft. inside diameter. How many gallons of oil, approximately, will it hold?

7. A piece of stove pipe is 30 in. in length and 8 in. inside diameter. How many cubic inches of air, approximately, are there in the stove pipe?

8. Find the weight of the water in a railroad tank 12 ft. in diameter and 16 ft. in depth, if the tank has 12 ft. of water in it.

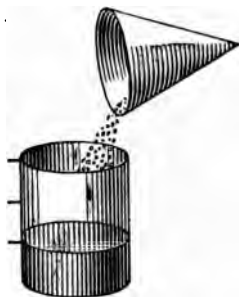
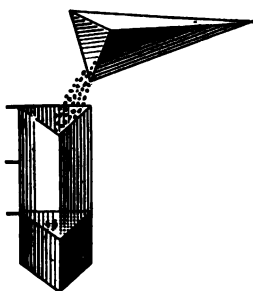
9. Ben's father has a windmill pump to pump the water to a trough for the cattle. The trough is 2 ft. wide, 15 ft. long and 18 in. in depth. How many barrels of water will it hold?

10. Estimate the number of tons of clover hay in a mow 50 ft. by 14 ft. by 16 ft.

11. A water wagon has a tank 12 ft. in length and  $3\frac{1}{2}$  ft. in diameter. Estimate quickly the number of barrels it holds.

# PYRAMIDS AND CONES

1. Fill a hollow pyramid with sand. Empty it into a prism having the same base and altitude. How often must the pyramid be filled and emptied to fill the prism? The volume of a pyramid, then, is what part of the volume of the prism?



2. Measure, in like manner, with a cone, the volume of a cylinder having the same dimensions. The volume of the cone is what part of the volume of the cylinder?

Observe: 1. That the dimensions of the pyramid and of the prism are the same, and that those of the cone and of the cylinder are the same.

2. That the volume of the pyramid is  $\frac{1}{3}$  that of the prism, and the volume of the cone is  $\frac{1}{3}$  that of the cylinder.

By geometry, it is shown that the volume of a pyramid is  $\frac{1}{3}$  of that of a prism having an equal base and an equal altitude. Hence

*The volume of a pyramid equals one third of the volume of a prism of like dimensions.*

*The volume of a cone equals one third of the volume of a cylinder of like dimensions.*



We have already learned that the volume of a prism or of a cylinder is found by multiplying its unit of measure by the product of the area of its base by its altitude. Hence

*The volume of a pyramid or of a cone is found by multiplying its unit of measure by one third the product of the altitude and the area of the base.*

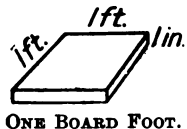
### Written Work

1. Find the volume of a cone whose altitude is 12 in. and whose base is 8 in. in diameter.
2. Find the volume of a pyramid whose base is 12 in. square and whose altitude is 30 in.
3. A square pyramid whose side is 18 in. is 32 in. high. Find its volume.
4. Find the volume of a cone whose altitude is 10 ft. and whose base is 6 ft. in diameter.
5. Find the contents of a rectangular pyramid 15 ft. high, the sides of whose base are 10 ft. and 12 ft., respectively.

### LUMBER

#### Measurement of lumber.

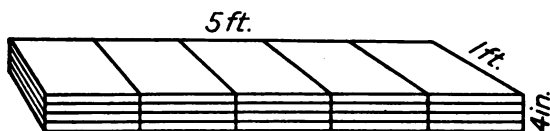
**Lumber** is any kind of sawed timber as boards, planks, sills, etc. The unit of lumber measure is the **board foot**; it is a board 1 ft. long, 1 ft. wide, and 1 in. thick. Draw it.



NOTE. — Boards less than 1 inch in thickness are measured as if they were 1 inch thick; boards over 1 inch in thickness are measured by their actual thickness in inches and fractions of an inch.

1. How many board feet are there in a board 1 ft. wide, 1 in. thick, and 3 ft. long? 5 ft. long?
2. How many board feet are there in a board 6 in. wide, 1 in. thick, and 3 ft. long? 10 ft. long?

3. How many board feet are there in a sill 5 ft. long, 1 ft. wide, and 4 in. thick ?



### Written Work

1. Find the number of board feet in a sill 18 ft. long, 10 in. wide, and 8 in. thick.

10 in. =  $\frac{5}{6}$  ft. One surface =  $18 \times \frac{5}{6} \times 1$  board foot, or 15 board feet.  
The sill contains  $8 \times 15$  board feet, or 120 board feet.

*The number of board feet in a piece of lumber is found by multiplying the number of board feet in one surface by the number of inches in thickness.*

Find the number of board feet in the following :

2. 1 board, 10 ft. long,  $1\frac{1}{2}$  ft. wide, and 1 in. thick.
3. 1 board, 16 ft. long,  $1\frac{1}{2}$  ft. wide, and  $\frac{3}{4}$  in. thick.
4. 2 boards, each 16 ft. long, 1 ft. wide, and  $\frac{1}{2}$  in. thick.
5. 6 boards, 15 ft.  $\times$  2 ft.  $\times$  1 in.
6. 4 boards, 16 ft.  $\times$   $1\frac{1}{2}$  ft.  $\times$   $\frac{1}{2}$  in.

How many feet of lumber are there in :

7. 1 plank, 12 ft. long, 1 ft. wide, and 3 in. thick ?
8. 1 sill, 15 ft. long,  $1\frac{1}{4}$  ft. wide, and 8 in. thick ?
9. 4 planks, 12 ft. long,  $1\frac{1}{2}$  ft. wide, and 2 in. thick ?
10. 2 pieces, 18 ft. by 1 ft. by 1 ft. ?

Find the number of feet of lumber in :

11. 10 planks, each 8 ft. long,  $1\frac{1}{2}$  ft. wide, and 3 in. thick.
12. 12 sills, each 20 ft. long and 10 in. square.
13. 20 joists, each 12 ft. long, 12 in. wide, and 3 in. thick.

14. 3 beams, each 40 ft. long and 10 in. by 12 in.
15. 30 scantlings, each 16 ft. long and 2 in. by 3 in.
16. The 1-inch flooring of two rooms, each 18 ft. by 20 ft.

### Buying and selling lumber.

Lumber is bought and sold by the *thousand board feet*. In practice the cost is computed at so much per board foot. Thus, \$20 per thousand feet (M.) is \$.02 *per board foot*.

Show that \$35 per M. = \$.035 per board foot.

\$60 per M. = \$.06 per board foot.

### Written Work

1. Estimate the cost of 378 ft. of lumber at \$26 per M.; of 6389 ft. at \$48 per M.; of 972 ft. at \$72 per M.; of 693 ft. at \$47 per M.

Find the cost, at \$35 per M., of :

2. 50 boards, 16 ft. long, 12 in. wide, and 1 in. thick.
3. 60 boards, 12 ft. long, 15 in. wide, and  $1\frac{1}{2}$  in. thick.
4. 100 boards, 15 ft. long, 6 in. wide, and  $\frac{3}{4}$  in. thick.
5. 75 boards, 18 ft. long, 10 in. wide, and 1 in. thick.
6. 45 boards, 16 ft. long, 5 in. wide, and 1 in. thick.

Find the number of board feet in :

7. 120 studding,  $2'' \times 4'' \times 11'$ .
8. 400 planks,  $2'' \times 1' \times 16'$ .
9. 300 boards,  $1'' \times 10'' \times 14'$ .
10. 600 boards,  $1'' \times 6'' \times 16'$ .
11. 100 boards,  $\frac{3}{4}'' \times 12'' \times 16'$ .

12. 15 sills,  $6'' \times 10'' \times 20'$ .
13. 250 joists,  $2'' \times 8'' \times 24'$ .
14. 70 sills,  $10'' \times 12'' \times 30'$ .
15. 125 sleepers,  $3'' \times 10'' \times 28'$ .
16. 200 boards,  $\frac{1}{2}'' \times 4\frac{1}{2}'' \times 16'$ .
17. 500 joists,  $2\frac{1}{2}'' \times 8'' \times 20'$ .
18. 325 planks,  $3'' \times 14'' \times 16'$ .
19. 300 sills,  $5'' \times 8'' \times 24'$ .
20. 50 posts,  $10'' \times 12'' \times 14'$ .
21. 400 studding,  $2'' \times 3'' \times 18'$ .
22. 500 boards,  $1\frac{1}{2}'' \times 10'' \times 16'$ .

23. Estimate the cost of the planks in examples 8 and 18, at \$.027 per board foot.

24. Estimate the cost of the sills in examples 12, 14, and 19, at \$.032 per board foot.

25. Estimate the cost of the studding in examples 7 and 21, at \$.026 per board foot.

### REVIEW PROBLEMS

1. A field containing 20 acres is 64 rods long. How wide is it?

2. The area of the floor of a schoolroom contains 1120 sq. ft. The air in the room occupies 16,800 cu. ft. What is the height of the ceiling?

3. How many tiles 6 in. square are required for a hall 40 ft. by 20 ft. 6 in.?

4. The side of a square is 20 inches. Find its area; its perimeter.

5. The edge of a cube is 18 inches. Find its surface; its contents.

6. How many cubical boxes whose edges are 6 in. can be put into a box 8 ft. 6 in. by 4 ft. 6 in. by 3 ft.?

7. How many cakes of soap 2 in.  $\times$  2 in.  $\times$  4 in. can be packed in a box 2 ft. long, 1 ft. wide, and 1 ft. high?

8. The edges of two cubes are respectively 10 inches and 12 inches. How much more surface has one than the other?

9. Find the cost of a farm, 480 rods long and 320 rods wide, at \$60 per acre.

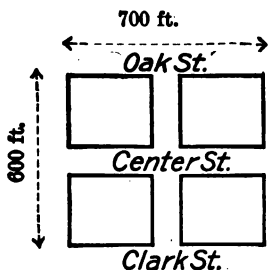
10. How much will it cost to put a wire fence around this farm at 50¢ per rod?

11. A ranchman bought one square mile of land at \$10 per acre. He put a fence around it and then divided it by fences into four equal square farms. Find the entire cost if the fence cost \$.40 per rod.

12. A level lot 60 ft. by 120 ft. has a dwelling erected on it 36 ft. by 42 ft. If the excavating averages 5 ft. and the removed earth is placed on the lot, to what height will it raise the grade of the lot?

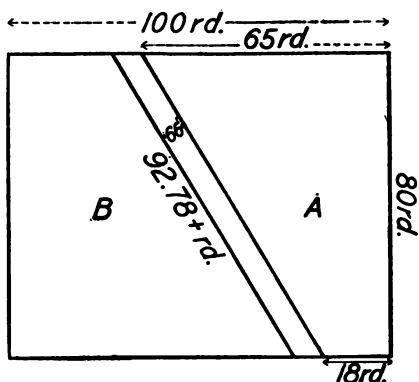
13. A certain town has a cylindrical water tank 20 ft. in diameter and 45 ft. in height. The gauge shows 30 ft. of water in the tank. Estimate the weight of the water.

14. How many yards of carpet, 1 yd. wide, are required for a room 24 ft. by 21 ft.?



15. This plot of ground is 700 ft. long and 600 ft. wide. Find the cost of grading streets 40 ft. in width run through the center each way, as here shown, at \$1.90 per square yard. Find the amount from the sale of all the lots (30'  $\times$  140') facing on Oak, Center, and Clark streets, at \$20 per front foot.

16. A railroad company owns a strip of land in the form of a parallelogram, 66 ft. wide and  $92.78 + \text{rd.}$  long through this farm. Find the area of A and the area of the part owned by the railroad. Find the area of B.



17. A wheel is 3 ft. in diameter. How many revolutions will it make in moving forward 942.48 ft.?

18. Lead is 11.35 times as heavy as water. Find the value of a cubic foot of lead at 5¢ per pound.

19. The base of a triangle is 30 ft. and its altitude is 23 ft. Find its area.

20. A barn is 80 ft. by 50 ft. It is 40 ft. to the base of the gable and 58 ft. to the top of the gable. How much will it cost to paint the sides and the gable ends at 8¢ per square yard?

21. If the slope of the roof of the barn in problem 20 is 32 ft. long, and it projects  $1\frac{1}{2}$  ft. at each end, how much will it cost to roof it at \$8 per square of 100 square feet?

22. A schoolroom is 40 ft. long and 30 ft. wide. Estimating 450 cubic feet of air to each person, what should be the height of the room to accommodate 39 pupils and their teacher?

23. How much will it cost to cement the floor of a cellar, 40 ft. by 20 ft., at 90¢ a square yard?

24. A street 50 ft. from curb to curb is opened for a distance of 300 yards. How much will it cost to excavate it, to a depth of 1 foot, at 40¢ per cubic yard?

25. How much will the curb of this street cost at 26¢ per linear foot?

26. A farmer built a circular silo 12 ft. in diameter and 24 ft. high. Find its contents in cubic feet.

27. How many blocks of ice, 2 ft.  $\times$  1 ft.  $\times$  1 ft., can be packed in a car 6 ft.  $\times$  8 ft.  $\times$  40 ft.? Ice is .92 as heavy as water. Find the weight of the ice.

28. A cistern is 4 ft. in diameter and 6 ft. deep. How many barrels, approximately, of water will it contain?

29. Estimate the weight of the water in a tank 8 ft. long, 6 ft. wide, and 2 ft. deep.

30. A vault is 5 ft. square and 6 ft. deep. How much will it cost to cement the sides and bottom at \$.50 per square foot?

31. A circular amusement park is 80 rd. in diameter. Find the cost of 1-inch boards for a tight board fence 8 ft. high, inclosing the park, at \$20 per M.

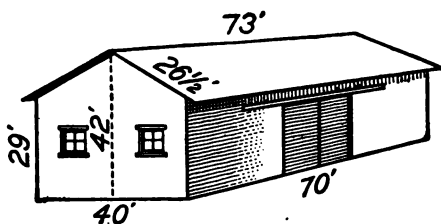
32. A corner lot in Denver, Colo., is 25 ft. by 100 ft. At \$25 per M., what will be the cost for 2-inch plank for a 10-foot sidewalk in front and on the side, including the corner?

NOTE.—Illustrate by diagram.

33. Mr. Ames owns a 50-ft. lot fronting on a street 60 ft. wide from curb to curb. The law compels him to pay  $\frac{1}{3}$  of the cost of paving the street in front of his lot. How much will it cost at \$2.90 per square yard?

34. A tank open at the top is 50 ft. long, 4 ft. wide, and 3 ft. deep. How much will the lead for lining it cost, at 8¢ per pound, estimating 4 pounds to a square foot?

35. Find the cost of painting the sides and ends of this hay barn at 25¢ a square yard; also the cost of staining the roof at 22¢ a square yard.



36. In excavating for a cellar 60 ft. long, 30 ft. wide, and 8 ft. deep, the material was evenly distributed over a lot 90 ft. by 40 ft. To what depth was the lot covered?

37. A two-story school building has 8 rooms 30 ft.  $\times$  32 ft. and a hallway 28 ft.  $\times$  15 ft., on each floor. How much will the flooring for the building cost at \$44 per M.?

38. From an artesian well 2 in. in diameter the water flows out at the rate of  $1\frac{1}{2}$  ft. per second. Find the number of barrels that flow out per hour.

39. One cubic inch of gold could be pounded into how many square inches of gold leaf  $\frac{1}{1000}$  of an inch in thickness?

40. A copper ingot containing 1 cubic foot is to be drawn into a copper wire  $\frac{1}{16}$  of an inch in diameter. Find the length of the wire when drawn.

41. A western farmer has a pile of corn in the ear, 400 ft. long. The pile at the end is in the form of an isosceles triangle, 12 ft. wide at the bottom, and the altitude of the pile is 6 ft. Find the number of bushels, allowing  $1\frac{1}{2}$  cu. ft. to a bushel.

42. What is the convex surface of a piece of stove pipe 2 ft. long and 8 in. in diameter?

43. The flow of water from the same source through two different pipes depends upon the area of the cross section of the openings. Compare the flow through a  $\frac{1}{2}$ -inch pipe with the flow through a  $\frac{3}{4}$ -inch pipe.



## TESTS FOR ACCURACY AND SPEED

**TO THE TEACHER.**—The short methods of computation learned in the preceding grades should be regularly used by the pupils, and increased skill and accuracy should be required.

### Written Work

Add the following :

**NOTE.**—In adding, group numbers into convenient combinations, as indicated in the preceding pages. Check all answers. Drill until each example can be added in  $1\frac{1}{4}$  minutes.

1.	2.	3.	4.	5.
\$ 167.34	\$ 428.99	\$ 175.13	\$ 436.74	\$ 829.94
43.26	160.50	436.18	234.62	4752.85
67.58	47.25	94.83	6758.41	661.54
987.53	675.83	2378.46	98.75	8675.83
49.76	67.24	671.42	497.61	672.44
8.39	654.91	916.45	839.84	29.73
557.81	9380.92	112.81	15.28	456.19
748.16	211.18	3819.29	755.18	5841.28
62.39	15.47	154.76	2639.43	75.41
822.16	357.23	35.72	288.65	746.39
721.50	1762.13	6167.31	7215.94	3284.67
18.52	55.61	834.25	3589.28	127.05
15.17	438.31	65.87	81.25	436.23
274.38	64.83	749.57	4728.93	8830.47
135.86	2461.72	938.41	13.49	921.11
<u>333.33</u>	<u>315.91</u>	<u>7184.67</u>	<u>6.78</u>	<u>5342.39</u>

Subtract and test :

6. $\begin{array}{r} 9500832 \\ 5362834 \\ \hline \end{array}$	9. $\begin{array}{r} \$ 28050.72 \\ 8493.84 \\ \hline \end{array}$	12. $\begin{array}{r} 3684021 \\ 986518 \\ \hline \end{array}$
7. $\begin{array}{r} 8029533 \\ 4960873 \\ \hline \end{array}$	10. $\begin{array}{r} \$ 95586.36 \\ 39679.85 \\ \hline \end{array}$	13. $\begin{array}{r} 2705420 \\ 647423 \\ \hline \end{array}$
8. $\begin{array}{r} 9780632 \\ 5928058 \\ \hline \end{array}$	11. $\begin{array}{r} \$ 79002.31 \\ 57220.90 \\ \hline \end{array}$	14. $\begin{array}{r} 970005 \\ 609099 \\ \hline \end{array}$

Multiply :

15. $\begin{array}{r} 8984 \\ 2169 \\ \hline \end{array}$	18. $\begin{array}{r} 8056 \\ 6907 \\ \hline \end{array}$	21. $\begin{array}{r} 8956 \\ 9547 \\ \hline \end{array}$	24. $\begin{array}{r} \$ 40.56 \\ 87.05 \\ \hline \end{array}$
16. $\begin{array}{r} 7564 \\ 6034 \\ \hline \end{array}$	19. $\begin{array}{r} 7650 \\ 4053 \\ \hline \end{array}$	22. $\begin{array}{r} 6528 \\ 4436 \\ \hline \end{array}$	25. $\begin{array}{r} \$ 39.42 \\ 56.31 \\ \hline \end{array}$
17. $\begin{array}{r} 9524 \\ 8069 \\ \hline \end{array}$	20. $\begin{array}{r} 3542 \\ 7009 \\ \hline \end{array}$	23. $\begin{array}{r} 5986 \\ 9608 \\ \hline \end{array}$	26. $\begin{array}{r} \$ 26.79 \\ 80.35 \\ \hline \end{array}$

Find the quotients and test:

27. $43160340 \div 862$	32. $\$ 58453.24 \div 857$
23. $2447592 \div 714$	33. $\$ 84546.85 \div 678$
29. $9268638 \div 534$	34. $\$ 227842.69 \div 586$
30. $7784648 \div 509$	35. $\$ 540401.87 \div 989$
31. $62752120 \div 890$	36. $\$ 443388.81 \div 489$

Simplify :

37. $\frac{2\frac{1}{2} - \frac{3}{8} + 2\frac{1}{4} - \frac{5}{6}}{4\frac{1}{4} + \frac{5}{8} - 1\frac{3}{4} + \frac{5}{12}}$	39. $\frac{2\frac{3}{5} \div \frac{13}{15} \times \frac{4\frac{1}{3} \div 6\frac{2}{3}}{5\frac{1}{3} \div \frac{8}{9} \times \frac{3\frac{3}{8} \div 2\frac{1}{4}}}$
38. $\frac{2\frac{3}{8} \times 3\frac{3}{6} \times \frac{5}{8}}{5\frac{1}{2} \times \frac{3}{11} \times \frac{5}{6}}$	40. $\frac{5\frac{1}{12} + 4\frac{1}{3}}{6\frac{3}{4} - 2\frac{5}{8}} \times \frac{11}{37}$

$$41. \frac{(\frac{4}{15} + 1\frac{1}{3}) \times (1\frac{7}{8} - \frac{1}{8})}{3\frac{3}{4} \times 1\frac{2}{3}}$$

$$43. \frac{(\frac{4}{15} + 1\frac{1}{3}) \div (3\frac{1}{9} \times \frac{4}{7})}{1\frac{2}{3} \times \frac{10}{21}}$$

$$42. \frac{\frac{8}{9} + (2\frac{1}{8} \div \frac{3}{4}) + \frac{5}{6}}{(3\frac{1}{2} - 1\frac{7}{8}) \div (1\frac{4}{5} \times \frac{1}{3})}$$

$$44. \frac{(\frac{5}{6} + \frac{3}{4}) \div (\frac{5}{12} - \frac{1}{3})}{3\frac{3}{4} \div 3\frac{1}{6}}$$

Add, as indicated, and test :

$$45. 8.07 + 72.5 + .039 + 87.$$

$$46. .45 + 5.07 + 8.557 + 6.5.$$

$$47. 40.937 + 89.09 + 41.005 + 11.9.$$

$$48. 62.093 + 20.98 + 16.909 + .5.$$

$$49. 12.015 + 9.99 + 12.001 + 2.2.$$

$$50. 12.3 + 15.024 + 8.023 + 18.014.$$

$$51. 3.7 + .001 + .65 + 11.4.$$

$$52. 15.025 + 10.01 + 9.009 + 16.087.$$

Subtract, as indicated, and test :

$$53. 28.7 - 8.95$$

$$57. 10.1 - 8.023$$

$$54. 9.009 - 7.04$$

$$58. 12.3 - 9.06$$

$$55. 219.75 - 39.009$$

$$59. 25.004 - 15.038$$

$$56. 40.275 - 12.056$$

$$60. 18.014 - 16.087$$

Find the products:

$$61. 1.005 \times .011$$

$$65. 39.75 \times .278$$

$$62. .375 \times 2.027$$

$$66. 2009 \times .06$$

$$63. 9.876 \times .786$$

$$67. 12.134 \times .025$$

$$64. 2.002 \times 4.004$$

$$68. 6.876 \times 4.37$$

Find the quotients:

$$69. 8.398 \div 3.7$$

$$73. 25.864 \div .777$$

$$70. 31.64 \div .056$$

$$74. .966 \div 46$$

$$71. .308 \div 14$$

$$75. .5 \div 125$$

$$72. 6.75 \div .009$$

$$76. 132.612 \div 15.42$$

## PROBLEMS FOR REVIEW

1. Reduce 25 rd. 4 yd. 2 ft. to feet.
2. How many minutes are there in 15 da. 11 hr. 15 min.?
3. How many feet are there in .875 of a mile?
4.  $\frac{5}{16}$  of a ton equal how many hundredweight and pounds?
5. A street is 1.85 miles long. What is its length in feet?
6. Find the cost of 3.5 gal. of sirup at 18¢ a pint.
7. How many minutes are there in 5.4 hr.?
8. Add 15 bu. 3 pk. 4 qt.; 9 bu. 7 qt.; 39 bu. 1 pk.; 25 bu. 2 pk. 5 qt.
9. A coal dealer sold four loads of coal weighing 1 T. 5 cwt. 15 lb.; 1 T. 7 cwt. 13 lb.; 1 T. 4 cwt. 85 lb.; 1 T. 5 cwt. 94 lb. Find the total weight.
10. Subtract 56 lb. 7 oz. (av.) from 72 lb. 3 oz. (av.).
11. Subtract 9 yr. 7 mo. 6 da. from 20 yr. 3 mo. 5 da., allowing 30 da. to a month.
12. How long a time elapsed from Feb. 22, 1732, to April 19, 1775?
13. Multiply 24 lb. 13 oz. (av.) by 28.
14. A wire fence inclosing a rectangular field 40 rd. 3 yd. by 15 rd. 4 yd. has three wires. Find the total length of wire in feet.

15. Find  $\frac{1}{6}$  of 36 bu. 6 qt.
16. Find  $\frac{1}{8}$  of 124 lb. 12 oz.
17. Find  $\frac{7}{8}$  of 272 ft. 6 in.
18. Find  $\frac{3}{4}$  of 36 bu. 3 pk. 4 qt.
19. Find  $\frac{2}{9}$  of 127 yd. 8 in.
20. Find  $\frac{5}{8}$  of 2 acres.
21. How many times are  $6\frac{1}{2}$  ft. contained in  $384\frac{1}{2}$  ft?
22. How many times will a wheel 10 ft. 8 in. in circumference turn in going 9 mi.?
23. A farm that contained 22 A. 50 sq. rd. was divided into lots, each containing 1 A. 10 sq. rd. Into how many lots was it divided?
24. An oil stove holding  $2\frac{3}{4}$  qt. of oil is filled from a can containing 2 gal. 3 qt. of oil. How many times can it be filled?
25. A field containing 17 A. and 60 sq. rd. was set off from a farm containing 120 A. and 40 sq. rd. The remainder was sold at \$72 an acre. How much was it sold for?
26.  $16\frac{2}{3}\%$  of a tub of butter containing 49 lb. 8 oz. was sold at 38¢ a pound. How much was received for the butter?
27. From a bin containing 54 bu. of grain, 20 bu. and 1 pk. were sold. What per cent of the grain remained?
28. A dealer bought 150 bbl. of apples at \$1.80 a barrel; but 6% were spoiled. The remainder were sold at \$2.50 a barrel. How much did the dealer gain or lose?
29. A man paid \$5000 for a house. For enlarging and repairing it, he spent 40% of this sum. He then sold it for \$7840. What per cent did he make?
30. A fruit merchant bought 120 bbl. of apples for \$360. He sold them at a gain of 55¢ a barrel. What was his gain per cent?

31. The window area of a schoolroom should be 20 % of the floor area. What should be the window area of a room 31 ft. 6 in. by 28 ft. ?

32. If 60 % of a farm containing 40 A. 120 sq. rd. was under cultivation, and 25 % of the remainder was woodland, how large was the woodland ?

33. Goods costing \$ 900 were marked at  $33\frac{1}{3}$  per cent above cost. They were sold at a discount of  $16\frac{2}{3}$  per cent on the marked price. What was the gain per cent ?

34. A house worth \$ 6000 is rented for \$ 42 a month. The taxes amount to \$ 80 ; the repairs to \$ 70 ; and other expenses to \$ 24. What per cent does the owner get on his investment ?

35. A commission merchant sold 240 bbl. of apples at \$ 5.50 a barrel, charging 3 % commission. If the freight charges were \$ 60, and cartage \$ 20, how much did the owner receive ?

36. A collector was employed to collect debts amounting to \$ 1250, his charge being 5 %. He was successful in collecting 82 % of the debts. What was his commission ?

37. A commission merchant sold 2000 bu. of corn at 90¢ a bushel. After deducting his commission and \$ 28 charges, he remitted to the owner \$ 1700. What was his rate of commission ?

38. A piano listed at \$ 1350 was bought at a discount of 20 % and 10 %. How much was it bought for ?

39. What will be the cost of a bill of goods listed at \$ 650 with discounts of 40 % and 5 % ; freight \$ 5.60 ?

40. Mr. Brown owns a house worth \$ 7200, assessed at  $\frac{3}{4}$  of its value. If the tax rate is 15 mills on the dollar, what is his tax ?

41. What per cent of the actual value of his house is his tax ?

42. A triangular lot 30 rd. by 40 rd. was cut off and sold from one corner of a square field containing 30 A. What per cent of the field remained?

43. How many gallons of water are there in a cistern 8 ft. in diameter and 9 ft. deep, if the cistern is  $33\frac{1}{3}\%$  full, considering  $7\frac{1}{2}$  gallons to equal one cubic foot?

44. A rectangular bin 6 ft. wide, 10 ft. high, and 20 ft. long is  $\frac{2}{3}$  filled with grain. How many bushels of grain are there, considering 1 bushel to equal  $1\frac{1}{4}$  cu. ft.?

45. A coal bin is 12 ft. long, 5 ft. 6 in. wide, and 6 ft. deep. Allowing 35 cu. ft. for a ton of hard coal, how much will it cost to fill the bin at \$6.50 a ton?

46. It is required to cover a walk 120 yd. long and 3 ft. wide with gravel to a depth of 4 inches. How many cubic yards are required?

47. It is desired to paint the curved surface of a circular iron tank 9 ft. deep and 10 ft. in diameter. How much will it cost at 42¢ a square yard?

48. At 35¢ a square foot how much will it cost to floor a semicircular platform 20 ft. in diameter?

49. Mr. Byers's farm is valued at \$18,000. He pays  $4\frac{2}{10}$  mills taxes on an 80% valuation of it. Find his tax.

50. An agent buys 30 tons of manure at \$1.50 per hundred, 20%, 10% off. Terms: 30 days net, or 2% for cash. If he pays cash, find the cost.

51. I have a note against Mr. Ford for \$500, bearing 6% interest, dated June 1, 1917, due in one year. If I discount the note at my bank March 1, 1918, at 7%, find the proceeds.

52. 560 yd. of cloth were bought at \$1.25 a yard and sold at \$2.25 a yard. A 60-day note for the amount was taken in payment and discounted at a bank at 6%. What was the gain?

## INTERPRETATION OF PROBLEMS

**TO THE TEACHER.**—Practice in the interpretation or explanation of problems should be continued throughout the book. The aim is to lead the pupils to form the habit of thinking out their problems before attempting the actual solutions. Practice in interpretation is designed to lead the pupils to see quickly what is required and what are the relations of the quantities in the problem. The following examples are simply suggestive.

1. An Iowa farmer buys cattle for \$1500, and sells them for \$2350. If the grazing and feeding are 20 % of the cost, find the per cent of profit on the sale.

*Required:* Per cent of profit on total outlay.

$20\% = \frac{1}{5}$ ;  $\frac{1}{5}$  of \$1500 = \$300, grazing and feeding expense.

$\$1500 + \$300 = \$1800$ , total outlay.

$\$2350 - \$1800 = \$550$ , profit.

$\frac{\$550}{\$1800} = .30\frac{1}{2}$ , or  $30\frac{1}{2}\%$ , profit.

2. A salesman, who received a salary of \$2400 and \$1500 expenses, sold \$75,000 worth of goods. In addition he received 2 % on all sales over \$60,000. What per cent of the selling price of the goods did it cost the firm to sell them ?

*Required:* Per cent of expense of sale on selling price.

$\$75,000 - \$60,000 = \$15,000$ ;  $2\%$  of \$15,000 = \$300.

$\$2400 + \$1500 + \$300 = \$4200$ , total expense of sale.

$\frac{\$4200}{\$75000} = .05\frac{2}{3}$ , or  $5\frac{2}{3}\%$ , expense.



3. A merchant bought 200 bbl. of apples at \$1.75 a barrel. If 12 % were spoiled and the remainder of the apples were sold at a loss of 8 %, for how much were the good apples sold ?

*Required :* Selling price of potatoes not spoiled.

12 % of 200 barrels = 24 barrels spoiled.

200 bbl. - 24 bbl. = 176 bbl., remainder.

$176 \times \$1.75 = \$308$ , cost price of remainder.

8 % of \$308 = \$24.64, loss.

$\$308 - \$24.64 = \$283.36$ , selling price.

4. A drugget 9 ft. by 12 ft. covers 50 % of the floor of a room  $13\frac{1}{2}$  ft. wide. Find the length of the room.

*Required :* Length of room, which equals floor area  $\div$  width.

108 sq. ft. = surface covered by drugget.

108 sq. ft. = 50 % or  $\frac{1}{2}$ , of floor area; therefore 100 % of floor area = 216 sq. ft.

$216 \div 13\frac{1}{2} = 16$ , no. of feet in length of floor.

Interpret the following problems :

5. How often can a conical cup that is 2 in. in diameter and 3 in. deep be filled from a cylindrical vessel that is 2 in. in diameter and 6 in. deep ?

*HINT.*— What is the relation of the cone to a cylinder of the same dimensions? Note that the cylinder in this problem is twice as deep as the cone.

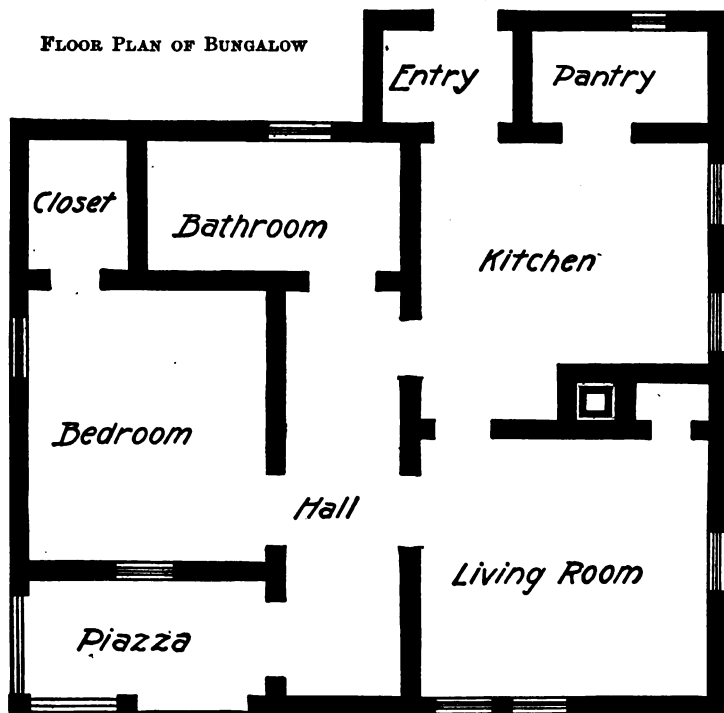
6. Which is the better offer, a piano listed at \$500, with discounts of 20 %, 10 %, and 5 %, or one listed at the same price with discounts of 5 %, 10 %, and 20 % ?

7. A salesman is offered a choice of a salary of \$3000, or \$2000 with 5 % commission on sales. If he accepts the latter offer and sells \$100,000 worth of goods a year, how much does he profit by his choice ?

## HOUSEHOLD PROBLEMS, SCHOOL SHOP WORK

**TO THE TEACHER.** — The application of what the pupil has learned to problems arising in the manual training, domestic science, and drawing departments should be continued. As stated before, the problems should be taken each year directly from the actual work in these departments. The use of working drawings and of drawing to scale should form part of the practice.

The bungalow represented in the plan is 30 ft. long and 29 ft. wide at the widest end.



Scale  $\frac{1}{4}$  in. to 1 ft.

1. On how wide a lot must it be built that it may be 6 ft. from one side of the lot and have a 12-foot space for a driveway on the other side?

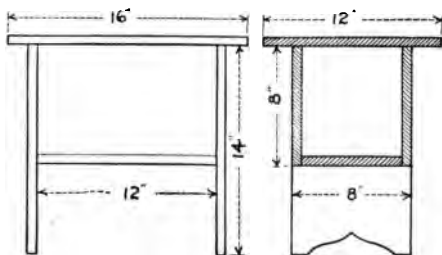
2. If the bungalow stands 20 ft. from the front line of the lot, the lot being 125 ft. deep, what will be the depth of the yard?

3. The living room is 12 ft. by  $10\frac{1}{2}$  ft. If a picture molding is put around the four walls of the living room, how much will it cost at 18¢ a foot?

4. How much will it cost to floor the hall, 17 ft. by 5 ft., and the bedroom,  $11\frac{1}{2}$  ft. by  $10\frac{1}{2}$  ft., with 1-inch flooring, at \$65 a thousand board feet, adding one fifth for waste?

5. What will be the cost of linoleum for the floor of the kitchen, 12 ft. by  $11\frac{1}{2}$  ft., making no allowance for the chimney; and for the pantry, 7 ft. by 4 ft., at 96¢ a sq. yd.?

1. What are the dimensions of the top of this blacking case?



2. What are the dimensions of each side?

3. What are the dimensions of each end?

4. What are the dimensions of the bottom?

5. If the top is cut from a 12" board and the other pieces from an 8" board 1" thick, how much will the material for a class of 20 cost at \$81 per M.? What is the cost for material for a box?

6. Reckoning a boy's time at 6¢ an hour, how much will the labor on the box be worth if the boy works from 1.15 P.M. to 2.45 P.M. the first day; from 1.20 P.M. to 3 P.M. the second day; and from 2.10 P.M. to 3.40 P.M. the third day?

7. What will be the cost, including labor and material?

## USING LETTERS TO REPRESENT NUMBERS

It is sometimes helpful to use letters, instead of figures, to represent quantities.

For example, the rule for finding interest can be stated as  $i = p \times r \times t$ ,  $i$  representing the interest,  $p$  the principal,  $r$  the rate, and  $t$  the time.

The area of a rectangle can be expressed as  $a = b \times h$ ,  $a$  representing the area,  $b$  the base, and  $h$  the height, or altitude, of the rectangle.

The circumference of a circle can be expressed as  $c = \pi \times d$ ,  $c$  representing the circumference,  $\pi$  the ratio, 3.1416, and  $d$  the diameter.

If we have the problem : "What number added to 31 will equal 56?" we can represent it as  $31 + (?) = 56$ ; or better, we can use a letter, as  $x$ , to represent the unknown number; thus,  $31 + x = 56$ . This means that 31 added to a number which we must find, and which, for convenience, we call  $x$ , equals 56.

If we represent a quantity, as an area, by  $a$ , we represent twice the area by  $2a$ , four times the area by  $4a$ . If  $a$  stands for 9 sq. ft.,  $2a$  represents 18 sq. ft.

If the principal is represented by  $p$ , 6 times the principal is  $6p$ , 12 times the principal  $12p$ , etc. If  $p = \$3000$ ,  $4p = \$12,000$ ,  $7p = \$21,000$ , etc.

When the area is  $a$ , half the area is  $\frac{1}{2}a$ , or  $\frac{a}{2}$ , one third the area is  $\frac{1}{3}a$ , or  $\frac{a}{3}$ . If  $a = 18$  sq. in.,  $\frac{a}{2} = 9$  sq. in., and  $\frac{a}{3} = 6$  sq. in.  $\frac{a}{2}$ , or  $\frac{1}{2}a$ , is read as one half  $a$ , or as  $a$  over 2.  $\frac{a}{3}$ , or  $\frac{1}{3}a$ , is read one third  $a$ , or  $a$  over 3.

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If  $p$  represents the principal,  $\frac{p}{4}$  represents  $\frac{1}{4}$  the principal,  $\frac{p}{3}$  one third the principal, etc.

If  $p = \$2700$ , what does  $\frac{p}{3}$  equal?

Two thirds of the area is represented by  $\frac{2}{3}a$ , or  $\frac{2a}{3}$ , three quarters of the area is represented by  $\frac{3}{4}a$ , or  $\frac{3a}{4}$ . If  $a$  is 24 sq. in.,  $\frac{2a}{3}$  is 16 sq. in., and  $\frac{3a}{4}$  is 18 sq. in.

$\frac{2a}{3}$ , or  $\frac{2}{3}a$ , is read as two thirds  $a$ , or  $2a$  over 3.

### Oral Work

1. Read the following:  $\frac{1}{2}x$ ,  $\frac{x}{3}$ ,  $\frac{3a}{5}$ ,  $\frac{3}{4}p$ ,  $\frac{2c}{7}$ .
2. How can you represent four fifths of the principal? Seven eighths? If  $p$  were \$1600, what would  $\frac{3p}{4}$  be?  $\frac{5p}{8}$ ?
3. If  $c$  represents the length of a circumference, what does  $4c$  represent?  $7c$ ?  $\frac{c}{3}$ ? If  $c = 6$  in., what does  $4c$  equal?  $7c$ ?  $\frac{c}{3}$ ?
4. If  $A$  represents area,  $b$  base, and  $h$  altitude, what does the following expression mean:  $A = b \times \frac{h}{2}$ ?
5. If  $\pi$  represents 3.1416,  $d$  diameter, and  $c$  circumference, read the following formula:  $c = \pi \times d$ .

7. If  $a$  represents the area of a triangle, what does  $5a$  represent?  $2a$ ?  $\frac{a}{4}$ ?  $\frac{3a}{4}$ ?

8. If  $a$  equals 12 sq. in., what does  $2a$  equal?  $5a$ ?  $\frac{a}{4}$ ?  $\frac{3a}{4}$ ?

9. If  $w$  represents the weight of a bushel of grain, what does  $10w$  represent?  $12w$ ?  $\frac{w}{5}$ ?  $\frac{w}{3}$ ?  $\frac{3w}{8}$ ?  $\frac{4w}{5}$ ? If  $w$  equals 40 lb., what does  $5w$  equal?  $\frac{w}{5}$ ?  $\frac{3w}{5}$ ?  $\frac{3w}{8}$ ?

10. If  $x$  represents the age of a man, what does  $3x$  represent?  $\frac{x}{6}$ ?  $\frac{3x}{7}$ ? If  $x$  is 42 years, what must  $2x$  be?  $\frac{x}{7}$ ?  $\frac{5x}{6}$ ? How can you represent three times the man's age?

11. If  $x$  represents the age of John, and Henry is twice as old, how can you represent Henry's age? If George is half as old, how can you represent his age?

12. If  $d$  is the distance between two cities, how can you represent the distance between two cities five times as far apart? one fourth as far apart? two thirds as far apart?

13. If  $d$  represents the distance from New York to San Francisco, and Chicago is three tenths as far from New York, how can you represent the distance from New York to Chicago?

14. If  $s$  is the speed of a railway train per hour, what is the speed per minute?

15. If  $a$  represents the age of John, and Tom is 6 years older, we can represent the age of Tom by  $a + 6$ , that is,  $a$  years plus 6 years.

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If George is  $x$  years old, and William is  $x + 5$  years old, how much older is William than George? If Mary's age is  $x + 3$  years, how much older is Mary than George? how much younger than William? If Henry's age is  $x - 8$  years, is he older or younger than George? how much?

16. Represent the speed of a railroad train by  $r$ . If another train runs 10 miles faster, how can you represent its speed? If still another train runs 9 miles slower, how can you represent its speed?

17. Represent the value of a house by  $h$ . How can you represent the cost of a house worth \$500 more? \$200 less?

18. Represent the weight of a load of hay by  $w$ . If the wagon weighs 1500 pounds less, what would represent its weight?

19. Represent John's age by  $n$ . What will his age be 12 years from now? What was it 12 years ago?

20. If Henry is twice as old as John, how can you represent his age? If he is 3 years more than twice as old, how can you represent his age?

21. Let  $x$  be the number of bushels of wheat in a bin. If 9 bu. are taken out and afterwards 12 bu. are put in, how can you represent the resulting quantity?

Let  $x$  = number of bushels in bin.

$x - 9$  = remainder after taking out 9 bu.

$x - 9 + 12$  = quantity after putting in 12 bu.

That is, the original quantity, less what was taken out, plus what was put in =  $x - 9 + 12$ .

22. Let  $x$  represent the number of pupils in a class; 25 were promoted from the class and 37 were then promoted into it. Represent the number in the class after promotions.

23. If a number is represented by  $x$ , how can you express twice the number increased by 40?

**24.** If John is  $x$  years old and George is  $\frac{1}{3}$  as old, how can you represent the sum of their ages?

$$\begin{array}{ll} \text{Let} & x = \text{John's age} \\ \text{then} & \frac{x}{3} = \text{George's age} \\ & x + \frac{x}{3} = \text{the sum of their ages} \end{array}$$

**25.** If Tom traveled  $x$  miles and Joe  $\frac{1}{4}$  as many, how can you represent the distance they both traveled?

**26.** If a train runs  $x$  miles in one hour, and  $\frac{3}{4}$  as far the second hour, how can you represent the distance the train travels in the two hours?

**27.** If a grocer sells  $x$  pounds of butter on Monday, and  $\frac{7}{8}$  as much on Tuesday, represent what he sells on both days.

**28.** If there are  $x$  girls in a class, and  $\frac{1}{2}$  as many boys, how can you represent the whole number of pupils in the class?

**29.** Suppose there are  $x$  boys in a class and  $\frac{2}{3}$  as many girls, how can you represent the whole number in the class?

**30.** If there are  $x$  bushels of grain in one bin and 25 % as much in another bin, how can you represent the grain in the second bin, remembering that 25 % equals  $\frac{1}{4}$ ?

**31.** If a horse costing  $x$  dollars is sold at a gain of  $33\frac{1}{3}\%$ , how can you represent the gain?

**32.** I buy a wagon for  $x$  dollars and sell it at a gain of 25 %. Represent the selling price.

**33.** I buy a wagon for  $x$  dollars, and sell it at a loss of 25 %. Represent the selling price.



## Written Work

$$\begin{aligned}\text{Let } a &= 36. \\ c &= 48. \\ d &= 64.\end{aligned}$$

$$\begin{aligned}p &= 2800. \\ x &= 60.\end{aligned}$$

What will be the values of the following?

- |                   |                    |                                         |
|-------------------|--------------------|-----------------------------------------|
| 1. $2a$           | 9. $x + 10$        | 20. $4x - x - 16$                       |
| 2. $3p$           | 10. $x - 10$       | 21. $x + \frac{x}{3}$                   |
| 3. $5c$           | 11. $2a + 4$       | 22. $2a + \frac{2a}{3}$                 |
| 4. $7d$           | 12. $3c - 6$       | 23. $p + \frac{p}{4}$                   |
| 5. $\frac{x}{2}$  | 13. $3x + 20$      | 24. $2d + \frac{3d}{8}$                 |
| 6. $\frac{3d}{4}$ | 14. $2a + 3a$      | 25. $\frac{2c}{3} + \frac{3c}{8} + 5$   |
| 7. $\frac{2c}{3}$ | 15. $c + 2c$       | 26. $\frac{3x}{10} + \frac{5x}{12} - 7$ |
| 8. $\frac{5a}{6}$ | 16. $2p - p$       |                                         |
|                   | 17. $5x - x$       |                                         |
|                   | 18. $6d - 2d$      |                                         |
|                   | 19. $3a - 2a + 10$ |                                         |

## THE EQUATION

## Oral and Written Work

- |                               |                     |
|-------------------------------|---------------------|
| 1. $i = p \times r \times t$  | 5. $8 + 4 = 12$     |
| 2. $a = b \times \frac{h}{2}$ | 6. $9 = 12 - 3$     |
| 3. $x + 6 = 9$                | 7. $5 + 7 = 4 + 8$  |
| 4. $x - 10 = 4$               | 8. $15 - 5 = 3 + 7$ |

Each of these expressions is an equation because it states that one quantity or combination of quantities is equal to another quantity or combination of quantities.

The part of the equation on the left of the equality sign is called the **first member** of the equation and the part on the right is called the **second member**.

$i$  is the first member of equation (1),  $x + 6$  of equation (3);  $p \times r \times t$  is the second member of (1) and 9 of (3).

What is the first member of each of the other equations? the second member?

$$9. \quad 18 + 6 - 3 =$$

$$12. \quad = 14 + 4 - 4$$

$$10. \quad 15 + 5 - 5 =$$

$$13. \quad = 24 + 8 + 2$$

$$11. \quad 20 - 8 + 12 =$$

$$14. \quad = 10 - 32 + 32$$

Complete (9) (10) (11) by writing the second member. Complete (12) (13) (14) by writing the first member.

Several important facts about equations may be learned by studying equations (5) (6) (7) (8) on p. 272. If you add 4 to the first member of (5), it is plain that you must add 4 also to the second member or else the two members will no longer be equal. Thus, if you write  $8 + 4 + 4 = 12$ , the members are not equal.

Prove also with (6) (7) (8) that if the same number is added to both members, the members will remain equal.

In the same way it is plain that if the same number is subtracted from both members, the members will remain equal. Thus, subtract 3 from each member of (5) and you have  $8 + 4 - 3 = 12 - 3$ . Prove this also with (6) (7) and (8).

If you multiply both members of (5) by 2, the equation will be  $16 + 8 = 24$ . The members are still equal. If both members of the equation are multiplied by the same number, the members will remain equal.

Multiply the members of (6) by 4.

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If you divide both members of (5) by 4, the equation will be  $2+1=3$ . If both members of an equation are divided by the same number, the members will remain equal.

Divide the members of (6) by 3.

You have now discovered these important rules relating to equations:

*The same number may be added to both members of an equation without destroying the equality.*

*The same number may be subtracted from both members of an equation without destroying the equality.*

*Both members of an equation may be multiplied by the same number without destroying the equality.*

*Both members of an equation may be divided by the same number without destroying the equality.*

Knowing these facts and employing letters to represent quantities whose value is unknown, you can, by means of the equation, solve many problems more easily than in any other way.

### **Solving equations by addition or subtraction.**

#### **Written Work**

1. What number added to 67 will give 96?  $x + 67 = 96$ .  
That is, an unknown number added to 67 will equal 96.

Subtract 67 from both members. The equation then is

$$x + 67 - 67 = 96 - 67.$$

$x + 67 - 67$  is simply  $x$ , as the 67 subtracted cancels the 67 added.

Therefore  $x = 96 - 67$ , or 29.

The number to be added to 67 is 29.

2. What number added to 49 will give 93?

Let  $x$  represent the number you wish to know.

The problem may now be put in the form of an equation

$$x + 49 = 93.$$

Subtract 49 from both members and the equation becomes

$$x + 49 - 49 = 93 - 49$$

$$x = 93 - 49$$

$$x = 44.$$

The required number is 44.

Find what  $x$  is equal to in the following equations :

3.  $6 + x = 10$

5.  $8 + x = 20$

4.  $x + 4 = 12$

6.  $x + 9 = 16$

Find the value of  $x$  in the equation :

7.  $x - 8 = 4$

Add 8 to each member and you have  $x - 8 + 8 = 4 + 8$ , or  $x = 12$ .

Find value of  $x$  in the following equations :

8.  $x - 3 = 10$

10.  $30 = x - 10$

9.  $x - 12 = 18$

11.  $x - 40 = 75$

Finding the value of  $x$ , or the unknown number, is called **solving an equation.**

Solve these equations :

12.  $5 + x = 12$

17.  $36 + x = 50$

13.  $7 + x = 15$

18.  $x + 27 = 45$

14.  $x + 12 = 20$

19.  $100 + x = 125$

15.  $x - 3 = 10$

20.  $90 + x = 120$

16.  $40 = x + 10$

21.  $75 + x = 100$

**Solving equations by division.**

### Written Work

1. If John is 3 times as old as George and the sum of their ages is 48 years, how old is George?

Let  $x = \text{George's age}$

then  $3x = \text{John's age}$

$x + 3x = \text{the sum of their ages, or 48 years}$

That is,  $x + 3x = 48.$

$x + 3x = 4x$

Therefore  $4x = 48$

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Divide both members by 4, that is, take one fourth of each member, and the equation is

$$x = 12.$$

George is 12 years of age.

2. A walks a certain number of miles in a day, B walks twice as far, and C walks three times as far. In all they walk 60 miles. How far does A walk?

Let	$x$ = number of miles A walks
then	$2x$ = number of miles B walks
and	$3x$ = number of miles C walks
and	$x + 2x + 3x = 60$ mi.
	$x + 2x + 3x = 6x$ .
Therefore	$6x = 60$ mi.

Divide both members by 6, that is, take one sixth of each member:

$$x = 10 \text{ mi., the distance A walks.}$$

Find the value of  $x$  in these equations:

3.  $5x = 25$

8.  $9x - x = 48$

4.  $8x = 64$

9.  $x + 4x + 3x = 96$

5.  $3x = 165$

10.  $x + 5x + x = 63$

6.  $15x = 90$

11.  $x + 3x - 2x = 20$

7.  $5x + x = 54$

12.  $15x - 4x + 7x = 90$

Transposition in equations.

### Oral Work

1.  $8 = 12 - 4$

4.  $9 + 7 = 16$

2.  $15 = 10 + 5$

5.  $16 - 5 = 11$

3.  $14 - 8 = 6$

6.  $18 + 3 = 21$

Add 4 to each member of (1) and you have  $8 + 4 = 12 - 4 + 4$ , or  $8 + 4 = 12$ . That is, the 4 has been transferred from the second member to the first and its sign has been changed from  $-$  to  $+$ .

Subtract 5 from both members of (2) and you have  $15 - 5 = 10$ . That is, subtracting 5 from each member gives the same result as transferring 5 from the second member to the first and changing its sign from + to -.

Add 8 to both members of (3), subtract 7 from both members of (4), add 5 to both members of (5), subtract 3 from both members of (6) and you have results as follows:

$$\begin{aligned} 14 - 8 &= 6; \text{ adding 8 to both members gives } 14 = 6 + 8 \\ 9 + 7 &= 16; \text{ subtracting 7 from both members gives } 9 = 16 - 7 \\ 16 - 5 &= 11; \text{ adding 5 to both members gives } 16 = 11 + 5 \\ 18 + 3 &= 21; \text{ subtracting 3 from both members gives } 18 = 21 - 3 \end{aligned}$$

It is plain that adding or subtracting a quantity from each member is the same as transferring the quantity from one member to the other and changing its sign. We may therefore transfer or transpose any quantity from one member of an equation to another if we change its sign.

This is called **transposition**.

### Oral Work

Find the value of  $x$  in the following equations, using transposition:

1.  $x - 6 = 10$ .

Transposing 6 gives

$$x = 10 + 6; x = 16.$$

2.  $2x + 8 = 24$

Transposing 8 gives

$$2x = 24 - 8; 2x = 16; x = 8.$$

3.  $4x + 3 - 2x = 15$

Transposing 3 gives

$$4x - 2x = 15 - 3; 2x = 12; x = 6.$$

## Written Work

Find the value of  $x$  in the following equations :

- |                         |                          |
|-------------------------|--------------------------|
| 1. $x + 20 = 30$        | 12. $9x - 5 = 22$        |
| 2. $x - 16 = 32$        | 13. $7x - 6 = 29$        |
| 3. $x - 12 = 3$         | 14. $3x + 2x + 4 = 34$   |
| 4. $x + 8 = 15$         | 15. $6x + 5x - 6 = 126$  |
| 5. $x - 9 = 27$         | 16. $x + 3x + 2 = 50$    |
| 6. $x + 14 = 20$        | 17. $6x - 2x + 4 = 112$  |
| 7. $x - 40 = 60$        | 18. $8x - 5x - 6 = 93$   |
| 8. $x + 35 = 65$        | 19. $5x - 4 + 3x = 100$  |
| 9. $x - 42 = 18$        | 20. $6x + 7 - 2x = 99$   |
| 10. $2x + 4 = 16$       | 21. $10x - 5 - 4x = 97$  |
| 11. $5x + 3 = 28$       | 22. $12x + 5 + 7x = 100$ |
| 23. $4x + 10 = 82 - 2x$ |                          |

In example 23 it is necessary to transpose two quantities. We transpose  $2x$  to the first member, so that all the quantities containing  $x$  may be in one member, and we transpose 10 to the second member so that the quantities not containing  $x$  may be in the other member. The equation then is,

$$4x + 2x = 82 - 10$$

$$6x = 72$$

$$x = 12$$

- |                                    |                          |
|------------------------------------|--------------------------|
| 24. $4x + 5 = 65 - 2x$             | 28. $6x - 12 = 13 + x$   |
| 25. $8x - 10 = 6x + 20$            | 29. $10x + 5 = 53 - 6x$  |
| 26. $4x + 1 = 100 - 5x$            | 30. $12x - 2 = 8x + 22$  |
| 27. $9x - 4x = 56 - 2x$            | 31. $15x + 5 = 145 - 5x$ |
| 32. $10x + 4x - 2x = 16 + 2x + 14$ |                          |

# Solving equations by multiplication.

## Written Work

If you have the equation  $\frac{x}{2} = 5$  and multiply both members by 2, you get  $\frac{x}{2} \times 2 = 5 \times 2$ , or  $x = 10$ .

If you have  $\frac{3x}{5} = 12$  and multiply both members by 5, you get  $\frac{3x}{5} \times 5 = 12 \times 5$ ;  $3x = 60$ ;  $x = 20$ .

Find the value of  $x$  in the following equations, by multiplying both members by the denominator of the fraction:

1.  $\frac{x}{3} = 20$

4.  $\frac{x}{2} = 2\frac{1}{2}$

7.  $\frac{2x}{3} = 6\frac{2}{3}$

2.  $\frac{x}{4} = 13$

5.  $\frac{3x}{5} = 36$

8.  $\frac{9x}{2} = 18$

3.  $\frac{x}{5} = 18$

6.  $\frac{8x}{3} = 24$

9.  $\frac{4x}{5} = 2.8$

10. John is  $\frac{1}{2}$  as old as his father, and George is  $\frac{2}{3}$  as old. The sum of the ages of John and George is 45 years. How old is the father?

Let  $x =$  the father's age

then  $\frac{x}{2} =$  John's age

and  $\frac{2x}{5} =$  George's age

The sum of the ages of John and George is 45 years, hence

$$\frac{x}{2} + \frac{2x}{5} = 45 \text{ (Multiply both members by the l. c. d., 10.)}$$

$$\frac{x}{2} \times 10 = 5x$$

$$\frac{2x}{5} \times 10 = 4x$$

$$45 \times 10 = 450$$

The equation is

$$5x + 4x = 450$$

$$9x = 450$$

$$x = 50, \text{ the father's age}$$



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11. Solve the equation  $\frac{3x}{4} + \frac{5x}{8} = 22$ .

(As the least common denominator is 8, multiply both terms by 8.)

$$\frac{3x}{4} \times \frac{2}{1} = 6x$$

$$\frac{5x}{8} \times \frac{1}{1} = 5x$$

$$22 \times 8 = 176$$

The equation is

$$6x + 5x = 176$$

$$11x = 176$$

$$x = 16$$

In all such problems, multiply both members by the least common denominator of the fractions.

Find the value of  $x$  in the following equations:

12.  $\frac{x}{3} + \frac{x}{2} = 10$

17.  $\frac{x}{3} - \frac{x}{6} = 9$

22.  $\frac{5x}{8} + 3 = 13$

13.  $\frac{x}{3} + \frac{x}{4} = 21$

18.  $\frac{3x}{8} - \frac{x}{4} = 12$

23.  $\frac{3x}{7} + 2 = 5$

14.  $\frac{x}{2} + \frac{x}{5} = 14$

19.  $\frac{5x}{6} - \frac{2x}{3} = 2$

24.  $\frac{5x}{6} - 4 = 6$

15.  $\frac{x}{2} + \frac{2x}{3} = 35$

20.  $\frac{2x}{7} - \frac{x}{14} = 9$

25.  $\frac{3x}{8} - 5 = 1$

16.  $\frac{3x}{4} + \frac{2x}{3} = 17$

21.  $\frac{2x}{3} + 4 = 12$

26.  $\frac{x}{2} - 2 = 4$

27.  $\frac{2x}{3} + \frac{x}{2} + 3 = 10$

31.  $\frac{3x}{4} + \frac{3x}{5} = 35 - \frac{2x}{5}$

28.  $\frac{3x}{4} + \frac{x}{3} - 5 = 8$

32.  $\frac{5x}{6} - 6 = \frac{5x}{9} + 4$

29.  $\frac{5x}{6} - 2 + \frac{x}{3} = 5$

33.  $7 = \frac{2x}{3} - \frac{x}{5}$

30.  $\frac{2x}{3} + 3 = 14 - \frac{x}{4}$

34.  $2x - 8 = \frac{2x}{3}$

$$35. 23 = \frac{x}{6} + \frac{3x}{5}$$

$$38. 2x - \frac{2x}{3} = x + 6$$

$$36. 3x - \frac{x}{2} = \frac{5}{16}$$

$$39. 2x - \frac{3x}{4} = 40 + \frac{x}{4}$$

$$37. x - \frac{3x}{4} = 3 - \frac{x}{8}$$

$$40. \frac{3x}{2} - 8 = \frac{x}{2} + 24$$

### PROBLEMS

1. What number added to 5 times itself equals 84?

Let  $x =$  the number  
 then  $5x = 5$  times the number  
 $x + 5x = 84$   
 $6x = 84$   
 $x = 14$ , the number required

2. What number increased by 4 times itself equals 125?

3. Two horses cost \$484. What is the cost of each, if one costs 3 times as much as the other?

4. 450 acres of land were divided among three sons. The second received twice as much as the first, and the third received three times as much as the first. How many acres did each receive?

5. Three men went into business with a capital of \$6300. The second furnished twice as much as the first, and the third furnished twice as much as the second. How much did each man furnish?

6. Separate 288 into two parts, one of which is 3 times the other.

7. The sum of two numbers is 6.25 and the larger number is 4 times the smaller. What are the numbers?

8. The age of a father is 5 times that of his son. If the sum of their ages is 48 years, how old is each?

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9. The difference between two numbers is 96, and the larger number is 4 times the smaller. What are the numbers?

10. A mother is 6 times as old as her daughter and the difference between their ages is 35 years. How old is each?

11. Twice a certain number increased by 36 is equal to 92. What is the number?

12. Henry is 6 years older than James, and the sum of their ages is 30 years. What is the age of each?

13. The sum of two numbers is 80, and the smaller is  $\frac{3}{5}$  of the larger. What are the numbers?

$$\begin{array}{ll} \text{Let} & x = \text{the larger number} \\ \text{then} & \frac{3x}{5} = \text{the smaller number} \\ \text{and} & x + \frac{3x}{5} = 80 \end{array}$$

$$5x + 3x = 400 \text{ (Multiplying both members by 5.)}$$

$$8x = 400$$

$$x = 50, \text{ the larger number}$$

$$\frac{3x}{5} = 30, \text{ the smaller number}$$

14. The sum of two numbers is 95 and one number is  $\frac{2}{3}$  of the other. What are the numbers?

15. What number increased by  $\frac{3}{4}$  of itself is equal to 77?

16. In a school of 45 pupils,  $\frac{2}{3}$  of the number of girls equals the number of boys. How many are there of each?

17. If a man's annual salary were increased by  $\frac{1}{3}$  of itself it would become \$2400. What is his salary?

18. If 70 marbles are divided between two boys so that one receives  $\frac{2}{3}$  of what the other receives, what number does each receive?

19. The difference between two numbers is 55 and the smaller number is  $\frac{2}{3}$  of the larger. What are the numbers?

20. When a certain number is diminished by  $\frac{2}{3}$  of itself, it becomes 35. What is the number?

21. If to Ned's age you add its  $\frac{1}{3}$  and  $\frac{1}{4}$ , the sum is 19 years. How old is he?

22. The sum of two numbers is  $6\frac{7}{8}$  and the smaller is  $\frac{1}{4}$  of the larger. What are the two numbers?

23. What is a teacher's monthly salary if  $\frac{1}{2}$  plus  $\frac{2}{3}$  of it is \$55?

24. Divide 120 into two numbers, one of which shall be  $2\frac{1}{2}$  times the other.

25. The sum of \$400 was paid for a horse and a wagon, the wagon costing  $\frac{2}{3}$  as much as the horse. How much did each cost?

26. A wagon loaded with hay weighed 4500 lb. The wagon weighed 1500 lb. less than the hay. Find the weight of each.

27. The sum of three numbers is 270. The second is double the first, and the third is twice the sum of the other two. What are the numbers?

28. In a certain school there were  $\frac{5}{8}$  as many boys as girls, and the girls exceeded the boys in number by 30. How many pupils were there in the school?

29. A watch and chain cost \$175. Find the cost of each, if  $\frac{1}{3}$  the cost of the watch equals twice the cost of the chain.

Let  $x$  = the number of dollars the watch costs

Then  $175 - x$  = the number of dollars the chain costs

Since  $\frac{1}{3}$  the cost of the watch = twice the cost of the chain,

$$\frac{x}{3} = 350 - 2x$$

$$x = 1050 - 6x \text{ (Multiplying both members by 3.)}$$

$$7x = 1050$$

$$x = 150, \text{ the number of dollars the watch cost}$$

$$175 - 150 = 25, \text{ the number of dollars the chain cost.}$$

## 284 USING LETTERS TO REPRESENT NUMBERS

30. Divide 26.35 in two parts, the smaller of which shall be  $\frac{2}{3}$  of the larger.

31. Three men own 230 A. of land. The share of the second is 10 A. less than that of the first, and the share of the third is 20 A. less than that of the second. Find the share of each.

32. The perimeter of a rectangular field 60 rd. wide is 300 rd. What is its length?

33. A rectangular field twice as long as it is wide has a perimeter of 480 rd. Find its dimensions.

34. Divide \$550 among three persons, giving to the first  $\frac{1}{2}$  as much as to the second and to the third  $\frac{1}{3}$  as much as to the second.

35. After paying  $16\frac{2}{3}\%$  and  $25\%$  of my debts, I still owe \$35. How much did I owe at first?

36. A horse was sold for \$320, the seller thereby gaining 60 % of what he paid for it. What was the cost price of the horse?

37. A man who spent each year 10 % of his salary for clothing and  $37\frac{1}{2}\%$  for board and other expenses, saved \$840. What was his salary?

38. A farmer paid for a cow 40 % of what he paid for a horse. How much did he pay for each, if the horse cost \$84 more than the cow?

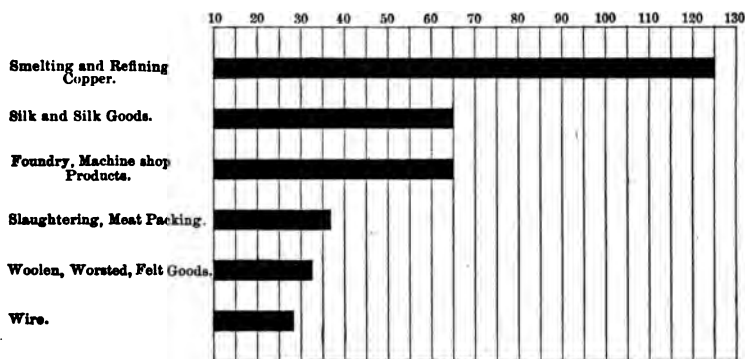
39. 40 % of a pole is in the water, 10 % in the mud, and the remainder above the water. If the part in water is 9 ft. longer than the part in the mud, what is the length of the pole?

40. A farmer sold 30 % of his farm to one purchaser, 28 % to another, and had 84 acres left. How large was his original farm?

41. A boy has 15 more than  $\frac{1}{3}$  as many marbles as his brother. If together they have 55, how many has each?

## GRAPHICAL PROBLEMS

The following is a graph showing the relative values one year of the products of the most important manufacturing industries of a certain state.



The black strips indicate the number of million dollars worth of the products of these manufactures. Each vertical division represents five million dollars of product.

1. What was the approximate value of the silk and silk goods industry?

2. How much greater in value was the production of copper products than of the woolen, worsted, and felt goods?

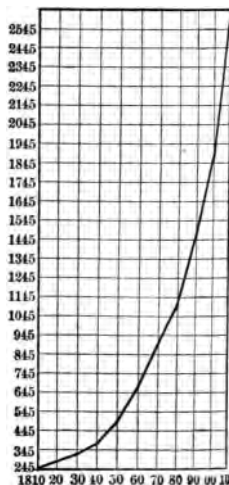
The graph most commonly used is the curve or broken line. The following is an illustration of this form, illustrating the growth in thousands of the population of a state by decades for one hundred years.

Each vertical division represents a decade, and each horizontal division, a growth in population of 100,000. For example, between 1810 and 1820, the growth was less than 50,000, while between 1850 and 1860, the growth was approximately 200,000.

3. In what decade was the growth greatest?

4. In what decade was the increase less than 100,000?

5. Show by a curve or a broken line the increase in the size of the graduating class in a high school whose graduating classes numbered as follows: 1912, 35; 1913, 38; 1914, 37; 1915, 41; 1916, 47; 1917, 52.



6. Trace a curve for the attendance of a school as follows: September, 775; October, 815; November, 800; December, 795; January, 790; February, 780; March 790; April, 800; May, 785; June, 770.

7. Trace a curve to illustrate the following changes in temperature for a certain day: 7 A.M. 40°; 9 A.M. 42°; 11 A.M. 46°; 1 P.M. 50°; 3 P.M. 53°; 5 P.M. 49°; 7 P.M. 46°.

8. Trace the curve of maximum temperature for a week as follows: Monday, maximum temperature, 29°; Tuesday, 35°; Wednesday, 43°; Thursday, 27°; Friday 24°; Saturday, 30°; Sunday, 43°.

9. Trace a curve to show the variation in the cotton crop of Texas: 1911, 4,256,000 bales; 1912, 4,880,000 bales; 1913, 3,945,000 bales; 1914, 4,592,000 bales; 1915, 3,175,000 bales; 1916, 3,725,000 bales. Use  $\frac{1}{8}$  in. to represent a change of 100,000 bales.

## STOCKS AND BONDS

### STOCKS

In very many cases, especially in extensive enterprises requiring large amounts of money, businesses are conducted by companies of persons, known as corporations or stock companies, such as the Standard Oil Company and the Pennsylvania Railroad Company.

A **corporation** is a company authorized by law to transact business as an individual. Its rights and duties are stated in its **charter**.

The money contributed to carry on the business is known as the **capital**, **capital stock**, or **stock** of the company.

The stock is divided into equal parts called **shares**. To every holder of stock is given a stock certificate, a written statement of the number of shares to which the person receiving the certificate is entitled and of the value of each share.

The **par value** of a share of stock is the amount specified on the face of the stock certificate.

The par value of the shares in different corporations varies from \$1 to \$100, though it is generally \$100. The persons who form the corporation determine the number of shares into which the capital stock shall be divided and the par value of the shares. Observe that the certificate on p. 288 gives the number and the par value of the shares.



**STOCK CERTIFICATE***Incorporated under the Laws of the State of Vermont**No. 25**20 Shares***Independent Marble Company**

*This certifies that.....James Wood.....is the owner of.....Twenty.....full-paid shares of the Capital Stock of One Hundred Dollars each of the Independent Marble Company.*

*Transferable only on the books of the Company by the holder, in person, or by an attorney upon the surrender of this certificate.*

*J. G. Meyer, President.*

*B. S. Smith, Secretary.*

*Montpelier, Vt., June 1, 1918.*

**A stockholder** is one who holds stock in a corporation.

1. A company was organized for business with a capital of \$300,000. How many shares of a par value of \$100 were there?

2. A mining company has a capital stock of \$75,000. How many shares of stock has it, if the par value of a share is \$25?

The body of men who manage the affairs of a corporation is the board of directors. They are elected by the stockholders.

A **dividend** is a part of the net profits divided among the stockholders, in proportion to the par value of their stock. These dividends are paid yearly, half-yearly, or quarterly, as the board of directors may determine.

The dividend is always a per cent of the par value of the stock. A 4% dividend means that 4% of the par value of the stock is divided among the stockholders.

3. If a company with a capital of \$600,000 declares a dividend of 5%, how much money will be required to pay the dividend?

4. If the dividend had been  $2\frac{1}{4}\%$  semiannually, how much would have been required to pay the dividend each year?

The stock of a company may be **preferred** stock or **common** stock. Before any dividend can be paid on the common stock, the dividend on the preferred stock, usually at a fixed rate of from 5% to 7%, must be paid. The dividend left after the preferred dividend has been paid, is divided among the common stockholders. Holders of preferred stock do not always have a vote in the election of directors.

5. A company with a capital of \$400,000, issued 1000 shares of preferred stock and the remaining 3000 shares as common stock, par value \$100 a share. What was the par value of the preferred stock?

6. What amount would be required to pay a 6% dividend on the preferred stock?

7. If the net profit to be used as a dividend were \$16,000, how much of this would be divided among the common stockholders?

Shares of stock are bought and sold like any other commodity of value. The transaction is generally through a **stock broker** whose business is the buying and selling of stocks. The charge made by brokers is known as **brokerage** and generally is  $\frac{1}{2}\%$  or  $\frac{1}{4}\%$  of the par value. While the ease with which stocks may be bought or sold is an advantage, their fluctuations in value, sometimes very great in a short time, often render them a dangerous form of investment.

The **market value** of a stock is the price at which it is selling.

The market values of the principal stocks are given in the daily papers of all the large cities. The following list shows how stocks are quoted in the newspapers :

American Produce	34½	Eastern R. R.	118½
American Silk	229½	General Locomotive	48
American Silk pf.	113½	General Locomotive pf.	104½
American Tin	233	General Mining	3½
Central Motor Company	325	Int. Rubber	100½
Central R. R.	30½	Ohio Central	121
Central R. R. pf.	47½	Universal Zinc	39½

The par value of each of the above stocks is \$100; but according to the above quotations, American Produce is selling for \$34½ a share, Central Motor Company for \$325, and General Mining for \$3.50.

The market value of a stock depends upon the demand for it. The demand is great or small as the dividends paid make it a good or a poor investment. A stock with high dividends will sell above par, while a stock with low dividends will sell for less than par.

The preferred stocks are marked pf. and in every case in the above list but one, exceed in value the corresponding common stock. The fact that their dividends must be taken from the profits before a dividend can be paid on the common stock makes the preferred stock generally a safer investment. In the exception it is evident that after the dividend on the preferred stock had been paid, the remaining profit was still so great as to pay a much larger dividend on the common stock than was paid on the preferred stock. The market value of this common stock is therefore much higher than that of the corresponding preferred stock.

Stocks selling for more than their par value are said to be above par. Stocks selling for less than the par value are said to be below par.

8. Which of the stocks in the above list are above par?
9. Which of the stocks in the above list are below par?

10. How much above par is the Central Motor Company ?

11. How much below par is the General Mining ?

*In the following problems, when no par value is mentioned, a par value of \$ 100 is to be assumed. Brokerage is not to be computed in this book, unless definitely stated.*

### Oral Work

Using the quotations on p. 290, find the cost of :

1. 10 shares of Central Motor Company.

SOLUTION. —  $10 \times \$ 325 = \$ 3250$ .

2. 50 shares of General Locomotive.

3. 100 shares of American Tin.

4. 10 shares of Central R. R.

5. 20 shares of Ohio Central.

6. 20 shares of American Produce.

### Written Work

Using the quotations on p. 290, find the cost of the following stocks :

1. 64 shares of Int. Rubber.

2. 560 shares of Central R. R. preferred.

3. 128 shares of General Mining.

4. 96 shares of Eastern R. R.

5. Find the cost of 143 shares of South American Coffee, par value \$ 50 each, at 78.

SOLUTION. —  $143 \times \$ 78 = \$ 11,154$ , market value of stock.

6. What will be the proceeds of the sale of 88 shares of stock, par value \$ 50 each, at  $101\frac{1}{8}$  ?

7. What would be realized from the sale of 21 shares of railway stock, par value \$100 each, at  $94\frac{1}{8}$ ?

8. Find the proceeds of the sale of 75 shares of milling stock, par value \$100 each, quoted at  $72\frac{3}{8}$ .

9. If I sell 125 shares of United States Chemical at 410, how much do I realize?

Find the cost of the following stock:

10. 16 shares, par value \$50, bought at  $51\frac{1}{2}$ .

11. 42 shares, par value \$50, bought at 45.

12. 220 shares, par value \$100, bought at  $100\frac{1}{2}$ .

13. 150 shares, par value \$50, bought at  $49\frac{5}{8}$ .

14. My broker purchased for me 256 shares of milling stock, par value \$100, at  $104\frac{1}{2}$ . How much did the stock cost?

15. How much is realized from the sale of 480 shares of gas stock, par value \$50, bought at 49?

#### Dividends and Incomes.

##### Oral Work

1. What is the income from \$1000 for 1 year at 6%?

2. What is the income from \$1000 invested in a manufacturing plant that pays 8% in dividends each year?

3. Why is a \$100 share of stock that pays \$12 in dividends each year worth more than \$100?

4. Why is a \$100 share of stock that pays only \$2 in dividends each year worth less than \$100?

NOTE. — Dividends are computed on the *par value* of the stock.

5. Find the dividend on 5 shares of stock, par value \$100, at 5%.

SOLUTION. — 5% of \$500 = \$25, dividend.

**Written Work**

1. What will my income be from 25 shares of stock paying 7% annual dividends?

2. I own 112 shares of railroad stock paying a quarterly dividend of  $1\frac{1}{4}\%$ . What is my income from this stock?

3. The Int. Electric pays a semiannual dividend of  $1\frac{1}{2}\%$ . What is the annual income, from this stock, of a person owning 160 shares, par value \$50 each?

4. 100 shares of stock were bought for \$10,300. If a dividend of 5% is declared, how much will the owner of the stock receive?

5. A dividend of 4% was declared by a company capitalized at \$700,000. How much will the owner of 45 shares of stock receive?

6. Mr. Johnston owns 100 shares of stock in a company whose capital is \$200,000. If a dividend of 8% is declared, find the amount of the check that will pay the whole dividend; the amount of Mr. Johnston's share.

7. Find the gain when 80 shares of Central Fuel Supply, which cost  $109\frac{1}{8}$ , are sold for  $116\frac{3}{4}$ .

8. 90 shares of Southern Sugar were bought at  $100\frac{1}{8}$  and sold at  $97\frac{1}{2}$ . What was the loss?

9. 58 shares of Western Electric were bought when the market price was  $123\frac{3}{8}$  and sold when the price was  $128\frac{1}{4}$ . What was the gain?

10. How much is gained by buying 470 shares of stock at 96 and selling at 98?

11. 320 shares of Florida Orange Company were bought at  $158\frac{3}{4}$ . One half was sold at 160 and one half at 156. How much was gained or lost?

12. 96 shares of Western Canning were bought at  $21\frac{1}{8}$ , and 72 shares at  $21\frac{7}{8}$ . All were sold at  $19\frac{3}{4}$ . How much was lost?

13. A man owning 100 shares of National Copper, worth 237 and paying a dividend of 12%, sold it, and bought with the proceeds a house, which netted him 6% in rent. Did he gain or lose in income and how much?

14. A man who owned 220 shares of Western Railway stock worth 96 sold it and invested 30% of the amount received for it in 72 shares of American Leather stock. What was the value of each share of the stock he bought?

### BONDS

When a corporation needs a large sum of money to carry on its business, instead of issuing more stock, it frequently issues a series of bonds payable at some future date, with interest.

**Bonds** are written obligations, under seal, by which corporations or governments bind themselves to pay specified sums at a fixed rate of interest, at or before the time specified in the bonds.

The bonds of **business corporations** are secured by mortgages on their property. Such mortgages authorize the sale of the property in case the conditions of the bonds are not fulfilled. The bonds of governments are without mortgages.

The terms *par value* and *market value* mean the same when applied to bonds as when applied to stocks.

A **coupon bond** is a bond with interest coupons attached. These coupons are detached when the interest is due, and the amount may be collected personally or through a bank. Coupon bonds are payable to the bearer.

## Coupon

\$ 30.	<b>ACME GLASS COMPANY</b>	\$ 30.
<p>will pay to Bearer at the</p> <p><b>Federal Trust Co., of Charleston, W. Va.</b></p> <p><i>on the first day of June, A.D. 1918,</i></p> <p style="font-size: 1.2em;"><i>Thirty Dollars</i></p> <p><i>in United States Gold Coin, being six months' Interest</i></p> <p><i>on Bond No. 501.</i></p> <p style="text-align: right;"><i>James Grine, Treasurer.</i></p>		

A **registered bond** is a bond registered on the books of the corporation issuing it. The interest when due is sent by check to the owner. Registered bonds are payable to the owner or to his assignee.

The name of a bond often indicates the government, municipality, or corporation by which it is issued, *its rate of interest*, and the time when the bond is *due*. Thus, "U. S. 4's, 1925," are United States 4% bonds, due in 1925; "Western Chemical 5's, 1920," are Western Chemical bonds, due in 1920, and bearing 5% interest; "U. S. Copper 6's" are United States Copper bonds, bearing 6% interest.

The market values of bonds are reported in the newspapers. The following are illustrations of such quotations:

U. S. 2's reg. 1930	100½
U. S. 4's coupon 1925	108½
American Railway 4½'s	96½
Southern Sugar 4½'s	103½
Western Chemical 5's 1920	117
Western Copper 4½'s	88



A Western Chemical bond, par value \$1000, was worth, when the above quotation was made, 117 % of \$1000, or \$1170. A Western Copper  $4\frac{1}{2}$  % bond, par value \$1000, was worth 88 % of \$1000, or \$880.

Bonds do not fluctuate in value as much as do stocks and are, on that account, safer for investment. The market value of private bonds is high or low, according as the property on which they are a mortgage is deemed a good or a poor security for their payment. Government bonds depend for their value on the strength and stability of the government issuing them. Thus U. S. 4's paying 4 % interest are quoted at  $8\frac{1}{2}$  above par because of the belief in the stability of the United States government. In such cases, the interest will be paid regularly and the bond itself is sure to be paid when it matures. If there is doubt as to either of these two considerations, the market value of the bond falls.

### Comparative Study

**Stockholders** are the *owners* of shares of stock representing their interest in the corporate property; **bondholders** are *creditors* who have loaned money to the corporation or the government. Bonds bear *interest* at a *fixed rate* and mature at a time specified in the bond; **stocks** continue while the corporation exists and pay *dividends* according to the earnings of the company.

### Written Work

1. Find the cost of fifteen \$1000 United States bonds at 103 $\frac{3}{4}$ .

SOLUTION. —  $1.03\frac{3}{4} \times \$15,000 = \$15,562.50$ .

Find the cost of the following :

2. 3 \$1000 General Industrial 5's at 78 $\frac{1}{2}$ .
3. \$23,000 Pacific Improvement 4's at 95 $\frac{3}{8}$ .
4. 22 \$1000 Int. Rubber 6's at 101 $\frac{1}{4}$ .

5. \$8000 U. S. Soap 5's at  $61\frac{1}{2}$ .
6. \$1000 Southern Gas 4's at 95.
7. \$12,000 Gen. Hide  $4\frac{1}{2}$ 's at  $88\frac{1}{2}$ .

Using the quotations on p. 295 find the proceeds of the sale of the following :

8. \$9000 American Railway.
9. \$16,000 U. S. 4's, coupon, 1925.
10. \$21,000 Southern Sugar  $4\frac{1}{2}$ 's.
11. \$7000 U. S. 2's, registered, 1930.
12. \$5000 Maine Ice Sec.  $3\frac{1}{2}$ 's were bought for  $84\frac{1}{2}$  and sold for  $83\frac{1}{8}$ . What was the loss ?

13. A house worth \$10,000 was traded for \$8000 Western Thread Company 4's at 91 and a cash payment. How much was the cash payment ?

14. What is the difference in value between \$10,000 Wolcott Mine at 55 and \$6000 Int. Lead at  $93\frac{1}{4}$  ?

15. A man sold \$20,000 Utah Mining at 94. He invested 60 % of the proceeds in land at \$80 an acre. How many acres did he buy ?

16. Find the annual income on the following bonds :  
\$5000 Pacific Improvement 4's. ( $4\%$  of \$5000 = \$200.)

17. \$4000 U. S. 4's, registered, 1925.
18. \$10,000 General Industrial 5's.
19. \$6000 U. S. 3's, coupon, 1918.
20. \$15,000 U. S. Chemical 4's.
21. \$20,000 American Produce  $4\frac{1}{2}$ 's.
22. \$10,000 U. S. 2's, registered.
23. \$4000 Central R. R.  $4\frac{1}{2}$ 's.
24. \$12,000 Int. Glass 5's.

25. \$5000 General Locomotive 4's.

26. A \$1000 5% bond, due in 10 yr., was purchased for \$1100. Find the average rate of interest on the investment, if the bond is held to maturity.

The total income on the bond for 10 yr. at 5% = \$500. Loss by redemption of bond \$1100 - \$1000 = \$100. Total income in 10 yr. = \$500 - \$100 = \$400. Average annual income =  $\frac{1}{10}$  of \$400 = \$40. Average annual rate =  $\$40 \div \$1100 = 3\frac{7}{11}\%$ .

27. A \$1000 4% bond, due in 5 yr., was bought for \$900. Find the average rate of interest, if the bond is held to maturity.

NOTE. — The purchaser gains \$100 when the bond is redeemed.

28. A \$1000 6% bond due in 5 yr. was bought for \$1150. Find the average rate of interest, if the bond is held to maturity.

29. A \$1000 4% bond due in 10 yr. was bought at \$800. Find the average rate of interest, if the bond is held to maturity.

## FIRE INSURANCE

**Insurance** is security against loss or damage.

A merchant owns a store, uninsured, valued at \$5000. If it is burned, who will bear the loss? An insurance company agrees to insure the store for \$4000 at 1% annually. In case the store is totally destroyed by fire, how much is the company expected to pay to the merchant? How much must the merchant pay annually to the company to guarantee this loss?

Insurance is of many kinds. Companies insure against loss through fire, accidents to ships, burglars, bad debts, accidents to people, etc. Life insurance is also an important business. In this book only fire insurance is considered.

The **policy** is a written contract between the person insured and the insurance company.

The **premium** is the sum paid for the insurance.

The **rate** is a specified number of cents or dollars per \$100 of insurance, or a certain per cent of the sum insured.

The **term** is usually a year or a period of years. **Short rates** are rates charged when the term is less than one year.

### Written Work

1. A frame dwelling is insured for \$2800 for one year at  $1\frac{3}{4}\%$ . Find the premium.

\$2800, amount insured

.01 $\frac{3}{4}$ , rate

2800

2100

\$49.00, premium

The premium is  $1\frac{3}{4}\%$  of the amount insured, or \$49.

To what term in *percentage* does the amount insured correspond? The premium? The rate?

What are the premiums on the following policies at the rates given?

2. \$14,500,  $1\frac{1}{4}\%$       4. \$11,400,  $2\frac{1}{4}\%$       6. \$18,900,  $3\frac{1}{4}\%$   
 3. \$9200,  $1\frac{1}{2}\%$       5. \$5600,  $2\frac{1}{2}\%$       7. \$16,700,  $1\frac{1}{4}\%$

What are the premiums on the following policies at the given premiums per \$100?

8. \$9600, \$ .75      10. \$7900, \$ .45      12. \$15,600, \$1.25  
 9. \$24,500, \$1.75      11. \$67,000, \$ .60      13. \$8700, \$ .90

14. A brick house is insured for \$4000 at 60¢ on \$100. Find the rate of premium and the annual premium.

15. If the three-year rate is twice the rate for one year, find the cost of insuring a brick dwelling for \$6500, for 3 years, when the annual rate is 45¢ per \$100.

16. A store building is insured for \$8500 at an annual premium of \$212.50. Find the rate per cent of premium and the annual cost per \$100 of insurance.

17. A school board pays annually \$45 for \$6000 of fire protection on a school building. Find the rate of premium.

18. The premium on a dwelling insured for \$5500 is \$38.50 for 3 years. Find the average rate for a year.

19. Mr. Lawrence wrote a check for \$31.50 to pay the insurance on his dwelling for 3 years. If the house cost \$2400 and was insured for  $\frac{3}{4}$  of its value, find the rate for the term.

20. A farmer insured his house for \$2700 at  $1\frac{1}{4}\%$ , his barn for \$1200 at  $\frac{7}{8}\%$ , and his furniture for \$900 at 1%. What premium did he pay?

21. The premium on 8000 bushels of corn, valued at 90¢ per bushel and insured for  $\frac{4}{5}$  of its value, is \$57.60. Find the rate of insurance.

22. A jewelry store is insured for \$20,000, and its contents, for \$27,000. The premium is \$705. What is the rate of insurance?

23. A clothier insured his stock of goods, valued at \$12,000, for 1 year at  $1\frac{1}{4}\%$ . At the end of 6 months he surrendered his policy. If the "short rate" for 6 months was 90¢ per \$100, how much premium was returned?

24. A farmer insured his buildings for \$3500 at  $1\frac{1}{2}\%$  for a term of 3 years. After he had paid the premium for 4 terms, the buildings were totally destroyed by fire. What was the farmer's loss? What was the company's loss?

25. A vessel worth \$27,000 is insured for  $\frac{3}{4}$  of its value at  $3\frac{1}{4}\%$ . In case of shipwreck, what is the company's loss? What is the owner's loss?

26. A business block valued at \$300,000 was insured in four different companies, the rate of each being 1%. The first company took \$50,000; the second, \$60,000; the third, \$90,000; and the fourth, the remainder. After the premiums had been paid four times, the block was damaged by fire to the amount of \$120,000. What was the loss of each company?

## GOVERNMENT INCOME AND EXPENDITURES

The revenues of our national government are of two kinds : *internal revenues* and *custom revenues* or *duties*.

**Internal revenues** consist of taxes on the manufacture or sale of malt liquors, tobacco, etc., and on incomes.

### Written Work

The internal revenue receipts for 1916 were \$512,723,000, derived from the following sources: from taxes on spirits, \$158,682,000; from taxes on fermented liquors, \$88,771,000; from taxes on tobacco, \$88,064,000; from income tax and other miscellaneous taxes, \$177,206,000.

1. What per cent of the total tax (to the nearest tenth per cent) was the tax on spirits?

2. What per cent of the total tax was the tax on fermented liquors?

3. What per cent of the total tax was the tax on tobacco?

4. What per cent of the total tax was the tax on miscellaneous articles and on incomes?

5. The receipts from the income tax in 1916 were \$124,937,000. Of this amount \$67,944,000 was paid by individuals. What per cent was paid by individuals? What per cent was paid by corporations paying \$56,973,000?

**Custom revenues**, or **duties**, are taxes on goods imported from foreign countries. These are collected at the custom-houses, which are maintained by the government at various *ports of entry*.

A **tariff** is a schedule of duties on imports fixed by our government.

## 302 GOVERNMENT INCOME AND EXPENDITURES

All merchandise brought into our country is classified as follows:

- (1) Merchandise on the **free list**, that is, free of duty.
- (2) Merchandise subject to an **ad valorem duty**, that is, a certain per cent of the cost of the goods, as shown by the invoice.

**Invoices** are statements showing the market price of goods, expressed in the money of the country where the goods are bought.

- (3) Merchandise subject to a **specific duty**, that is, to a certain amount per yard, pound, bushel, etc., without regard to value.

- (4) Merchandise subject to both an **ad valorem** and a **specific duty**.

For example, the duty on perfumery is 40¢ a pound *specific* and 60% *ad valorem*.

*Duties are not computed on parts of a dollar. If the invoice shows a number of cents less than 50, they are rejected in computation of duties: while 50 cents or more are counted as another dollar.*

**Tare** is an allowance made for the weight of the boxes or bags used in packing. It is deducted before duties are computed.

### Comparative Study

The **cost** of imported goods corresponds to what term in *percentage*?

The **rate** in duties corresponds to what term in *percentage*?

The **duty** corresponds to what term in *percentage*?

### Written Work

Find the duty on the following imports:

1. 1650 lb. of hops, specific duty 16¢ per pound.
2. \$6250 worth of clocks at 30% *ad valorem*.
3. \$10,000 worth of uncut diamonds, *ad valorem* duty 10%.

4. 2400 pounds of butter ; duty  $2\frac{1}{2}\%$  per pound.
5. \$5000 worth of blankets at 25 % ad valorem.
6. 800 yd. Brussels carpet valued at \$1.65 a yard. Duty 25 % ad valorem.
7. 500 boxes cigars (each box weighing 1 lb. and containing 100 cigars) invoiced at \$3.50 a box. Duty 25 % plus \$4.50 per pound.

Explain the different kinds of duty in this problem.

8. The specific duty on grape sugar is  $1\frac{1}{8}\%$  per pound. If the duty is \$135, find the number of pounds imported.

9. A department store in Buffalo imported from London flannels invoiced at \$6400 at an ad valorem duty of 30 %. How much duty was paid ?

10. A merchant imported from Sheffield, England, 12 gross table-knives costing \$3 a dozen. If the duty was 30 % ad valorem, and transportation cost \$9.60, find the cost per dozen, delivered.

11. A certain painting in Rome was purchased for 50,000 lire (\$.193). Find the cost when delivered in New York, if the duty was 15 % and freight and insurance cost \$49.75.

12. Find the entire cost of importing 3000 lb. of worsted yarn, invoiced at \$1800, if the freight charges were \$11.75; duty 18 % ad valorem.

13. Brown and Company imported 1200 sacks of chocolate each containing 90 lb. Find the duty at 2 % per pound.

The expenses of the national government are for the support of the army and the navy, for pensions, the care of the Indians, interest on the public debt, etc.



## 304 GOVERNMENT INCOME AND EXPENDITURES

The disbursements in 1915 were as follows: War department, \$172,923,000; navy department, \$141,836,000; pensions, \$164,388,000; interest on public debt, \$22,908,000; Indians and miscellaneous items, \$222,664,000.

14. What per cent of the total expenditure (to the nearest tenth per cent) was the war department's expense? the navy department's?

15. In 1910 the interest on the public debt was \$21,426,000. What was the per cent of increase in 1915?

16. The expenditure of the navy department was, in round numbers, \$56,000,000 in 1900 and \$142,000,000 in 1915. What was the per cent of increase from 1900 to 1915?

17. In 1900 the expenditure for pensions was approximately 15% less than in 1915. About how much was it in 1900?

**Taxes.** — The expenses of state and local governments are largely met by taxation. For example, the principal items of a state revenue of \$18,000,000 were as follows: tax on railroad corporations, \$4,400,000; tax on miscellaneous corporations, \$2,400,000; state school tax, \$5,500,000; local tax on railroads, \$2,297,000; inheritance tax, \$904,000. The outlay was for the support of the educational institutions, the care of criminals and dependents, and the salaries of state officials.

18. If the state school tax is  $2\frac{3}{4}$  mills on each dollar of taxable property, what will be the school tax of a county whose valuation is \$120,000,000?

The following is a statement of the principal sources of revenue in the city of Warren: taxation of property, \$289,500; excise licenses, \$22,000; franchise taxes, \$10,200; municipal licenses, \$7,000; rent of city properties, \$5,000.

19. What per cent of the receipts of the city of Warren is derived from excise license, if the total receipts are \$321,500?

20. A county with a valuation of \$90,000,000, pays a state school tax of  $2\frac{3}{10}$  mills. 90 % of this tax is returned to the county for use in support of its schools. How much does the county receive?

21. What was the tax rate in the city of Warren if the valuation amounted to \$16,000,000? (See p. 304.)

22. How much would a property owner pay, having a property worth \$35,000, assessed at 60 % of its value?

The principal expenditures in the city of Warren were for lighting, \$23,500; streets, \$44,000; police department, \$23,100; salaries of city officials, \$16,400; fire department, \$16,000.

23. \$250,000 worth of bonds paying  $4\frac{1}{2}$  % interest were issued in the year 1911. How much did that increase the tax rate for the next year?

24. A new school building costing \$120,000 was erected. Bonds at 4 % were issued to pay for this, and 2 % of the par value of the bonds issued was set aside each year as a sinking fund. What increase in the tax rate did this cause?

25. Of the expense for streets, 8 % was for cleaning, 23 % for sprinkling, and 33 % for regrading and rebuilding streets with gravel. What was the expenditure for each item?

26. Of the expense for the fire department, \$900 was for fuel, \$3400 for salaries, and \$4200 for supplies. What per cent of the expense was caused by each of these items?

The schools of some states are supported partly by local taxation and partly by money furnished by the state, derived from the state school taxes and other sources.

27. \$133,000 is required for the current expenses of a system of schools.  $50\frac{1}{2}$  % is raised by local taxation and the remainder is the state appropriation. What is the amount of each?

## 306 GOVERNMENT INCOME AND EXPENDITURES

28. If the valuation of the city is \$14,000,000, what is the rate of the local school tax (\$67,165)?

29. Of the total expense of \$133,000, \$79,000 is for teachers' salaries; \$6500 for fuel, and \$6000 for repairs. What per cent of the total is each of these items?

30. An increase of \$4000 was made in teachers' salaries; a building for storing coal was erected at an expense of \$2000, and \$2500 was spent in erecting fire escapes. How much did this add to the tax rate on \$14,000,000?

The general plan of distribution of the state appropriation in a certain state is to give each school district \$600, if it has a supervising principal, \$400 for each high school teacher, \$200 for each elementary school teacher, and a certain amount for each day's attendance of a pupil, depending in amount principally on the state school tax raised in the county.

31. A school district had a supervising principal, 15 high school teachers, and 70 elementary school teachers. Its yearly attendance was 419,000 days. The apportionment for one day's attendance was  $9\frac{15}{100}$ ¢. How much did this district receive from the state?

32. A school numbering 820 pupils was in session 188 days and had an average attendance of 91%. How much should it receive from the state for attendance, if the apportionment is  $8\frac{7}{10}$ ¢ for one day's attendance for each pupil?

33. How much would a school district receive from the state if it employed 10 high school teachers, 32 elementary teachers, and a supervising principal, the attendance numbering 193,631 days, and the apportionment for a day's attendance being  $7\frac{4}{10}$ ¢?

34. The tax rate in a certain city is 17 mills on the dollar. Find the tax on a property valued at \$12,500.

## LONGITUDE AND TIME

**Meridians** are imaginary lines passing north and south from one pole of the earth to the other.

The **equator** is an imaginary line passing around the earth midway between the poles.

These imaginary lines aid in locating places on the earth and in determining differences in time.

Observe that the equator is a circumference of a circle; therefore distances along it are measured in degrees.

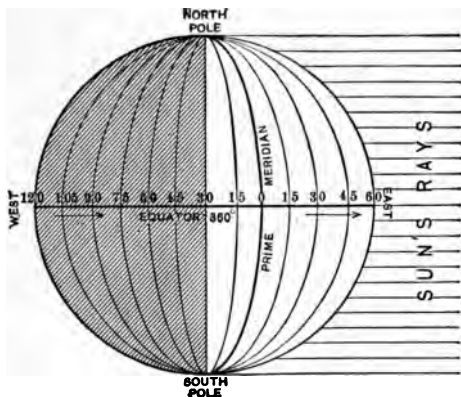
The **prime meridian** is a meridian from which time and place, east and west, are reckoned.

The meridian passing through the Royal Observatory at Greenwich, England, is the prime meridian in common use.

**Longitude** is the distance east or west of this prime meridian measured in degrees. (See p. 111.)

Places *east* of this prime meridian have *east longitude*; places *west* of this prime meridian have *west longitude*.

From the time the sun's rays are vertical over any meridian until they are vertical again it is 24 hours. Therefore any



point passes through  $360^\circ$  in one rotation of the earth on its axis.

Since  $360^\circ$  of longitude pass under the sun's vertical rays during 24 hours, how many degrees pass during 12 hours? 1 hour?

Since  $\frac{1}{24}$  of  $360^\circ$ , or  $15^\circ$ , pass under the sun's rays in 1 hour, 1 hour of time corresponds to  $15^\circ$  of longitude.

Since  $15^\circ$  of longitude correspond to 1 hour of time,  $\frac{1}{60}$  of  $15^\circ$ , or  $\frac{1}{4}^\circ$ , or  $15'$ , of longitude, correspond to 1 minute of time, and  $15''$  of longitude correspond to 1 second of time.

Table of relation between *longitude* and *time* :

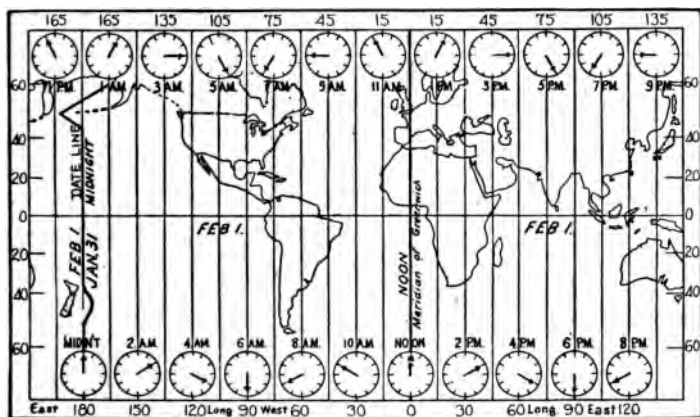
$360^\circ$ of longitude	correspond to 24 hours of time
$15^\circ$ of longitude	correspond to 1 hour of time
$15'$ of longitude	correspond to 1 minute of time
$15''$ of longitude	correspond to 1 second of time
$1^\circ$ of longitude	corresponds to 4 minutes of time
$1'$ of longitude	corresponds to 4 seconds of time

When the sun's rays are vertical on the 90th meridian, all places on that meridian have noon.

The rotation of the earth from west to east makes the sun appear to move from east to west. The map on p. 309 shows that when it is noon at Greenwich, it is *before* noon or *earlier* at all places *west*, because the sun's rays are not yet vertical on any meridian west of the prime meridian. It is *after* noon or *later* at all places *east*, because the sun's rays have already been vertical on all meridians east of the prime meridian.

Examine the map. What time is it on the meridian of Greenwich?  $45^\circ$  east of Greenwich?  $45^\circ$  west of Greenwich? In traveling from London to New York would a watch be set *forward* or *backward*? about how much? About how much change in time must be made in traveling from Cal-

cutta westward to San Francisco? from Honolulu eastward to Cape Town?



MAP SHOWING NOON, FEBRUARY 1, AT GREENWICH.

What is the difference in degrees between a place  $30^{\circ}$  east longitude and a place  $45^{\circ}$  east longitude? What is the difference in time and which has the earlier time?

Table of Longitude of Some Important Places

London	$0^{\circ} 5' 48''$ W.	Denver	$104^{\circ} 58' 0''$ W.
New York	$74^{\circ} 0' 3''$ W.	Cape Town	$18^{\circ} 28' 45''$ E.
Trenton	$74^{\circ} 46' 0''$ W.	Honolulu	$157^{\circ} 50' 36''$ W.
Newark	$74^{\circ} 10' 0''$ W.	Tokyo	$139^{\circ} 44' 30''$ E.
Pittsburgh	$80^{\circ} 2' 0''$ W.	Manila	$120^{\circ} 58' 6''$ E.
Washington	$77^{\circ} 3' 0''$ W.	Canton	$113^{\circ} 16' 30''$ E.
Chicago	$87^{\circ} 36' 42''$ W.	Berlin	$13^{\circ} 23' 44''$ E.
San Francisco	$122^{\circ} 25' 42''$ W.	Rome	$12^{\circ} 27' 14''$ E.
Boston	$71^{\circ} 3' 50''$ W.	Paris	$2^{\circ} 20' 15''$ E.

Longitudes are given to the nearest seconds.

## Written Work

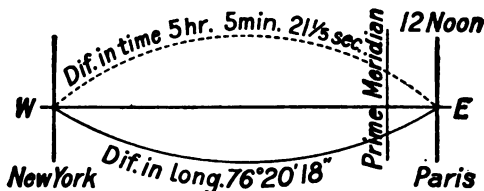
1. When it is noon, solar time, at Paris, what is the solar time at New York?

$$\begin{array}{r}
 2^{\circ} \ 20' \ 15'' \text{ E.} \\
 74^{\circ} \ 0' \ 03'' \text{ W.} \\
 15 \overline{) 76^{\circ} \ 20' \ 18''} \\
 \underline{5^{\circ} \ 5' \ 21\frac{1}{2}''} \\
 5 \text{ hr. } 5 \text{ min. } 21\frac{1}{2} \text{ sec.}
 \end{array}$$

Since the earth rotates  $15^{\circ}$  in 1 hr.,  $15'$  in 1 min., and  $15''$  in 1 sec., the difference in time is as many hours, minutes, and seconds as there are degrees, minutes, and seconds in  $\frac{1}{15}$  of the difference in longitude,  $76^{\circ} \ 20' \ 18''$ .

$\frac{1}{15}$  of  $76^{\circ}$  is  $5^{\circ}$ , with  $1^{\circ}$ , or  $60'$ , remaining;  $60' + 20' = 80'$ ;  $\frac{1}{15}$  of  $80' = 5'$ , with  $5'$  remaining;  $5' = 300''$ ;  $300'' + 18'' = 318''$ ;  $\frac{1}{15}$  of  $318'' = 21\frac{1}{2}''$ .

The difference in time is 5 hours, 5 minutes,  $21\frac{1}{2}$  seconds. Since New York is *west* of Paris, the time in New York is *earlier*; that is, when it is noon at Paris, it is 6 o'clock, 54 min., and  $38\frac{1}{2}$  sec. A.M. at New York.



What is the difference in longitude between the two places? the difference in time? In going west from Paris to New York would a traveler set his watch forward or backward? how much?

Find difference in degrees, difference in time, and which place has earlier time:

- | PLACES                                  | PLACES                                  |
|-----------------------------------------|-----------------------------------------|
| 2. $74^{\circ}$ W. and $45^{\circ}$ W.  | 6. $120^{\circ}$ W. and $30^{\circ}$ E. |
| 3. $120^{\circ}$ W. and $75^{\circ}$ W. | 7. $90^{\circ}$ W. and $30^{\circ}$ E.  |
| 4. $15^{\circ}$ W. and $45^{\circ}$ E.  | 8. $135^{\circ}$ E. and $30^{\circ}$ E. |
| 5. $30^{\circ}$ E. and $60^{\circ}$ E.  | 9. $45^{\circ}$ W. and $60^{\circ}$ E.  |

When the sun's rays are vertical on the meridian of Washington, find the solar time in the following places :

- |                   |              |              |
|-------------------|--------------|--------------|
| 10. Newark        | 13. Paris    | 16. Berlin   |
| 11. Chicago       | 14. Rome     | 17. New York |
| 12. San Francisco | 15. Honolulu | 18. Trenton  |

19. When it is midnight (solar time) on the last day of the year in Trenton, how much of the old year (solar time) remains to the people of Honolulu ?

20. A ship sets sail from Liverpool for New York, Jan. 10. When in longitude  $34^{\circ} 6' 10''$  W., its chronometer reads 2.30 P.M., Jan. 15. Find the difference in the readings between the ship's time and the meridian time of New York.

21. Berlin meridian time is 6 hr. 44 min. and  $1\frac{1}{4}$  sec. later than Chicago meridian time. What is the longitude of Berlin?

hr.	min.	sec.	"	"	"
6	44	$1\frac{1}{4}$	101	0	26 difference in longitude
		15	87	36	42 W. (Chicago)
101	0	26	13	23	44 E. (Berlin)

15 times the difference in time expressed in hours, minutes, and seconds corresponds to the difference in longitude expressed in degrees, minutes, and seconds.

$15 \times 1\frac{1}{4}$  sec. = 26 sec.;  $15 \times 44$  min. = 660 min., or 11 hr.;  $15 \times 6$  hr. = 90 hr.; 90 hr. + 11 hr. = 101 hr.

Therefore  $15 \times 6$  hr. 44 min.  $1\frac{1}{4}$  sec. corresponds to  $101^{\circ} 0' 26''$  of longitude.

This difference in longitude would not tell us whether Berlin is east or west of Chicago; but as Berlin has *faster* time than Chicago, it must be east of it. Chicago is  $87^{\circ} 36' 42''$  west of the prime meridian. Therefore Berlin must be  $13^{\circ} 23' 44''$  east of the prime meridian.



22. The "Treaty of Portsmouth" between Japan and Russia was signed at Portsmouth, N. H., Sept. 5, 1905, at 47 minutes past 3 P.M., 75th meridian time. What was the corresponding solar time at Petrograd, Russia,  $30^{\circ} 17' 51''$  E., and at Tokyo, Japan,  $139^{\circ} 44' 30''$  E.?

Tokyo is east of the 75th meridian  $214^{\circ} 44' 30''$ , therefore its time is 14 hr. 18 min. 58 sec. faster than Portsmouth, which has 75th meridian time. Counting this time forward from 3.47 P.M., Sept. 5, gives 5 minutes and 58 seconds past 6 o'clock A.M., Sept. 6, Tokyo solar time.

23. The President of the United States takes the oath of office at 12 noon, 75th meridian time. Find the solar time and date in each of the following places for the inauguration of March 4, 1917: Honolulu; San Francisco; Manila.

### INTERNATIONAL DATE LINE

The nations have agreed upon the 180th meridian, with slight changes as shown on page 309, as the place where the new day always begins. The calendar is set forward one day on ships crossing this line sailing *westward*; the calendar is set back one day on ships crossing this line sailing *eastward*.

1. A ship sets sail from San Francisco for Manila, Oct. 9, at 9 A.M., 120th meridian time; it arrives at Manila, Oct. 27, at 9 A.M., meridian time. How long is the voyage?

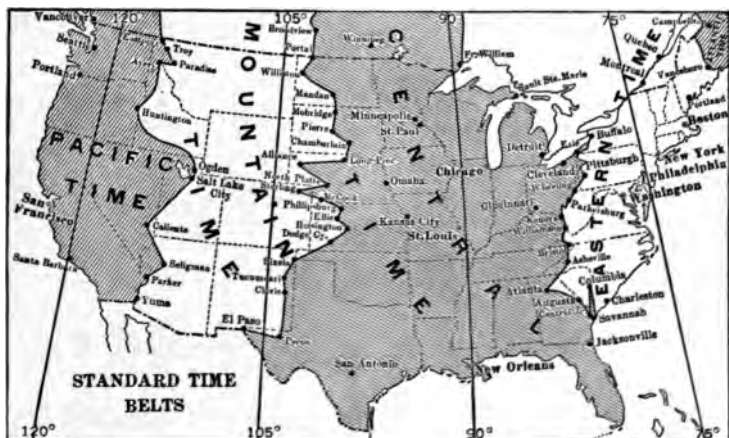
2. The same ship sets sail from Manila, Nov. 3, at 3 P.M., meridian time, and arrives at San Francisco, Nov. 23, at 3 P.M., 120th meridian time. How long is the voyage?

### STANDARD TIME

The railroads of the United States in 1883 agreed upon a system of **standard time** and divided our country into four time belts, as shown on the map. In the **eastern** time belt, all trains keep the time of the 75th meridian, known as

**eastern** standard time. In the **central** time belt, all trains keep the time of the 90th meridian, known as central standard time. In the **mountain** time belt, all trains keep the time of the 105th meridian, known as mountain standard time. In the **Pacific** time belt, all trains keep the time of the 120th meridian, known as Pacific standard time.

Each railway has selected the most convenient towns on its route, as is shown on the map, to change from the standard time of one belt to the standard time of another belt. The time in any belt is 1 hour *faster* than the time in the belt west of it, or 1 hour *slower* than the time in the belt east of it. Correct time is telegraphed each day to all parts of the United States from the Naval Observatory at Washington.



1. What is the difference in time between closing of the election polls at 7 P.M. in New York and 7 P.M. in Seattle, on the day for choosing presidential electors?

2. A telegraph message sent from Philadelphia at 11 A.M. was delivered in San Francisco at 8.45 A.M. of the same day. Why could this be true?

## GENERAL REVIEW

### Oral Work

1. A boy buys oranges at 25 cents a dozen and retails them at 3 for 10 cents. Find his gain per cent.

2. Mr. Henderson was earning \$3.50 per day of 10 hours. His wages were reduced 20%. Find the rate of wages per hour he then earned and his daily wages.

3. A man purchased 30 shares of steel stock at 998. How much did he pay for the stock?

4. A merchant buys apples at 60 cents a bushel and retails them at 30 cents a peck. Find his gain per cent.

5. A can do  $\frac{3}{4}$  of a piece of work in 10 hours and B can do  $\frac{2}{3}$  of it in 9 hours. In how many hours working together can they do the work?

6. If I sell  $\frac{3}{4}$  of a pound of butter for  $\frac{4}{5}$  of the cost of a pound, what per cent do I gain?

7. What per cent is lost by selling goods at  $\frac{5}{6}$  of their cost?

8. A merchant buys goods at discounts of 20%, and 5%, and sells them at the list price. Find the per cent of profit.

9. Read as per cents:  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$ ,  $1\frac{1}{2}$ ,  $\frac{3}{4}$ ,  $\frac{1}{16}$ ,  $\frac{1}{12}$ , .16 $\frac{2}{3}$ , .05, .06 $\frac{1}{4}$ , .37 $\frac{1}{2}$ , .40, .25.

10. What is the interest on \$750 for 4 months at 6%?

11. A grocer bought 20 barrels of apples at \$2 a barrel. How much will he gain if he sells them at an advance of 25%?

12. A coal merchant pays \$2 a ton for soft coal. The freight and delivery cost him 50¢ a ton. If he retails the coal at \$3 a ton, find his per cent of profit on the entire cost of the coal.

13. Wishing to find whether a corner was square, a man measured 8 feet along one side from the corner and 6 feet along the other side from the corner. What should be the distance between the ends of the lines if the corner is square?

14. Explain why, when it is 12 o'clock M. at Washington, it is approximately 9 o'clock A.M. at San Francisco and 5 o'clock P.M. at London (standard time).

15. What part of 3 bu. is 3 pk.?

16. If a man's salary is \$1600, and he saves  $37\frac{1}{2}\%$  of it each year, how much does he save in 3 years?

17. 30%, 10% off is equivalent to what single commercial discount?

18. What will be the cost of 3 gal. 2 qt. of oil at 16¢ a gallon?

19. A retailer bought pencil tablets at \$2.40 per hundred and sold them at 5 cents each. Find the gain per cent.

20. At 4¢ a foot, find the cost of picture molding for a room 25 ft. by 30 ft.

21. The surface of a cube is 96 square inches. Find its volume.

22. A house and lot cost \$6000. It rents for \$360 a year. The taxes average 1%, insurance  $\frac{1}{2}\%$ , and repairs \$30. What is the per cent of net income?

23. What is the interest on \$800 for 2 yr. and 3 mo. at 4%?

24. What is the difference in longitude of two places whose difference in time is 1 hr. 4 min.?
25. A farmer bought a wagon for \$120, and sold it for \$140. What was his per cent of profit?
26. How many yards of gingham, worth  $37\frac{1}{2}$ ¢ a yard, can I buy for \$3?

### Written Work

1. A 60-day note for \$1000 is discounted at a bank at 6 % on the day of issue. Find the proceeds.
2. A cow is tied in the corner of a square field by a rope 2 rd. long. Find the extent of the surface over which it can graze.
3. What is the amount of \$1575 for 3 yr. 6 mo. at 6 %?
4. A plowman by measurement found that he had plowed a strip 4 rd. in width around a rectangular field 40 rd. long and 20 rd. wide. How many acres had he plowed?
5. There are 50 persons in a schoolroom 36 ft. long, 30 ft. wide, and 15 ft. high. How many cubic feet of air space are there for each person? The law fixes the minimum of fresh air at 30 cubic feet per minute per person. How frequently must the room be filled with fresh air to meet this requirement?
6. Find the commission, at 5 %, on 2240 lb. of butter sold at 35 ¢ a pound.
7. A man buys an automobile for \$1500. He pays for it by mortgaging his house for \$1500, paying interest at  $5\frac{1}{2}$  %. At the end of two years he sells his automobile for \$700. Not counting the running expenses, what has been the cost per year of his machine?

8. A ton of coal occupies 42 cu. ft. How much will it cost to fill a bin 12 ft. long, 6 ft. wide, and 5 ft. high with coal at \$6.50 a ton?

9. It cost \$560 to fence a field 80 rd. long and 60 rd. wide. How much less will it cost to fence a field 50 rd. square with the same kind of fence?

10. A man borrows \$3680 at 5% and immediately lends it at  $6\frac{1}{2}\%$ . How much does he gain in two years?

11. How much does a man receive at the end of 6 wk. 3 da. and 6 hr., if his wages are \$12 a week? Reckon 6 days to a week and 8 hours to a day.

12. A town assessed at \$2,450,000 must raise a tax of \$7766.25. The poll tax is \$825. Find the tax rate, if the collector is allowed 5% for collecting.

13. West Point is in longitude  $74^{\circ} 57'$ , and St. Louis in longitude  $90^{\circ} 15'$ . When it is 4 P.M. in West Point, what time is it in St. Louis?

14. I bought 1 pk. 2 qt. 1 pt. of berries for \$ $2\frac{1}{2}$ . At what price per quart must I sell them to gain  $12\frac{1}{2}\%$ ?

15. A farmer having 6 bu. 8 qt. of cranberries, lost by decay 7 pk. 7 qt. What per cent had he left?

16. My bill for plastering and painting was \$190, and  $\frac{3}{8}$  the cost of the painting plus \$12 equals  $\frac{3}{10}$  the cost of the plastering. Find the cost of the painting; of the plastering.

17. A piece of steel in the form of a cylinder is 4 ft. long and 2 in. in diameter. How long is it when rolled into a bar 1 in. square?

18. A line 28 inches long is divided into two parts, one of which is 3 times as long as the other. How long is each

19. A man bought a tenement containing 8 apartments for \$35,000. His taxes, repairs, and insurance cost him \$1157 annually. He rented 4 suites at \$40 a month, 2 suites at \$35 a month, and 2 suites at \$28 a month. What per cent did he realize on his investment?

20. What is the balance of a bill of \$85.50 after two deductions have been made; the first of 10% on the \$85.50, and the other of 5% on what remained?

21. How much will a block of stone weigh which is 6 ft. long, 3 ft. 6 in. wide, 4 ft. 4 in. high, if each cubic foot weighs 180 lb.?

22. A coal dealer bought 360 T. of coal, 2240 lb. to the ton, at \$5.75 a ton. He sold the coal at \$6.25 a ton, 2000 lb. to the ton. How much did he gain?

23. If books are bought for \$1.50 less  $33\frac{1}{3}\%$  and 10%, at what price per copy must they be sold to gain  $43\frac{1}{3}\%$ ?

24. A square log of wood is 15 in. broad and 11 in. thick. What length of it will contain 10 cubic feet?

25. A furniture dealer buys 120 tables @ \$5.50 and gives in payment a 90-day note dated April 21, 1917. If the note is discounted May 3, at 6%, what will be the proceeds?

26. The Rose and Ward Company imported from England 10 cases of woollen dress goods, invoiced at \$2000 a case. Find the duty at 35% ad valorem.

27. The duty on cigars is \$4.50 per pound and 25% ad valorem. Find the duty on 1000 one-pound boxes of cigars, invoiced at \$6 a box?

28. A watch marked \$80 was sold at a discount of 25% and 5% for cash. Find the cash selling price.

29. I sold my horse for \$ 180 and took in payment a 90-day note bearing interest at 5 %, dated Feb. 1, 1917. On April 5, I had the note discounted at the bank at 6 %. What were the proceeds of the note ?

30. An importer bought 1000 pieces of goods in Europe at \$ 30 per piece. The duty on the goods was 50 % ad valorem, and the freight and other charges were \$ 2000. For how much per piece must the goods be sold to yield a profit of 20 % ?

31. I have a rectangular field, the perimeter of which is 240 rods. It is twice as long as it is wide. How many acres does it contain ?

32. A, B, & C have 1400 acres of land. If  $\frac{1}{2}$  of A's share is equal to  $\frac{1}{3}$  of B's, and  $\frac{5}{6}$  of B's share is equal to  $\frac{2}{3}$  of C's, how many acres has each ?

33. \$ 7500 worth of goods were bought for cash and sold on a 4 months' note at  $16\frac{2}{3}$  % advance. The note was discounted the same day at 6 %. Find the proceeds of the note.

34. A 90-day note for \$ 640, without interest, is discounted at the bank at 6 % on the day of issue. Find the proceeds of the note. What would be the proceeds, if the note read "with interest" at 6 % ?

35. I buy furniture listed at \$ 4400, getting trade discounts of 20 %, 10 %, and 5 %. I sell at 40 % above cost, taking in payment a 120-day note without interest. I have the note discounted at 6 % on the day of sale. How much do I gain ?

36. How many building lots, each 75 ft. by 125 ft., can be laid out in 2 A. 92 sq. rd. 37 sq. yd. and how much are they worth at \$ 350 a lot ?

37. A steel ingot is 16 in. square and 8 ft. long. What length of steel bar will it make 4 in. thick and 6 in. wide ?



38. Find the cost, at 27 ¢ per load, to excavate the ground for a cellar 24 ft. wide and 42 ft. long, the ground being excavated to a depth of  $7\frac{1}{2}$  ft. at one end and 2 ft. 4 in. at the other end.

39. I bought land at \$62.50 an acre and sold it at 20 % profit, making thereby \$8100. How many acres did I buy?

40. How much United States money would be required to pay 25 % of a debt of £ 961 16s. 8d., a pound sterling being worth \$4.8665?

41. A train ran 361 miles in  $9\frac{1}{2}$  hours. What was its average rate?

42. How many cubic yards of broken stone will be required to build a road 16 ft. wide, 8 in. deep, and 1 mi. in length?

43. A bankrupt's liabilities are \$15,000 and his assets \$9000. How much does a creditor receive whose claim is \$6000, no allowance being made for court costs?

44. How many cakes of soap 4 in. long, 2 in. wide, and  $1\frac{1}{2}$  in. thick can be packed in a box 2 ft. long, 1 ft. wide, and 1 ft. deep?

45. Ten pounds of potatoes contain 7.83 lb. of water and 2.17 lb. of solid matter. How much water and solid matter would a bushel of 60 lb. contain?

46. Find the sum of the quotients:

$3 \div 3$	$3 \div .03$	$300 \div 30$
$3 \div .3$	$.003 \div 3$	$30 \div 300$
$.03 \div 3$	$30 \div .03$	$.03 \div 30$

47. The tax rate in a town was 14 mills on a dollar. What would be the tax on property worth \$250,000?

48. The perimeter of a square field is 320 rods. How many acres does it contain?

49. A man has a life insurance policy for \$5000 on which he pays an annual premium of \$28.40 per thousand. How much does he pay in premiums in five years?

50. If the tax rate is \$.75 on \$100, what is the tax on property valued at \$4,800, assessed at  $\frac{1}{2}$  of its value?

51. Sound travels 1120 feet per second. The thunder from a flash of lightning was heard 11 seconds after the flash was seen. How many miles distant was the cloud?

52. Hats, costing \$33 per dozen, were sold for \$3.50 each. What was the per cent profit on each hat?

53. Find the cost of the following bill of lumber:

30 pieces  $30' \times 6'' \times 12''$ , at \$30 per M.

60 pieces  $24' \times 6'' \times 8''$ , at \$24 per M.

120 pieces  $18' \times 4'' \times 6''$ , at \$20 per M.

150 pieces  $16' \times 3'' \times 4''$ , at \$18 per M.

54. A barn valued at \$1800 is insured for  $\frac{3}{4}$  of its value, at  $1\frac{1}{2}\%$ , for a term of 3 years. Find the average annual cost of insurance.

55. \$200 was borrowed at 5% on Oct. 1, 1917, and was paid July 1, 1918. What was the amount paid at that time?

56. On a bill of \$450 two successive discounts of 20% and 15% were given. What was the amount of the discount?

57. At \$13.50 a thousand, what is the tax on property assessed for \$12,480?

58. A broker paid \$6307.50 for stock at  $105\frac{1}{8}$ . How many shares did he buy?

59. At  $4\frac{1}{2}\%$ , what is the interest on \$750 from Jan. 12, 1914, to July 9, 1918?

60. A street 40 rd. long and 40 ft. wide is to be graded down on an average  $1\frac{1}{2}$  ft. How much will the excavating cost at 27¢ a cubic yard?

61. A coal dealer bought 260 tons of coal at \$5.50 a long ton and sold it at \$7 a short ton. Find the gain.

62. A wholesale merchant imports from Belgium 30,000 yd. of Brussels carpet at \$1 a yard. The duty is 25 % ad valorem. Find the cost in the United States, including the duty.

63. I bought 50 shares of railroad stock at  $156\frac{1}{4}$  and sold it at  $166\frac{3}{4}$ . Find my gain.

64. The assessed valuation of a town is \$875,000. What rate must be levied to raise \$10,937.50?

65. The residence of Mr. Daniels valued at \$8000 was insured for 3 years at 90 ¢ per \$100 on 80 % of its valuation. How much was the premium paid?

66. Each of two farmers has a ten-acre field of potatoes. One farmer sprays to prevent blight, using on each acre 200 gal. of mixture costing 30 ¢ a gallon. His yield is 210 bu. an acre. The other neglects to spray his field and gets but 135 bu. an acre. If the potatoes are sold at \$1.85 a bushel, what is the profit due to spraying?

67. A train running 30 mi. an hour is 54 min. in going from one city to another. If it makes 3 stops of 4 min. each, how far apart are the cities?

68. The wholesale list price of a stove is \$38, and the retail dealer is allowed 20 %, 10 %, and  $3\frac{1}{2}$  % off. What price does he pay for the stove?

69. What would have been the price if there had been one discount of 30 %?

70. In an examination, 150 questions were asked each of 5 pupils. The first answered 140; the second, 135; the third and fourth, 120 each; and the fifth, 110. Find the average per cent of the class.

71. A farm roller 8 ft. long and  $2\frac{1}{2}$  ft. in diameter will pass over how much surface in 100 revolutions?

72. The floor of a room is 18 ft. by 15 ft. Find the cost of covering it with matting a yard wide, at  $29\frac{1}{2}\phi$  a yard, no allowance being made for matching.

73. A grocer pays \$16 for 5 bu. of cranberries and sells them so as to gain 30%. What is the selling price of the cranberries per quart?

74. A debt of \$450 was paid at the expiration of 2 yr. 4 mo. The rate of interest being 5%, what was the amount paid?

75. The face of one side of a cube is a surface of 100 square feet. Find the volume.

76. A tax of \$6181.40 is to be assessed upon a town. There are 620 persons subject to a poll tax of \$1.25 each. The property assessment is \$675,800. Find the tax on Mr. Anderson's property, which is assessed at \$4750, if he pays for one poll.

77. A man buys a house and lot for \$3500; he pays \$1000 cash and gives a mortgage for the balance at 6%. At the end of 9 mo. he sells the house and lot for \$4500, paying the interest due on the mortgage at the time of sale. How much does he realize on his investment?

78. A farmer sold 200 bu. of apples at  $75\phi$  a bushel, 10 doz. eggs at  $36\phi$  a dozen, and 40 lb. of butter at  $32\phi$  a pound. He invested  $\frac{2}{3}$  of the proceeds in coal and deposited the balance in the bank. How much did he deposit?

79. Find the amount of a note for \$550 at 6%, given Dec. 1, 1914, and paid May 15, 1917.

80. Express as decimals and add the following:  $\frac{3}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{3}$ ,  $\frac{1}{8}$ ,  $\frac{7}{8}$ .

81. Simplify:  $\frac{(\frac{4}{15} + 1\frac{1}{3}) \div (3\frac{1}{2} \times \frac{4}{5})}{1\frac{2}{3} \times \frac{3}{20}}$ .

82. A farmer buys 24 head of cattle, at \$80 a head; and after losing 2, sells the remainder at \$105 a head. What per cent does he gain or lose?

83. On a bill of goods amounting to \$900, I am offered a discount of 25%, or two successive discounts of 15% and 10%. Which would be more advantageous for me to accept, and how much more?

84. My gas meter, Jan. 1, registered 11,800 cu. ft.; Feb. 1, 85,800 cu. ft. I paid the bill before Feb. 10, receiving a discount of 2¢ on the even thousand. At 27¢ a thousand cubic feet, how much did I pay for the gas used in January?

85. Simplify:  $\frac{7\frac{1}{2} + \frac{3}{4}}{5\frac{3}{8} - 1\frac{1}{4}} \times \frac{22}{49}$ .

86. I imported from Canada 7500 yd. of flannel valued at \$1.25 a yard, ad valorem duty 30%. Find the amount of duty paid.

87. The total assessed valuation of a school district in a certain state was \$3,009,251. The amount to be raised for state and county taxes was \$17,886.41, and for local tax \$84,601.84. What was the rate of taxation?

88. A street  $\frac{1}{2}$  mi. long and 30 ft. wide is paved and curbed. The paving costs \$3 a square yard, and the curbing 80¢ a linear foot. Find the entire cost.

89. Find the sum of the quotients:

$.01 \div .001$	$.1 \div 10$	$.001 \div .1$
$10 \div .01$	$.01 \div 10$	$.001 \div .01$

90. The oxygen in the air is to the nitrogen as 21 to 79. Find the number of cubic feet of each gas in a schoolroom whose inside dimensions are 30 ft.  $\times$  24 ft.  $\times$   $12\frac{1}{2}$  ft.

91. A boy buys a newspaper route for \$75. He pays \$10 down, and agrees to pay the balance in weekly payments by using  $\frac{1}{2}$  of his profits. If he has 250 customers and makes 4¢ a week on each customer, how many weeks will it require to pay the debt?

92. A street  $\frac{1}{2}$  mi. long and 60 ft. wide is paved with granite blocks and curbed on both sides. The paving costs \$2.75 per square yard and the curbing, 60¢ a lineal foot. Find the entire cost.

93. At 24¢ a pound, find the cost of lining a tank, without a top, 5 ft. 8 in. long, 4 ft. wide, and 6 ft. deep, with zinc weighing 5 lb. to the square foot.

94. A man paid \$8000 for a house and lot. If taxes are \$85 and other expenses are \$35 per year, what monthly rent must he get for it to make 6% on the cost of the property?

95. A town expends for improvements \$6894. The assessed valuation is \$480,000. Find the rate levied to cover the expense, including the collector's commission estimated at \$306.

96. A swimming pool is 90 ft. long, 40 ft. wide, and averages 5 ft. in depth. How many gallons of water does it contain when full?

97. A ranchman bought a tract of land in the form of a rectangle  $4\frac{1}{2}$  mi. in length by  $2\frac{1}{2}$  mi. in width. Find the number of acres in it.

98. A house valued at \$6000 was insured for  $\frac{3}{4}$  of its value for a period of 3 years. If the rate was 50 cents per \$100 of valuation, what was the amount of the premium paid?

**99.** A commission merchant sold the following at the rates of commission indicated. What was his total commission?

1500 bu. corn @ \$.90; rate of commission  $1\frac{1}{2}\%$ .

90 doz. eggs @ \$.25; rate of commission 11%.

800 bu. produce @ \$.85; rate of commission 10%.

60 bbl. apples @ \$3.50; rate of commission 5%.

**100.** John Gordon bought a lot from George Ball for \$3500. He paid \$2000 cash, and gave a 3 months' promissory note with interest at 6%, for the balance. Write the note.

**101.** An agent sold 1720 bu. oats at 50¢ a bushel and charged \$34.40 commission. What was the rate of commission?

**102.** An insolvent debtor pays 40¢ on the dollar. How much will a creditor receive whose claim is \$960, after paying his attorney 10% for collecting it?

**103.** A rectangular field whose width is  $\frac{1}{2}$  of its length, has a perimeter of 240 rd. What is the length of each side?

**104.** What is the interest on \$427 for 2 yr. 6 mo. 21 da. at 4%?

**105.** Stock was bought at  $85\frac{1}{8}$  and sold at  $109\frac{7}{8}$ . What was the profit on 130 shares?

**106.** From one tenth take one thousandth; multiply the remainder by 10,000; divide the product by one million, and write the answer in words.

**107.** A drugget 12 ft. by 15 ft. covers  $\frac{3}{8}$  of the floor of a room 15 ft. wide. Find the length of the room.

**108.** A rectangular piece of land is 80 rd. long and 24 rd. wide.  $16\frac{2}{3}\%$  is planted with potatoes,  $33\frac{1}{3}\%$  with corn, and the remainder is in pasture. How many acres are there in each part?

109. A house that had been insured for \$3000 at  $1\frac{3}{4}\%$  for each term of 3 years, was destroyed by fire in the ninth year. How much did the money received exceed the premiums paid?

110. A natural gas company declares a semiannual dividend of  $4\%$  on a capital stock of \$150,000. Find the yearly dividend of a stockholder who owns 36 shares, par value \$50 a share.

111. A merchant in Denver, Colo., buys a New York draft for \$600, at  $\frac{1}{4}\%$  exchange, and mails it in payment of a bill in Memphis, Tenn. Find the amount paid for the draft.

112. A house worth \$12,800 is insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ . The furniture, worth \$3600, is insured for  $\frac{3}{8}$  of its value at  $1\frac{3}{4}\%$ . What is the total premium?

113. A commission merchant sold 600 bbl. of apples at \$1.75 a barrel. He sent the owner \$1013.25, after deducting his commission. Find the rate of commission.

114. A bought 80 shares of railroad stock at 125. His annual dividends amounted to \$400. What per cent did he receive on his investment?

115. A lawyer in collecting a note of \$3000, compromised by taking  $80\%$  and charged  $5\%$  for his fee. Find his commission.

116. A building that cost \$40,000 was insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ . After two premiums had been paid, the building was totally destroyed by fire. What was the entire loss to the owner?

117. From a vessel containing 5 gal. 3 qt. 1 pt. of vinegar  $10\%$  was used. How much remained in the vessel?

118. A piano listed at \$650 was sold at a discount of  $40\%$  and  $20\%$ . If the freight was \$3.25 and drayage \$5, what was the net cost of the piano?



119. A merchant sold a bill of goods amounting to \$3600 and took a 90-day note for it. Fifteen days later he sold the note at a bank at 6% discount. How much did he receive for the note?

120. How many times can a conical cup  $2\frac{1}{2}$  in. in diameter and 3 in. deep be filled from a cylindrical vessel  $2\frac{1}{2}$  in. in diameter and 9 in. deep?

121. Property worth \$8400 is insured for  $\frac{3}{4}$  of its value at  $\frac{3}{4}$  of one per cent. What will be the loss, including premium, in case of total destruction by fire?

122. A coat that cost \$25 was marked at an advance of 20%. It was afterward sold at a discount of 10% from the marked price. What was the actual per cent of profit in the transaction?

123. A San Francisco banker discounts a draft for \$3000 payable at Portland, Oregon, 90 days after sight, exchange  $\frac{1}{10}$ %, discount 8%. Find the proceeds.

124. A merchant imported 20 rolls of Axminster carpet, each containing 84 yd., valued at \$2 a yard. Find the duty at 35% ad valorem.

125. A cable message was sent at 6.15 A.M. from New York to London. It was delivered 23 minutes 20 seconds after being sent. At what time was it delivered?

126. Mr. Coll leased some property for three years at \$2400 per year. His commission for leasing was 1% of the first year's rent and his commission for collecting was 5% of each year's rent. Find the agent's entire commission for the three years, if all the rent was collected.

127. A commission merchant was offered \$1800 per year salary and 2% on all sales above \$40,000, or 5% on all sales. He chose the latter and sold \$85,000 worth of goods. Did he gain or lose, and how much, by so doing?

**128.** A dealer bought 4 loads of hay at \$22.50 a ton. The loads, including the wagons, weighed 4456 lb., 3920 lb., 4330 lb, and 4260 lb., respectively. The wagons averaged 1180 lb. in weight. Find the value of the hay.

**129.** A plot containing 11 A. 88 sq. rd. was divided into building lots 66 ft.  $\times$  115 $\frac{1}{2}$  ft. The lots were sold at \$250 each. How much was received for them?

**130.** In a bin at different times the following quantities of oats were put: 16 bu. 3 pk., 24 bu. 2 pk., 44 bu. 1 pk. 6 qt. At 64¢ a bushel, find the value of the contents of the bin.

**131.** If you leave Concord, N. H., at 1 P.M. and in 24 hours are 15° 30' west of Concord, what will be the time of the place at which you have arrived?

**132.** If a wagon body is 10 ft. long, 4 ft. wide, and 3 ft. deep, how much will a load of corn be worth at 90¢ a bushel?

**133.** A farmer's herd of 15 cows averaged 24 lb. of milk per head daily. The milk contained 3.5% butter fat, which was worth 25¢ a pound. How much did the farmer receive for his milk in a month of 30 days?

**134.** A commission merchant sold 1700 bbl. of apples at \$3.50 a barrel. He paid \$112 for freight, \$14.25 for cartage, and \$12.75 for storage. After deducting his commission, he sent the owner \$4026. What was the rate of commission?

**135.** The quarterly dividend on 60 shares of stock is \$75. What is the rate of dividend each quarter?

**136.** Ames Brothers, brokers, bought for me 10 shares of United Produce, selling at 129 $\frac{7}{8}$ , par value \$100. Find the total cost of the stock, including  $\frac{1}{8}$ % brokerage.

**137.** A clerk made the following deposits in a savings bank, at 4% interest, payable January 1 and July 1: January 1, 1916, \$500; July 1, 1916, \$350; July 1, 1917, \$200. On January 2, 1917, he drew out \$100. What was his balance July 1, 1918?

**138.** A commission merchant sold 60 lb. of print butter for \$21.60. His commission was 5%. How much did the shipper receive a pound for his butter?

**139.** Mr. Adams bought a property for \$20,000. He expended \$4000 in improvements. The repairs each year averaged \$250, the insurance and taxes  $2\frac{1}{2}\%$  on  $\frac{1}{2}$  of the original cost of the property. For how much a year must he rent the property to realize 6% net on his investment?

		1916	1917	PER CENT OF INCREASE	DECREASE
<b>140.</b>	Apples, per barrel	\$ 1.75	\$ 3.00		
<b>141.</b>	Beans, per basket	2.50	5.00		
<b>142.</b>	Beets, per barrel	1.50	4.00		
<b>143.</b>	Cabbage, per ton	11.00	160.00		
<b>144.</b>	Carrots, per barrel	1.25	2.75		
<b>145.</b>	Celery, per crate	1.25	3.00		
<b>146.</b>	Cranberries, per box	1.50	1.25		
<b>147.</b>	Pineapples, per box	2.00	1.25		
<b>148.</b>	Potatoes, per barrel	4.00	8.00		
<b>149.</b>	Tomatoes, per crate	1.00	1.50		
<b>150.</b>	Turnips, per barrel	1.00	2.75		

The above table shows the change from 1916 to 1917 in the price of fruits and vegetables. Find to the nearest tenth per cent the per cent of increase or decrease in each case.

## APPENDIX

**TO THE TEACHER.**—The following subjects, which were purposely omitted from the body of the book, are here presented for the convenience of teachers who think it desirable to teach them.

### GREATEST COMMON DIVISOR

A **common divisor** of two or more numbers is a number that exactly divides each of them; thus, 4 is a common divisor of 16 and 24.

The **greatest common divisor** (g. c. d.) of two or more numbers is the greatest number that exactly divides each of them; thus, 9 is the g. c. d. of 27 and 36.

#### Written Work

1. Find the greatest common divisor of 56, 98, 154.

$$\begin{array}{r|rrr} 2 & 56 & 98 & 154 \\ 7 & 28 & 49 & 77 \\ \hline & 4 & 7 & 11 \end{array}$$

g.c.d. =  $2 \times 7$ , or 14.

*As the g. c. d. of two or more numbers is the product of all their common prime factors, divide the numbers by their common prime factors. In the same way divide the quotients until they are prime to each other. The divisors 2 and 7 are all the common prime factors of the numbers. Hence, the g. c. d. of 56, 98, and 154 is  $2 \times 7$ , or 14.*

Find the g. c. d. of :

2. 42, 63, 189

7. 84, 56, 210

3. 54, 216, 360

8. 22, 110, 132

4. 48, 60, 96

9. 42, 84, 175

5. 84, 252, 512

10. 17, 68, 85

6. 21, 48, 78

11. 432, 720, 864

## LEAST COMMON MULTIPLE

A **common multiple** of two or more numbers is a number that is exactly divisible by each of them; thus, 36 is a common multiple of 6 and 9.

The **least common multiple** (l. c. m.) of two or more numbers is the least number that is exactly divisible by each of them; thus, 18 is the l. c. m. of 6 and 9.

### Written Work

1. Find the l. c. m. of 18, 32, and 40.

$$18 = 2 \times 3 \times 3$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$40 = 2 \times 2 \times 2 \times 5$$

$$\text{l. c. m.} = 2^5 \times 3^2 \times 5, \text{ or } 1440.$$

*The l. c. m. of two or more numbers is the product of all their prime factors, each factor being used as often as it occurs in any number.*

2 occurs 5 times as a factor in 32.

It must, therefore, be used 5 times in the l. c. m. 3 occurs twice as a factor in 18; it must, therefore, be used twice in the l. c. m. 5 occurs once as a factor in 40; it must, therefore, be used once in the l. c. m. Hence, the l. c. m. of 18, 32, and 40 is  $2^5 \times 3^2 \times 5 = 1440$ .

2. Find the l. c. m. of 12, 36, 54, and 63.

$$\begin{array}{r} 2 \overline{)12} \quad 36 \quad 54 \quad 63 \\ \quad 3 \overline{)18} \quad 27 \quad 63 \\ \quad \quad 3 \overline{)6} \quad 9 \quad 21 \\ \quad \quad \quad 2 \quad 3 \quad 7 \end{array}$$

Since 12 is a divisor of 36 the l. c. m. of 36, 54, and 63 is also a multiple of 12. 12 may therefore be rejected from the work.

$\text{l. c. m.} = 2^2 \times 3^3 \times 7 = 756$ . Divide any two of the numbers by a common prime factor. Then divide the quotients in like manner until the quotients are prime to each other. The product of the divisors and the last quotients is the l. c. m.

Find the l. c. m. of :

3. 24, 48, 72

7. 30, 60, 105

11. 144, 180, 240

4. 36, 70, 105

8. 32, 48, 96

12. 85, 51, 255

5. 32, 40, 48

9. 45, 70, 90

13. 120, 225, 540

6. 25, 35, 56

10. 48, 64, 72

14. 98, 42, 126

## PRACTICAL MEASUREMENTS

### PLASTERING AND PAINTING

In plastering, painting, and kalsomining, the unit of measure is the **square yard**.

NOTE.—In some localities an allowance is made for openings and baseboards, but there is no uniform rule in practice. Any allowance should always be specified in the contract.

There are either 50 or 100 laths in a **bundle**. A bundle of 100 is generally estimated to cover 5 square yards of surface.

#### Written Work

1. How many square yards of plaster are necessary to cover the ceiling of your classroom?
2. Find the cost of painting both sides of a tight board fence 150 ft. long and 8 ft. high, at 20 ¢ a square yard.
3. Allowing nothing for openings, how much will it cost to kalsomine the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 12 ft. high, at 12 ¢ a square yard?
4. A storeroom is 75 ft. long, 20 ft. wide, and 15 ft. from floor to ceiling. It has a door in the end 7 ft. by  $3\frac{1}{2}$  ft., and a window 8 ft. by 3 ft. How many bundles of laths, each containing 100, are required for the sides, ends, and ceiling, making full allowance for openings?

5. How much will it cost to plaster the walls and ceiling of a storeroom, 40 ft. by 18 ft. and 12 ft. high, at 8¢ a square yard for lathing, and 24¢ a square yard for plastering, deducting  $\frac{1}{2}$  the area of 2 doors, each 9 ft. by 4 ft., and of 4 windows, each  $6\frac{1}{2}$  ft. by  $3\frac{1}{2}$  ft. ?

6. A building 90 ft. by 24 ft. contains 3 stories, each 13 ft. high. The first story is plastered on the sides and rear and the other stories on all four sides. The second and third stories each have 3 windows in the front, each 8 ft. by  $3\frac{1}{3}$  ft., and each 2 windows in the rear, each 8 ft. by 3 ft. If the ceilings are sheet iron, find the cost of the plastering, at 33¢ per square yard, deducting for all openings.

7. In modern business buildings metal laths are used. Estimate the cost of metal laths, for the building in example 6, at 25¢ per square yard.

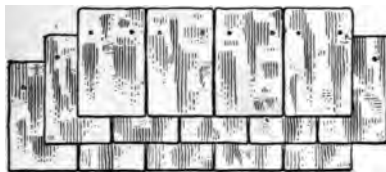
### ROOFING AND FLOORING

In roofing, tiling, and flooring, the unit of measure is the square of 100 square feet.

#### Written Work

1. Each of the two slopes of a roof is 60 ft. long and 20 ft. wide. Find the cost of covering them with tar paper at \$5.60 per square.

2. The floor of a hallway 30 ft. by 12 ft. is inlaid with 2-inch square tile. Find the number necessary.



In roofing with slate, each course of slate is partly overlapped. Each slate as here shown is 10 in. by 16 in. and has 4 in. exposed to the weather.

3. How many square inches of each slate are exposed?

4. If a 10-inch by 16-inch slate is exposed 4 inches to the weather, find the number of slates necessary to lay a square (10 ft. by 10 ft.).

5. If slate 10 in. by 16 in. is laid 6 in. to the weather, find the number necessary to lay a square. Find the weight of a square of slate at  $4\frac{1}{2}$  lb. per square foot.

6. Each slope of a roof is 40 ft. by 20 ft. Find the number of slates, 10 in. by 16 in., exposed 4 in. to the weather, required for this roof, allowing nothing for breakage. Find the cost of the slates at \$5.50 per square.

There are 250 shingles in a **bunch**.

Shingles average 16 inches in length and 4 inches in width. The exposed surface of a shingle laid  $4\frac{1}{2}$  inches to the weather is, therefore, 18 square inches. Without waste 8 shingles will lay one square foot, and 800 shingles will lay 100 square feet or 1 *square*. Allowing for waste, 4 bunches, or 1000 shingles are estimated to lay a square.

7. Allowing nothing for waste, how many bunches of shingles are required to cover a barn roof 35 ft. in width on each side and 70 ft. in length. Find the cost at \$1.00 per bunch.

8. A roof has 14 squares. Allowing for waste, find the cost of the shingles for it, at \$1.25 per bunch.

**Flooring** is frequently estimated by the *square*.

9. How much will it cost, at \$5.00 per square, to lay the floor of a hall 30 ft. by 60 ft., adding  $\frac{1}{6}$  for waste?

10. Estimate the number of squares of flooring required for two floors of a store room 25 ft. by 60 ft.



**PAPERING AND CARPETING**

The unit of measure in wall paper is the **single roll**, which is 8 yards in length and usually 18 inches in width. A **double roll** is 16 yards in length.

In approximating the number of rolls, paper hangers generally deduct from the perimeter of the room the width of the doors and windows. The remaining number of feet divided by  $1\frac{1}{2}$  ft. (18 in. =  $1\frac{1}{2}$  ft.) gives the number of strips required for the surface of the wall. Dividing the total number of strips by the number that can be cut from a double roll gives the number of double rolls required. Fractional parts of a roll are not sold. The ends of the rolls are generally sufficient to paper the surfaces above and below the doors and windows. Border is sold by the *linear yard*.

Carpet, matting, and border are sold by the **linear yard**. Oil cloth and linoleum are sold by the linear yard or by the **square yard**. Ingrain carpets are usually 1 yard wide, other carpets are generally 27 inches wide.

Liberal allowance must be made for loss in matching.

**Written Work**

1. Estimate the number of double rolls of paper required for a ceiling 18 ft. by 22 ft., strips running lengthwise.

$22 \text{ ft.} = \text{length of one strip.}$

$16 \text{ yd.} = 48 \text{ ft.}; 48 \text{ ft.} \div 22 \text{ ft.} = 2$ , the number of *whole* strips in a double roll.

$18 \text{ ft.} + 1\frac{1}{2} \text{ ft.} = 12$ , the number of strips required.

$12 \div 2 = 6$ , the number of double rolls required.

2. A room 15 ft. by 22 ft. is 11 ft. from baseboard to ceiling. It has four openings  $3\frac{1}{2}$  ft. by 7 ft. How many double rolls of paper are needed for it, strips on ceiling, lengthwise?

3. The room in problem 2 has a plate rail extending around it between the openings. Find the cost of this rail at 30¢ per foot.

4. How much carpet 27 in. wide, laid the long way of the room, is required for a room 18 ft. long and 15 ft. wide, allowing 12 in. on each strip except the first for matching?

6 yd. = the length of one strip.

27 in. =  $2\frac{1}{4}$  ft.; and 15 ft.  $\div$   $2\frac{1}{4}$  ft. =  $6\frac{3}{4}$ , therefore

7 = the number of strips.

$7 \times 6$  yd. = 42 yd.

$6 \times 12$  in. = 72 in., or 2 yd., the waste on 6 strips.

42 yd. + 2 yd. = 44 yd. of carpet required.

5. Explain why it takes fewer yards of carpet to cover a room 18 ft. by 27 ft. with ingrain carpet (1 yard wide) than with Brussels carpet (27 inches wide). Laying the carpet the long way of the room, how many yards of each would it take, if 10 in. were allowed on each strip, except the first, for matching?

6. The widths of certain floors are 15 ft.,  $13\frac{1}{2}$  ft.,  $15\frac{3}{4}$  ft., 18 ft., 16 ft. Estimate the number of strips of ingrain carpet necessary to cover each room.

7. Estimate the number of strips of Brussels carpet necessary to cover a room 15 ft. wide; a room  $13\frac{1}{2}$  ft. wide;  $15\frac{3}{4}$  ft. wide; 18 ft. wide; 16 ft. wide.

8. Find the cost of covering a kitchen  $13\frac{1}{2}$  ft. by 12 ft. with linoleum, 4 yd. wide, at \$1.60 per yard, if the linoleum is laid the long way of the room.

9. Estimate the difference in cost between covering a room 18 ft. by  $20\frac{1}{4}$  ft. with carpet 27 inches wide, at \$1.45 a yard, laid lengthwise, allowing 12 inches on each strip except the first for matching, and covering the room with carpet 1 yd. wide at 85¢ a yard, laid in the same way, allowing 12 inches on each strip, except the first, for matching.

# PERCENTAGE AND INTEREST

## PERCENTAGE

Finding the number when a per cent of it is given.

### Oral Work

1. If  $\frac{1}{5}$  of a number is 10, what is the number? If 20% of a number is 10, what is the number?

2. If  $33\frac{1}{3}\%$  of a man's loss is \$300, how much does he lose?

What is the number of which :

3. 10 is  $66\frac{2}{3}\%$ ?      7. 40 is  $16\frac{2}{3}\%$ ?      11. \$48 is 10%?

4. \$100 is  $33\frac{1}{3}\%$ ?      8. 100 is  $12\frac{1}{2}\%$ ?      12. \$12 is  $16\frac{2}{3}\%$ ?

5. 500 is 50%?      9. 300 is  $37\frac{1}{2}\%$ ?      13. 30 is  $33\frac{1}{3}\%$ ?

6. 800 is 60%?      10. 500 is 25%?      14. \$10 is  $83\frac{1}{3}\%$ ?

### Written Work

1. Find the number if 33% of it is 3135.

$$(a) 3135 \div .33 = 9500$$

The unknown number is the base. Since the percentage equals the base multiplied by the rate, the base equals the percentage divided by the rate,  $3135 \div .33 = 9500$ , the number. Test: .33 of 9500 = 3135.

2. Find the number if  $8\frac{1}{3}\%$  of it equals 250.

$$(b) 8\frac{1}{3}\% = \frac{1}{12}$$

$$250 \times 12 = 3000$$

$8\frac{1}{3}\%$  of the number equals  $\frac{1}{12}$  of it.

$\frac{1}{12}$  of the number is  $12 \times 250$ , or 3000.

Test:  $\frac{1}{12}$  of 3000 = 250.

The base equals the percentage divided by the rate.

Find the number if:

- |                                       |                                        |
|---------------------------------------|----------------------------------------|
| 3. 8% of it is \$2.40                 | 8. 6% of it is \$72                    |
| 4. 12% of it is \$3.60                | 9. 32% of it is \$3.60                 |
| 5. $12\frac{1}{2}$ % of it is \$91    | 10. 45% of it is \$3.60                |
| 6. $37\frac{1}{2}$ % of it is \$27    | 11. $62\frac{1}{2}$ % of it is \$35.50 |
| 7. $33\frac{1}{3}$ % of it is \$42.50 | 12. $87\frac{1}{2}$ % of it is \$28.28 |
13. A has \$3612, which is  $87\frac{1}{2}$ % of what B has. How much has B?

$$87\frac{1}{2}\% \text{ of B's} = \$3612, \text{ A's.}$$

14. If \$14 is 25% of A's salary, find his salary?
15. After a battle 70% of a regiment, or 644 men, were left. How many men were there in the regiment at first?
16. I drew from the bank \$1500, or  $83\frac{1}{3}$ % of my deposit. How much was my deposit?
17. If a man rents a house for \$752 per year, which is 16% of its value, what is the value of the house?
18. A teacher's expenses are \$30 a month, and this amount is  $37\frac{1}{2}$ % of his salary. How much does he save?
19. The number of pupils in attendance at school in a certain town is 576, which is 96% of the enrollment. What is the enrollment?

**Finding a number when the number plus the rate of increase is given**

### Written Work

1. What number increased by 17% of itself equals 585?

100% of itself	1	= the number
100% of itself + 17% of itself, or 117% of the number		= 585, amount
1% of the number		= $\frac{1}{117}$ of 585, or 5
100% of the number		= $100 \times 5$ , or 500

*Divide the sum by one plus the rate.*

What number increased by:

2. 8% of itself is 324?
3. 30% of itself is 260?
4.  $37\frac{1}{2}$ % of itself is 550?
5.  $\frac{3}{4}$ % of itself is  $201\frac{1}{2}$ ?
6. 50% of itself is 69?
7. 250% of itself is 105?
8.  $16\frac{2}{3}$ % of itself is 1050?
9. 70% of itself is 510?
10. I gained 35% by selling an article for \$4.05. How much did it cost?
11. A laborer had his wages twice increased 10%. If he now receives \$2.42 a day, what were his wages before they were increased?
12. A property sold for \$4025, which was an increase of 15% of the cost. How much did the property cost?
13. A receives \$1600 salary, which is 60% more than B receives. What salary does B receive?
14. W. H. Richmond bought a jewelry store for a certain sum and increased the stock 27% of the purchase price. He found that the whole investment amounted to \$5969. What was the purchase price of the store?
15. The land surface of the District of Columbia is 60 square miles, which is 500% more than the water surface. What is the water surface?

Finding a number when the number minus the rate of decrease is given.

### Written Work

1. What number diminished by 16% of itself equals 168?

100% of the number	= the number
100% of the number — 16% of the number,	
or 84% of the number	= 168 (difference)
1% of the number	= $\frac{1}{84}$ of 168, or 2
100% of the number, or the number	= $100 \times 2$ , or 200

*Divide the difference by one minus the rate.*

What number diminished by :

2. 45% of itself equals 55?      6.  $18\frac{1}{2}\%$  of itself equals 325?  
3. 18% of itself equals 246?      7. 23% of itself equals 308?  
4.  $62\frac{1}{2}\%$  of itself equals 27?      8. 95% of itself equals 25?  
5. 50% of itself equals 22.5?      9. 10% of itself equals  $4\frac{1}{2}$ ?

10. John has \$35, which is  $12\frac{1}{2}\%$  less than his brother has. How much has his brother?

11. Mrs. Lee spent \$24 for a coat, which was  $33\frac{1}{3}\%$  less than the cost of a suit. Find the cost of both.

12. After losing  $8\frac{1}{3}\%$  of his money, a man had \$352 left. How much had he at first?

13. What number decreased by 35% of itself equals \$1300? \$520? \$6500?

14. A school enrolls 249 boys, which is 17% less than the number of girls it enrolls. How many pupils are there in the school?

15. A lady when shopping spent \$15 of her money for a hat, which was 25% less than the amount she spent for a coat. How much did she spend for the coat?

16. The fraction  $\frac{2}{15}$  is 40% less than what fraction?

17. If a certain number is increased 40% of itself, and this sum is diminished by 50% of itself, the result is 700. Find the number.

18. I sold two lots for \$1200 each; on one I gained 25% and on the other I lost 25%. Did I gain or lose and how much?

19. The population of a town in 1916 was 14,000, which was  $12\frac{1}{2}\%$  less than the population in 1917. What was the population in 1917?

## PROBLEMS IN SIMPLE INTEREST

## Finding the principal.

## Written Work

1. What principal invested at 4 % per annum will yield an annual income of \$200 ?

Since the interest on \$1 for 1 year at 4 % is \$.04, as many dollars must be invested to yield \$200 per year as \$.04 is contained times in \$200.  $\$200 \div \$.04 = 5000$ . Hence, \$5000 must be invested.

*The principal equals the given interest divided by the interest on \$1 for the given time at the given rate.*

2. What principal at  $4\frac{1}{2}\%$  will gain \$213.75 interest in 4 yr. 9 mo.?

3. What principal at 5 % will gain \$120.70 interest in 3 yr. 6 mo. 18 da.?

4. What principal at 8 % will gain an interest of \$163.20 from Sept. 30, 1916, to June 12, 1918?

5. A man gave his note April 1, 1916, at 6 %. When he settled the note Aug. 13, 1918, he paid \$195.25 interest. What was the principal, or face of the note?

## Finding the rate.

## Written Work

1. At what rate must \$500 be invested to yield \$75 interest in 2 yr. 6 mo. ?

Since the interest on \$500 for  $2\frac{1}{2}$  yr. at 1 % is \$12.50, it will require a rate of as many per cent to yield \$75, as \$12.50 is contained times in \$75, or 6 %.

*The rate equals the given interest divided by the interest for the given time at 1 %.*

2. The interest on \$1125 for 3 yr. 4 mo. 24 da. is \$229.50. What is the rate per cent ?

3. The interest on \$1800 for 4 yr. 8 mo. 16 da. is \$424. What is the rate?

4. At what rate will \$2460 give \$682.65 interest in 5 yr. 6 mo. 18 da.?

**Finding the time.**

### Written Work

1. In what time will \$450 at 6% yield \$90 interest?

Since the interest on \$450 for 1 yr. at 6% is \$27, the required time is as many years as \$27 is contained times in \$90, or  $3\frac{1}{3}$  yr.

*The time in years equals the given interest divided by the interest at the given rate for 1 year.*

2. In what time will \$275 gain \$55 interest at 6%.

3. In what time will any principal double itself (that is, gain 100% of itself) at 5%? at 6%? at 8%?

4. In what time will any principal treble itself (that is, gain 200% of itself) at 5%? at 6%? In what time will it quadruple itself at 8%? at 10%?

5. A note of \$500 at 5% interest was paid May 1, 1918, the interest amounting to \$63.75. When was the note given?

### MERCHANTS' RULE FOR PARTIAL PAYMENTS

In some localities the following rule is used on notes running a year or less.

*Find the amount of the principal from the time it begins to bear interest to the date of settlement.*

*Find the amount of each payment from the time it was made to the date of settlement.*

*From the amount of the principal, subtract the sum of the amounts of the payments. The result will be the balance due.*

Make and solve a problem applying this rule, if it is used in your locality.



# RATIO AND PROPORTION

## RATIO

1. The quotient of  $30 \div 10$  is 3. Compare 30 with 3 in such a way as to show how many times 3 is contained in 30. What, then, is the relation of 30 to 3?

Ratio is the relation of two similar numbers as expressed by the quotient of the first divided by the second.

2. What is the ratio of 10 to 8? of 12 to 4? of 3 to 6? of 2 yd. to 8 yd.? of \$12 to \$3?

Since the division of two similar numbers gives an *abstract quotient*, all ratios are *abstract*.

The **sign** of ratio is a colon : placed between the numbers. Thus, the ratio of 12 to 3 is written  $12 : 3$ . It is read, "the ratio of 12 to 3." It may be written also  $12 \div 3$ , or  $\frac{12}{3}$ .

The **terms** of a ratio are the numbers compared. The first is the **antecedent**; the second the **consequent**.

$$12 : 6 = \frac{\text{antecedent}}{\text{consequent}} = 12 \div 6 = \frac{\text{dividend}}{\text{divisor}} = \frac{12}{6} = \frac{\text{numerator}}{\text{denominator}}$$

Since the antecedent of a ratio may be regarded as the numerator, and the consequent as the denominator of a fraction, *both terms of a ratio may be multiplied or divided by the same number without changing the value of the ratio.*

Find the ratio of :

- |                                   |                                    |                                    |                                    |
|-----------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 3. 10 to 5                        | 7. 18 to 9                         | 11. 40 to 10                       | 15. 50 to 25                       |
| 4. 5 to 15                        | 8. 27 to 9                         | 12. 8 to 24                        | 16. 24 to 8                        |
| 5. 8 to 2                         | 9. 35 to 5                         | 13. 2 to $\frac{1}{2}$             | 17. 4 to $\frac{1}{4}$             |
| 6. $\frac{1}{2}$ to $\frac{1}{4}$ | 10. $\frac{2}{3}$ to $\frac{1}{3}$ | 14. $\frac{5}{8}$ to $\frac{1}{8}$ | 18. $\frac{3}{4}$ to $\frac{1}{4}$ |

Find the value of the

- |                                    |    |
|------------------------------------|----|
| 1. 125 : 25                        | 5. |
| 2. 6.25 : 25                       | 6. |
| 3. $\frac{3}{4}$ to $\frac{1}{4}$  | 7. |
| 4. $\frac{1}{8}$ to $1\frac{1}{2}$ | 8. |

RATIO AND

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What is  $\frac{1}{2}$  of  $(6 \times 4)$ . What

Find the value of  $2\frac{1}{2}$  his assets \$10,000.

- |                      |                          |
|----------------------|--------------------------|
| 1. $36 : 6 = 24 ; ?$ | Then, $36$ of paper, how |
| 2. $15 : 25$         | are yards?               |
| 3. $?$               | Then flour               |
|                      | when                     |

SIM

**Proportion** is an equation.  $1\frac{1}{2} = \frac{3}{2}$  is a proportion.

**Proportion** is generally written by a double colon :: between the ratios. Thus,  $12 : 6$  as  $8 : 4$ , is written,  $12 : 6 = 8 : 4$ , or  $12 : 6 :: 8 : 4$ .

A proportion may be read in two ways ; thus,  $12 : 6 = 8 : 4$  is read, "The ratio of 12 to 6 is equal to the ratio of 8 to 4," or, "12 is to 6 as 8 is to 4."

The **extremes** are the *first* and the *fourth* terms of a proportion ; the **means** are the *second* and the *third* terms.

In  $15 : 5 = 12 : 4$ , the extremes are 15 and 4 ; the means, 5 and 12.

Find the product of the means ; then the product of the extremes :

- |                      |                                 |                                 |
|----------------------|---------------------------------|---------------------------------|
| 1. $8 : 4 = 10 : 5$  | 3. $24 : 4 = 36 : 6$            | 5. $\frac{2}{3} = \frac{8}{9}$  |
| 2. $15 : 3 = 30 : 6$ | 4. $\frac{3}{8} = \frac{6}{16}$ | 6. $\frac{1}{5} = \frac{3}{15}$ |

Observe how the product of the extremes in each proportion compares with the product of the means.

*In every proportion the product of the means is equal to the product of the extremes.*

## Written Work

Find the value of  $x$ , the unknown term :

1.  $36 : 6 = 24 : x$

Then, 36 times  $x$ , or  $36x = 144$ , and once  $x$ , or  $x = 4$ .

2.  $15 : 25 = x : 40$

Then,  $25x = 15 \times 40$ , or 600, and  $x = 24$ .

3.  $60 : 15 = 75 : x$

8.  $\frac{3}{5} : \frac{2}{3} = 9 : x$

4.  $75 : x = 90 : 18$

9.  $\frac{5}{8} : x = 25 : 8$

5.  $40 : x = 72 : 18$

10.  $7.5 : 1.5 = 2.5 : x$

6.  $x : 30 = 8 : 48$

11.  $6.25 : 2.5 = x : 1$

7.  $x : 45 = 7 : 63$

12.  $60 : 150 = 36 : x$

13. If 8 lamps cost \$48, how much will 20 lamps cost ?

Let  $x$  = number of dollars 20 lamps cost

48 = number of dollars 8 lamps cost

Since *ratio* is the relation of two similar numbers, 8 lamps and 20 lamps form one ratio, and \$48 and  $x$  dollars, the other ratio.

Write as the second ratio  $48 : x$ . Since 20 lamps cost more than 8 lamps,  $x$  dollars represents a larger sum than \$48. Therefore, as the larger number is the consequent of the second ratio, the larger number must be made the consequent of the first ratio. The proportion, therefore, is,

$$8 : 20 = 48 : x$$

$$8x = 960$$

$$x = 120$$

14. It is estimated that 25 men can build a bridge in 18 days. How long at the same rate will it take 15 men to build it ?

15. How much will 30 dozen eggs cost when 70 dozen cost \$42 ?

16. It is estimated that 90 men are necessary to grade a certain street in 45 days. If only 81 men are hired to do the work, how long will it take them ?

17. The ratio is  $\frac{3}{2}$ . The first term is  $\frac{1}{2}$  of  $(6 \times 4)$ . What is the second term?

18. A bankrupt's debts are \$32,000, and his assets \$10,000. How much will be paid on a claim of \$6150?

19. If \$2.25 is paid to clean  $35\frac{1}{2}$  square yards of paper, how much at the same rate will it cost to clean  $65\frac{1}{2}$  square yards?

20. A bakery sells 5¢ loaves weighing 16 oz. when flour is \$6 a barrel. What size loaves should they sell at 5¢ when flour is \$8 a barrel?

21. A map is drawn on a scale of 100 miles to  $\frac{3}{4}$  of an inch. What distance is represented on the map by  $\frac{5}{16}$  of an inch?

22. If the interest on \$500 for 6 months is \$15, how much is the interest on the same sum for 1 year 4 months?

### PARTITIVE PROPORTION

**Partitive proportion** is the process of separating a number into parts proportional to two or more numbers.

#### Written Work

1. Separate 180 into parts proportional to 1, 2, and 3.

**SOLUTION.** — Since the parts are in the ratio of 1, 2, and 3, then  $1 + 2 + 3$  or 6 parts = 180.

The 1st number =  $180 \div 6$ , or 30

The 2d number =  $2 \times 30$ , or 60

The 3d number =  $3 \times 30$ , or 90

**Test:**  $30 + 60 + 90 = 180$ ;  $30 : 60 : 90 = 1 : 2 : 3$ .

2. A man and two boys earn \$162 and agree to divide it as follows : 3 parts to the man, 2 parts to the first boy, and 1 part to the second boy. How much should each receive?

3. The receipts of a street railway in one month are \$15,600, and the expenses are to the profits as 1 to 2. Find the expenses and the net savings.

## POWERS AND ROOTS

1.  $3 \times 3 = 9$ .  $4 \times 4 = 16$ .  $5 \times 5 \times 5 = 125$ .
2. Name the two equal factors that produce 9. 16.
3. Name the three equal factors that produce 125.
4. What two equal factors produce 25? 36? 49? 81?
5. What three equal factors produce 8? 27? 64?
6. How many times is 5 used as a factor in 5? 25? 125?
7. How many times is 3 used as a factor in 3? 9? 27?

A **power** of a number is the product obtained by taking the number one or more times as a factor. Thus, 9 is a power of 3, and 8 is a power of 2.

The *first* power of a number is the number itself. The *second* power of a number is called the **square** of the number. Thus, 16 is the square of 4. The *third* power of a number is called the **cube** of the number. Thus, 64 is the cube of 4.

8. What is the square of 4? 5? 6? 7? 8? 9?
9. What is the cube of 2? 3? 5? 6? 7?
10.  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ . What, then, is the square of  $\frac{2}{3}$ ?
11.  $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$ . What, then, is the cube of  $\frac{3}{4}$ ?
12.  $.3 \times .3 = .09$ .  $.5 \times .5 = .25$ . What, then, is the square of .3? of .5?

*The square of a fraction is found by squaring both terms, the cube of a fraction, by cubing both terms.*

13.  $3 \times 3 = 9$ . The square of 3 is indicated thus,  $3^2$ ;  
 $4 \times 4 \times 4 = 64$ . The cube of 4 is indicated thus,  $4^3$ .

14. How much is  $5^2$ ?  $6^2$ ?  $7^2$ ?  $5^3$ ?  $6^3$ ? What is the value of  $(\frac{3}{4})^2$ ?  $(\frac{2}{5})^3$ ?  $.4^2$ ?  $.4^3$ ?  $(2\frac{1}{2})^2$ ?

An **exponent** is a small figure placed at the right of, and a little above the number, to indicate the number of times it is to be taken as a factor; thus,  $4^3 = 4 \times 4 \times 4 = 64$ . Exponent, 3.

### Written Work

1. Square 6, 7, 8, 9, 10, 12, 15.
2. Cube 3, 4, 5, 6, 7, 8, 9, 10, 12.
3. Square 30, 50, 60, 80, 120.
4. Cube 20, 30, 40, 50, 100.
5. Find the value of  $6^2$ ,  $8^2$ ,  $9^2$ ,  $5^3$ ,  $6^3$ ,  $7^3$ .
6. Find the value of  $(\frac{3}{5})^2$ ,  $(\frac{2}{5})^3$ ,  $(\frac{3}{4})^2$ ,  $(\frac{4}{5})^3$ ,  $(\frac{5}{7})^2$ ,  $(1\frac{1}{2})^2$ .
7. Square .3, .04, .05, .6, .06. Cube .4, .04, .6.

Find the value of:

- |            |             |                         |
|------------|-------------|-------------------------|
| 8. $15^2$  | 11. $22^2$  | 14. $.75^2$             |
| 9. $16^2$  | 12. $25^2$  | 15. $(1\frac{1}{5})^2$  |
| 10. $18^2$ | 13. $6.5^2$ | 16. $(16\frac{1}{2})^2$ |

Find the number of square units in a surface whose side is:

- |            |                 |                 |
|------------|-----------------|-----------------|
| 17. 15 in. | 20. 8 ft. 6 in. | 23. 10 yd.      |
| 18. 25 ft. | 21. 5 in.       | 24. 5 yd. 2 ft. |
| 19. 16 yd. | 22. 8.5 in.     | 25. 6 mi.       |

Find the number of cubic units in a volume whose edge is:

- |           |                 |                  |
|-----------|-----------------|------------------|
| 26. 8 in. | 28. 3 ft. 3 in. | 30. 1 yd. 10 in. |
| 27. 2 ft. | 29. 42 in.      | 31. 12 ft. 4 in. |

## EXTRACTING ROOTS

A **root** of a number is one of the equal factors that produce that number. Thus, 3 is a root of 9.

The **square root** of a number is one of its *two* equal factors. Thus, 4 is the square root of 16.

The **cube root** of a number is one of its *three* equal factors. Thus, 4 is the cube root of 64.

1. What is the square root of 25? 36? 49? 81? 100?

2. What is the cube root of 8? 64? 125? 216?

The root of a number is generally indicated by writing the number under the *radical or root sign*  $\sqrt{\quad}$ , and placing a figure called the *index* in the angle of the sign; thus,  $\sqrt[3]{27}$  denotes the cube root of 27. The square root is indicated by  $\sqrt{\quad}$  without the index.

*The root of a fraction equals the root of the numerator divided by the same root of the denominator.*

Find the required root: Thus,  $\sqrt{64} = \sqrt{8 \times 8} = 8$ .

3.  $\sqrt{16}$

8.  $\sqrt{\frac{64}{100}}$

13.  $\sqrt[3]{216}$

18.  $\sqrt[3]{512}$

4.  $\sqrt[3]{27}$

9.  $\sqrt{49}$

14.  $\sqrt[3]{1000}$

19.  $\sqrt{144}$

5.  $\sqrt{\frac{49}{81}}$

10.  $\sqrt[3]{125}$

15.  $\sqrt{121}$

20.  $\sqrt{169}$

6.  $\sqrt{25}$

11.  $\sqrt[3]{\frac{27}{125}}$

16.  $\sqrt[3]{343}$

21.  $\sqrt{2500}$

7.  $\sqrt[3]{64}$

12.  $\sqrt{81}$

17.  $\sqrt[3]{.064}$

22.  $\sqrt{8100}$

Some perfect powers and their roots.

Memorize:

$\sqrt{1} = 1$

$\sqrt{36} = 6$

$\sqrt[3]{1} = 1$

$\sqrt[3]{216} = 6$

$\sqrt{4} = 2$

$\sqrt{49} = 7$

$\sqrt[3]{8} = 2$

$\sqrt[3]{343} = 7$

$\sqrt{9} = 3$

$\sqrt{64} = 8$

$\sqrt[3]{27} = 3$

$\sqrt[3]{512} = 8$

$\sqrt{16} = 4$

$\sqrt{81} = 9$

$\sqrt[3]{64} = 4$

$\sqrt[3]{729} = 9$

$\sqrt{25} = 5$

$\sqrt{100} = 10$

$\sqrt[3]{125} = 5$

$\sqrt[3]{1000} = 10$

**Finding the root of a perfect square or cube by factoring.****Written Work**

1. Find the square root of 1225.

When factored  $1225 = 5 \times 5 \times 7 \times 7$ .Arranged into *two* like groups  $1225 = (5 \times 7) \times (5 \times 7)$ .

$$\sqrt{1225} = 5 \times 7, \text{ or } 35.$$

Find the root of each number, as indicated, by factoring :

- |                  |                       |                       |                       |
|------------------|-----------------------|-----------------------|-----------------------|
| 2. $\sqrt{225}$  | 7. $\sqrt{784}$       | 12. $\sqrt{1764}$     | 17. $\sqrt{5184}$     |
| 3. $\sqrt{576}$  | 8. $\sqrt{1296}$      | 13. $\sqrt[3]{13824}$ | 18. $\sqrt[3]{15625}$ |
| 4. $\sqrt{441}$  | 9. $\sqrt[3]{4096}$   | 14. $\sqrt{3136}$     | 19. $\sqrt[3]{32768}$ |
| 5. $\sqrt{196}$  | 10. $\sqrt{2304}$     | 15. $\sqrt[3]{5832}$  | 20. $\sqrt{4096}$     |
| 6. $\sqrt{1600}$ | 11. $\sqrt[3]{42875}$ | 16. $\sqrt[3]{8000}$  | 21. $\sqrt[3]{19683}$ |

**SQUARE ROOT****Comparing roots and periods.**

The **squares** of the smallest and the largest integers composed of one, two, and three figures are as follows :

$$1^2 = 1$$

$$10^2 = 100$$

$$100^2 = 10000$$

$$9^2 = 81$$

$$99^2 = 9801$$

$$999^2 = 998001$$

1. Separate each of these squares into periods of two figures each, beginning at the right ; thus, 99' 80' 01.

2. How does the *number of periods* in each square compare with the *number of figures* in the corresponding roots ?

*The number of periods of two figures each, beginning at units, into which a number can be divided equals the number of figures in the root.*

NOTE. — The left-hand period may contain only one figure.

3. How many figures are there in the square root of 4225? of 12544? of 133225? of 810000?



## Written Work

1. Square 25.  $25 = 20 + 5$ , hence it may be squared in two ways, thus:

$$\begin{array}{r} 25 = 20 + 5 \\ 25 = 20 + 5 \\ \hline 125 = 20 \times 5 + 5^2 \\ 500 = 20^2 + 20 \times 5 \\ \hline 625 = 20^2 + 2(20 \times 5) + 5^2 \end{array}$$

The square of 25 has three partial products:

$$25^2 = \left\{ \begin{array}{l} (1) \quad 20^2 = 400 \\ (2) \quad 2(5 \times 20) = 200 \\ (3) \quad 5^2 = 25 \end{array} \right\} = 625$$

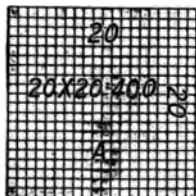
2. Find the square root of 625.

$$\begin{array}{r} 6'25 \quad | \quad 20 \\ 20^2 = 4 \ 00 \\ \hline \text{Trial divisor, } 2 \times 20 = 40 \quad | \quad 225 \\ \quad \quad \quad \quad \quad \quad \quad \quad | \quad 5 \\ \quad \quad \quad \quad \quad \quad \quad \quad | \quad \hline \text{Complete divisor, } = 45 \quad | \quad 225 \end{array}$$

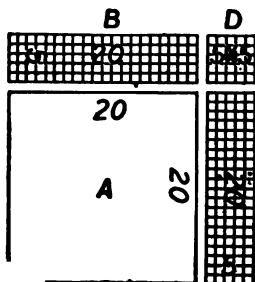
In practice:

$$\begin{array}{r} 6'25 \quad | \quad 25 \\ \quad \quad \quad \quad \quad | \quad 4 \\ 45 \quad | \quad 225 \\ \quad \quad | \quad \hline \end{array}$$

Since 625 has two periods, its square root is composed of two figures, tens and ones. Since the square of *tens* is *hundreds*, 6 hundreds must be the square of at least 2 tens. Two tens or 20 squared is 400, as shown in figure A; and  $625 - 400$  leaves a remainder of 225. The root 20, therefore, must be so increased as to exhaust this remainder and keep the figure a square.



The necessary additions to enlarge A, and keep it a square, are the two equal rectangles B and C, and the small square D.



B, C, and D contain 225 square units; and since the area of D is small, if 225 is divided by 40, the combined length of B and C, the quotient will indicate the approximate width of these additions. The quotient is 5; the entire length of B, C, and D is  $20 + 20 + 5 = 45$  units; and the area of the additions is 5 times  $45 \times 1$  sq. unit, or 225 sq. units. Since these three additions exhaust the remaining 225 sq. units, and keep the figure a square, the side of the required square is 25 units, and the square root of 625 is 25.

3. Square  $35(30 + 5)$ ; then find the square root of 1225.

$$\begin{array}{rcl} \text{Partial Products: } \left\{ \begin{array}{l} (1) \quad 30^2 = 900 \\ (2) \quad 2(5 \times 30) = 300 \\ (3) \quad 5^2 = 25 \end{array} \right\} & 1225 & \\ & \underline{12 \overline{) 25}} & 30 \\ & 9 \ 00 & 5 \\ \text{Trial divisor, } 2 \times 30 = 60 & \underline{3 \ 25} & 35 \\ & 5 & \\ \text{Complete divisor, } & \underline{65} & 3 \ 25 \end{array}$$

**Study of Problem**

1. How many periods in 1225?
2. How many figures, then, will be in its root?
3. What is the largest partial product in 1225?
4. What is the square root of 900?
5. What two

partial products are contained in the 325 remaining?

6. 325 is composed principally of which partial product?  $2(5 \times 30)$ .

325 is composed principally of 2 times the first figure of the root by the second. Hence, if 325 is divided by  $(2 \times 30)$ , the quotient will give approximately the second figure of the root.

7. What is the quotient? the second figure of the root?

8. What must be added to the trial divisor  $(2 \times 30)$  to form the complete divisor?

Since  $5 \times 65 = 325$ , 5 is the second root figure, and  $30 + 5$ , or 35, is the square root of 1225.

4. Find the square root of 2134.44. Separate the number into periods, left and right from the decimal point.

$$\begin{array}{rcl} & 21'34'.44 & \underline{46.2} \\ 40^2 = 16 \ 00 & & \\ \text{Trial divisor, } 2 \times 40 = 80 & \underline{5 \ 34} & \\ & 6 & \\ \text{Complete divisor, } & \underline{86} & 5 \ 16 \\ \text{Trial divisor, } 2 \times 460 = 920 & \underline{18 \ 44} & \\ & 2 & \\ \text{Complete divisor, } & \underline{922} & 18 \ 44 \end{array}$$

$$\begin{array}{rcl} & 21'34'.44 & \underline{46.2} \\ \text{In practice:} & & \\ & 16 & \\ 86 & \underline{5 \ 34} & \\ & 5 \ 16 & \\ & \underline{18 \ 44} & \\ 922 & \underline{18 \ 44} & \end{array}$$

*Beginning at units, separate the number into periods of two figures each.*

*Find the largest square in the first period on the left, and write its root as the first figure in the required root. Subtract its square from the period and annex the second period to the remainder.*

*Double the root found for a trial divisor. Divide the remainder, omitting the last figure, by this trial divisor, and annex the quotient to the trial divisor and also to the root.*

*Multiply the complete divisor by the second figure of the root, and subtract the product from the remainder.*

*Double the root already found for another trial divisor, and proceed as above until all the periods have been used.*

**NOTE.**—When a naught occurs in the root, annex a naught to the trial divisor, bring down another period, and proceed as before.

In extracting the square root of a decimal or a whole number and a decimal, point off into periods of two figures each, the whole number toward the left and the decimal toward the right of the decimal point.

In extracting the square root of a fraction, extract the square root of the numerator and of the denominator if both terms are perfect squares; or reduce the fraction to a decimal, and extract the root of the decimal.

Solve the following :

- |                 |                   |                              |                    |
|-----------------|-------------------|------------------------------|--------------------|
| 5. $\sqrt{484}$ | 9. $\sqrt{1089}$  | 13. $\sqrt{\frac{64}{100}}$  | 17. $\sqrt{.0225}$ |
| 6. $\sqrt{576}$ | 10. $\sqrt{1849}$ | 14. $\sqrt{\frac{196}{225}}$ | 18. $\sqrt{4.41}$  |
| 7. $\sqrt{676}$ | 11. $\sqrt{2601}$ | 15. $\sqrt{\frac{169}{324}}$ | 19. $\sqrt{6.25}$  |
| 8. $\sqrt{961}$ | 12. $\sqrt{3969}$ | 16. $\sqrt{.25}$             | 20. $\sqrt{.0625}$ |

Find the square root of :

- |            |             |                        |             |
|------------|-------------|------------------------|-------------|
| 21. 315    | 26. 178.25  | 31. 96.8256            | 36. .8464   |
| 22. 525    | 27. 14884   | 32. $192\frac{33}{64}$ | 37. .012996 |
| 23. 1156   | 28. 26569   | 33. $540\frac{9}{16}$  | 38. .000566 |
| 24. 4356   | 29. 136161  | 34. $3192\frac{1}{4}$  | 39. .43681  |
| 25. 210.25 | 30. 20.7936 | 35. $56\frac{34}{64}$  | 40. 112225  |

## METRIC SYSTEM OF WEIGHTS AND MEASURES

By the United States system of money we may write 5 dollars, 9 dimes, and 7 cents thus, \$5.97, because there is a uniform ratio between the dollar and the dime, the dollar and the cent, the dollar and the mill; the dollar being **10** times the dime, **100** times the cent, and **1000** times the mill.

As a mill is  $\frac{1}{1000}$  of a dollar, a cent  $\frac{1}{100}$  of a dollar, and a dime  $\frac{1}{10}$  of a dollar, *United States money* is based on a **decimal system**.

By the **English long measure**, 12 in. = 1 ft., 3 ft. = 1 yd., and  $5\frac{1}{2}$  yd. = 1 rd.; thus, we see there is no uniform ratio between the rod and the yard, the rod and the foot, and the rod and the inch, the rod being  $5\frac{1}{2}$  times the yard,  $16\frac{1}{2}$  times the foot, and **198** times the inch.

By the **English measure of weights**, 16 oz. = 1 lb., 100 lb. = 1 cwt., 20 cwt. = 1 ton. The ton equals **20** times the hundredweight, **2000** times the pound, and **32000** times the ounce.

The **metric system** was devised by the *French* government in an effort to establish a system of weights and measures that would be on a uniform decimal scale, so that a unit of one denomination might be changed to a unit of another denomination by simply moving the decimal point.

The **meter** is the fundamental unit of the metric system. Its length (about 39.37 in.) was meant to be .0000001 of the distance from the equator to the pole. Though an error has since been discovered in the measurement of the distance from the equator to the pole, the standard unit has not been changed. The original standard is a bar of platinum 39.37 inches in length, deposited in the archives in Paris.

From the meter every other unit of measure or weight is derived. Thus, the unit of **weight** is the **gram**, which equals the weight of 1 cubic centimeter of pure water.

Draw a cube .01 of a meter on an edge and state the length of the edge in inches.

The unit of **capacity** is the **liter** (lêter), which contains 1 cubic decimeter.

Draw a cube .1 of a meter on an edge and state the length of the edge in inches.

The metric system is now in use in most of the civilized countries except Great Britain and the United States, and in the latter it is in use in some of the departments of the government. It is the official system adopted by Congress for our island possessions. It is universally used by scientists. The United States by a vote of Congress permitted its use in 1866.

Observe :

The **meter** measures length.

The **square meter** measures surface.

The **cubic meter** measures solids or volume.

The **gram** measures weight.

The **liter** measures capacity.

**Latin prefixes.**

To express .1 of a meter, .1 of a gram, and .1 of a liter, we prefix **deci** to each of the words, meter, gram, and liter. Thus, *decimeter* means  $\frac{1}{10}$  of a meter ; *decigram*,  $\frac{1}{10}$  of a gram ; and *deciliter*,  $\frac{1}{10}$  of a liter.

To express .01 of a meter, gram, and liter, we prefix **centi** to each of the words, meter, gram, and liter.

To express .001 of a meter, gram, and liter, we prefix **milli** to each of the words, meter, gram, and liter.

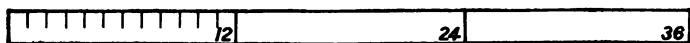
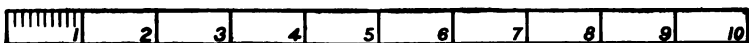
**NOTE.**—From these Latin prefixes we get our words *dime*, *cent*, and

**Greek prefixes.**

To express **10** times a meter, **10** times a gram, and **10** times a liter, we prefix **deca** to each of the words, meter, gram, and liter. Thus, *decameter* means **10** times a meter; *deca-gram*, **10** times a gram; and *decaliter*, **10** times a liter.

To express **100** times a meter, gram, and liter, we prefix **hecto** to each of the words, meter, gram, and liter.

To express **1000** times a meter, gram, and liter, we prefix **kilo** to each of the words, meter, gram, and liter.

**METRIC MEASURES OF LENGTH****Comparison of Fundamental Units of Measures of Length***1 Yard,**1 Meter***Table of Long Measures**

<b>1 millimeter (mm.)</b>	= .001 of a meter
<b>1 centimeter (cm.)</b>	= .01 of a meter
<b>1 decimeter (dm.)</b>	= .1 of a meter (nearly 4 in.)
<b>1 meter</b>	= 39.37 in.
<b>1 decameter (Dm.)</b>	= 10 meters
<b>1 hectometer (Hm.)</b>	= 100 meters
<b>1 kilometer (Km.)</b>	= 1000 meters (.6 mi.)

*In metric long measure 10 times one unit of any denomination equals one unit of the next higher denomination.*

The denominations most frequently used are given in black-faced type.

Approximately :

$$1 \text{ yard} = \frac{11}{12} \text{ meter}$$

$$1 \text{ mile} = 1.6 \text{ kilometer}$$

The **kilometer** is used for measuring long distances; the **meter**, for short distances and for measuring cloth, etc.; and the **millimeter** is used in the sciences and to show very small measurements, as the thickness of wire, etc.

### Written Work

1. What decimal parts of a meter are expressed by the Latin prefixes? What multiples are expressed by the Greek prefixes?

2. Draw a line one meter long. Show the number of decimeters in a meter; the number of centimeters; the number of millimeters.

3. Explain why  $5 \text{ m.} = 50 \text{ dm.} = 500 \text{ cm.} = 5000 \text{ mm.}$

4. In the metric system the fundamental operations are decimal or multiple operations. Thus, 8 m. 5 dm. 6 cm. 25 mm. are added in this manner.

Added in meters:

$$\begin{array}{rcl} 8 \text{ m.} & = & 8 \text{ m.} \\ 5 \text{ dm.} & = & .5 \text{ m.} \\ 6 \text{ cm.} & = & .06 \text{ m.} \\ 25 \text{ mm.} & = & .025 \text{ m.} \\ \hline & & 8.585 \text{ m.} \end{array}$$

Added in millimeters:

$$\begin{array}{rcl} 8 \text{ m.} & = & 8000 \text{ mm.} \\ 5 \text{ dm.} & = & 500 \text{ mm.} \\ 6 \text{ cm.} & = & 60 \text{ mm.} \\ 25 \text{ mm.} & = & 25 \text{ mm.} \\ \hline & & 8585 \text{ mm.} \end{array}$$

5. Add 1 m., 3 dm., 6 cm., 3mm. Add 6.5 m., .25 mm., 65 dm.

6. The distance between two towns is 5 Km. and  $45\frac{1}{2}$  m. After a bicyclist has traveled 3 Km. 57.5 m., how much of the distance remains to be traveled?

7. The distance from Paris to Calais is 295.32 Km. Express this distance approximately in miles.

8. How many meters of ribbon are necessary to make 150 badges, each 25 cm. in length?

9. The distance from Erie, Pa. to Buffalo, N.Y. is 112.651 Km. Express the distance approximately in miles.

10. Reduce the decimal in the last problem to meters and lower denominations.

11. The distance from New York to San Francisco is 4000 miles. Approximate this distance in kilometers.

### METRIC MEASURES OF SURFACE

#### Comparison of Fundamental Units of Square Measure

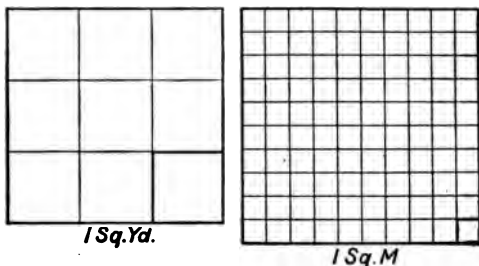


Table of Square Measures

1 sq. millimeter (sq. mm.)	= .000001 sq. meter
1 sq. centimeter (sq. cm.)	= .0001 sq. meter
1 sq. decimeter (sq. dm.)	= .01 sq. meter
1 sq. meter (sq. m.)	= 1.196 sq. yd.
1 sq. decameter (sq. Dm.)	= 100 sq. meters
1 sq. hectometer (sq. Hm.)	= 10000 sq. meters
1 sq. kilometer (sq. Km.)	= 1000000 sq. meters
	(nearly .4 sq. mi.)

*In metric measure of surface 100 times one unit of any denomination equals one unit of the next higher denomination.*



**Land Measure.**

The standard unit used for measuring is the **are** (âr) containing 100 square meters = 119.6 square yards.

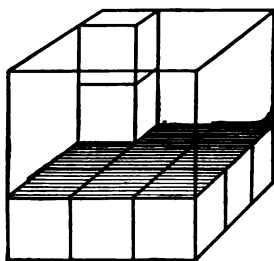
Table :

1 centare = 1 sq. meter.

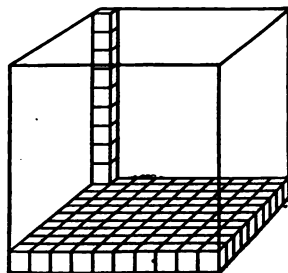
1 are = 100 sq. meters (nearly 120 square yards)

1 hectare = 10000 sq. meters (nearly  $2\frac{1}{2}$  acres).

The **square meter** is used in measuring ordinary surfaces, such as are found in houses, lots, farms, etc. ; the **square kilometer** for measuring areas of countries and their divisions into states, counties, etc.

**METRIC MEASURES OF VOLUME****Comparison of Fundamental Units of Cubic Measure**

1 CUBIC YARD



1 CUBIC METER

**Table of Solid or Cubic Measures**

1 cu. millimeter (cu. mm.)	= .000000001 cu. meter
1 cu. centimeter (cu. cm.)	= .000001 cu. meter
1 cu. decimeter (cu. dm.)	= .001 cu. meter
1 cu. meter	= 1.308 cu. yd.

*In metric measure of volume 1000 times one unit of any denomination equals one unit of the next higher denomination.*

In measuring wood 1 cu. meter is called a **stere**.

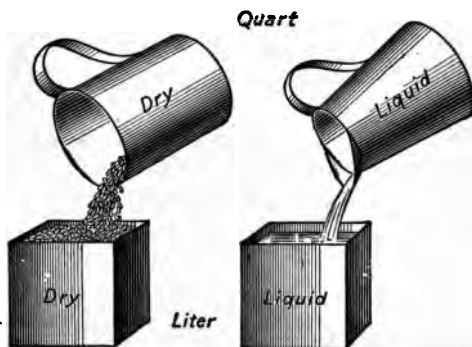
The **cubic meter** is the practical unit of measure of volume for all purposes.

### Written Work in Square and Cubic Measures

1. Find the number of square meters in the floor of your schoolroom.
2. At 27 cents per cubic meter, find the cost of excavating a cellar 10 m. by 18 m. by  $1\frac{1}{2}$  m.
3. How much will it cost to paint one side of a tight board fence 25 meters long and 3 meters wide at \$10 per square decameter?
4. How many square meters of linoleum will be required to cover the floor of a hall 8 meters long and 3 meters wide?
5. How many steres are there in a pile of wood 2 meters high, 2 meters wide, and 6 meters long?

### METRIC MEASURES OF CAPACITY

#### Comparison of Fundamental Units of Capacity



**Table of Measures of Capacity**

1 milliliter (ml.)	= .001 of a liter
1 centiliter (cl.)	= .01 of a liter
1 deciliter (dl.)	= .1 of a liter
1 liter	= 1.0567 liquid quarts = .908 dry quart
1 decaliter (Dl.)	= 10 liters
1 hectoliter (Hl.)	= 100 liters (nearly 2.84 bu.)
1 kiloliter (Kl.)	= 1000 liters

*In metric measure of capacity 10 times one unit of any denomination equals one unit of the next higher denomination.*

The liter is used for liquid and dry measures.

**METRIC MEASURES OF WEIGHT****Comparison of Units of Weight****1 Ounce****1 Gram****1 Lb.****1 Kilo****Table of Measures of Weight**

1 milligram (mg.)	= .001 of a gram
1 centigram (cg.)	= .01 of a gram
1 decigram (dg.)	= .1 of a gram
1 gram	= .03527 of an oz. Avoir.
1 decagram (Dg.)	= 10 grams
1 hectogram (Hg.)	= 100 grams
1 kilogram (Kg.)	= 1000 grams (2.2 lb.)
1 myriagram (Mg.)	= 10000 grams
1 quintal (Q.)	= 100000 grams
1 tonneau (T.)	= 1000000 grams (nearly 2205 lb.)

*In the metric measure of weight 10 times one unit of any denomination equals one unit of the next higher denomination.*

The *gram* is the weight of 1 cubic centimeter of water, the *kilogram*, of 1 cubic decimeter, and the *metric ton*, of 1 cubic meter; the *gram* is used by druggists and chemists; the *kilogram* (usually called the kilo) for weighing small articles; and the *metric ton* for large, heavy articles.

#### Table of Metrical Equivalents

1 cu. mm. of water weighs 1 mg. and measures .001 ml.
1 cu. cm. of water weighs 1 g. and measures 1 ml.
1 cu. dm. of water weighs 1 Kg. and measures 1 l.
1 cu. m. of water weighs 1 T. and measures 1 Kl.

#### Written Work

Things sold in the United States and in England by the *quart* are sold in countries using the metric system by the *liter*.

1. Estimate the number of liters in a tank  $2\frac{1}{2}$  m. long,  $\frac{3}{4}$  m. in width, and  $\frac{1}{2}$  m. in depth.

2. A cylindrical tank 6 m. in diameter and 10 m. in height is  $\frac{3}{4}$  full. Estimate the number of liters it contains. Estimate the weight in metric tons.

3. A Paris milkman retailed on an average 55 liters of milk daily at 50 centimes a liter. Find the amount of his sales in our own money for 30 days. (50 centimes = 10 ¢.)

4. The rainfall in a certain place in one week was 1 dm. Find the number of liters that fell on 3 hectares of land.

5. A horse eats 4 liters of oats 3 times a day. How many hectoliters does it eat in 60 days?

6. From an olive orchard, 4.5 Kl. of olive oil was put up in bottles holding .5 l. How many bottles were used?

7. Find the amount in United States money from the sale of 1000 Hl. of wheat at 20 francs per hectoliter. (1 fr. = \$.193.)

8. An ice dealer retails blocks of ice .8 m. in length, .3 m. in width, and .2 m. in thickness. The weight of ice is .92 that of the same volume of water. Find its cost at 1¢ per kilo.

9. Change a cubic meter of water to liters.

10. Find the weight of a barrel of flour in kilos.

11. A stone 8 ft. by 3 ft. by 2 ft. contains how many cubic meters?

12. If stone is 2.9 times as heavy as the same volume of water, find the weight of the stone in kilos.

13. The Washington Monument is 555 feet high. Express its height in meters.

14. Find the cost of laying a cement walk .025 Km. in length and 1.5 m. in width, at \$1.70 per square meter.

15. Mr. James bought a tract of land in the Philippine Islands, 3 Km. in length and 2.5 Km. in width, at \$15.75 per hectare. Find the cost of inclosing this land with wire fence at 10¢ per meter.

16. A hallway is 12 m. in length and 5 m. in width. Estimate the number of tiles 1 cm. square necessary to cover it.

17. A railroad in building a retaining wall used 52,000 cu. m. of stone. Find its weight in metric tons if stone is 2.7 times as heavy as the same volume of water.

18. A city sewer is 1.3 Km. in length, 1.2 m. in width, and averages 3 m. in depth. Estimate the number of cubic meters of earth removed.

19. A certain kind of cloth costs 90¢ per meter + 25% ad valorem duty. For how much must it be marked in United States money to gain 25% on the yard? (Use 1 yd. =  $\frac{1}{1.1}$  m.)

# TABLES OF MEASURES

## Liquid Measures

4 gills	= 1 pint
2 pints	= 1 quart
4 quarts	= 1 gallon
1 gal.	= 4 qt. = 8 pt.

The gill is now seldom used.

The standard unit of liquid measure is the **gallon**.

1 gallon	= 231 cubic inches
1 cubic foot	= nearly $7\frac{1}{4}$ gallons
$31\frac{1}{2}$ gallons	= 1 barrel
63 gallons	= 1 hogshead
} in measuring the capacity of cisterns and vats	
1 gallon of water	weighs nearly $8\frac{1}{4}$ pounds
1 cubic foot of water	weighs nearly $62\frac{1}{2}$ pounds

## Dry Measures

2 pints	= 1 quart
8 quarts	= 1 peck
4 pecks	= 1 bushel
1 bu.	= 4 qt. = 32 pt. = 64 pt.

Our standard unit, the **Winchester bushel**, used for measuring shelled grains, = 2150.42 cu. in., or nearly  $1\frac{1}{4}$  cubic feet. In form it is a cylinder  $18\frac{1}{2}$  inches in diameter and 8 inches deep.

The dry gallon = 268.8 cu. in.

The heaped bushel = 2747.71 cu. in., or nearly  $1\frac{1}{4}$  cu. ft.

The standard **English bushel** = 2218.192 cu. in.

## Measures of Length

12 inches	= 1 foot
3 feet	= 1 yard
$5\frac{1}{2}$ yards	} = 1 rod
$16\frac{1}{2}$ feet	
320 rods	} = 1 mile
5280 feet	
1 mi.	= 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

The standard unit of length is the **yard**.

A **nautical mile (knot)** = 6080.27 ft. or nearly 1.15 common miles.

A **league** = 3 nautical miles; a **fathom**, used in measuring the depth of water, = 6 ft.; a **hand**, used in measuring the height of horses, = 4 in. A **furlong** =  $\frac{1}{8}$  mi.

**Measures of Surface**

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30 $\frac{1}{4}$ square yards	= 1 square rod
160 square rods	} = 1 acre
43560 square feet	
640 acres	= 1 square mile
1 mile square	= 1 section
36 square miles	= 1 township

The acre is not a square unit like the square foot, the square yard, etc. When in the form of a square, it is nearly 209 feet on a side.

**Surveyors' Measures**

**Surveyors and engineers** formerly used the *Gunter's Chain*. It is 66 feet long and divided into 100 links of 7.92 inches each. The tables are as follows:

Length	Surface
7.92 inches = 1 link	16 square rods = 1 square chain
100 links = 1 chain	10 square chains = 1 acre
80 chains = 1 mile	

They now generally use a steel tape 50 ft. to 100 ft. long divided into feet and tenths of a foot; or a chain 50 ft. to 100 ft. long having links each a foot in length, divided into tenths of a foot.

*Land Measure* is computed by dividing the number of square feet of surface by 43560, the number of square feet in an acre, and changing the decimal of an acre to square rods, etc.

**Measures of Volume**

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

A cubic yard of earth is considered a load.

A cord of 4 foot wood is a pile of wood 8 feet long and 4 feet high, the sticks averaging 4 feet in length, making 128 cubic feet in the pile.

A cord of short wood is a pile of wood 8 feet long and 4 feet high, the number of cords in a pile being computed by multiplying the length of the pile in feet by the height in feet, and dividing the product by 32.

**Avoirdupois Weight**

16 ounces = 1 pound  
 100 pounds = 1 hundredweight  
 2000 pounds = 1 ton  
 2240 pounds = 1 *long* ton  
 1 T. = 20 cwt. = 2000 lb. =  
 32000 oz.

The standard unit of weight is  
 the pound = 7000 grains.  
 1 Av. oz. = 437½ grains.

* 60 pounds = 1 bu. of wheat or  
 potatoes [or rye  
 * 56 pounds = 1 bu. of shelled corn  
 * 32 pounds = 1 bu. of oats  
 * 48 pounds = 1 bu. of barley  
 * 70 pounds = 1 bu. of corn in ear  
 196 pounds = 1 bbl. of flour  
 200 pounds = 1 bbl. of beef or pork

* In most states.

The long ton is used in the United States custom houses and in the  
 wholesale transactions in coal and iron. The long cwt. = 112 lb.

**Troy Weight**

24 grains = 1 pennyweight  
 20 pennyweights = 1 ounce  
 12 ounces = 1 pound  
 1 pound = 12 oz. = 240 pwt. = 5760 gr.

The unit generally used for weighing diamonds, gems, etc., is the  
 carat, which is about 3.2 Troy grains. It is used also to express the  
 fineness of gold. 18 carats fine means  $\frac{14}{18}$  pure gold and  $\frac{4}{18}$  baser metal.

The Troy pound = 5760 grains

The Troy ounce = 480 grains

**Apothecaries' Weight**

This is used only in filling  
 medical prescriptions.

20 grains = 1 scruple — sc. or  $\mathfrak{D}$   
 3 scruples = 1 dram — dr. or  $\mathfrak{z}$   
 8 drams = 1 ounce — oz. or  $\mathfrak{℥}$   
 12 ounces = 1 lb. or  $\mathfrak{℔}$

**Counting**

12 things = 1 dozen (doz.)  
 12 dozen = 1 gross (gro.)  
 12 gross = 1 great gross  
 20 things = 1 score.

**Apothecaries' Liquid Measures**

This is used only in filling  
 medical prescriptions.

60 minims (m) = 1 fluid dram (f $\mathfrak{z}$ )  
 8 fluid drams = 1 fluid ounce (f $\mathfrak{℥}$ )  
 16 fluid ounces = 1 pint (O)  
 8 pints = 1 gal. (Cong.)

**Stationers' Measures**

24 sheets = 1 quire  
 20 quires = 1 ream  
 Paper is frequently sold by the  
 pad or bulk of 100, 500, or 1000  
 sheets, or by the pound.



**Measures of Time**

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
12 months	} = 1 common year
365 days	
366 days	= 1 leap year
10 years	= 1 decade
100 years	= 1 century

Thirty days have September,  
 April, June, and November.  
 All the rest have thirty-one  
 Save February, which alone  
 Has 28, and one day more  
 We add to it one year in four.

The true solar year is 365 days, 5 hr., 48 min., 46 sec. The standard unit of time is the **day** which is divided into 24 hours counting from midnight to midnight. In business transactions 30 days are considered a month, and 12 months are regarded as a year.

The centennial years divisible by 400, and all other years divisible by 4, are **leap years**.

**Measures of Angles and Arcs**

60 seconds	= 1 minute
60 minutes	= 1 degree
360 degrees	= 1 circumference
1 right angle	= 90 degrees

**United States Money**

10 mills	= 1 cent
10 cents	= 1 dime
10 dimes	= 1 dollar
10 dollars	= 1 eagle

**English Money**

4 farthings	= 1 penny	= \$.02025
12 pence	= 1 shilling	= \$.243
20 shillings	= 1 pound	= \$4.8665

The unit of English money is the **pound**.

The value in United States money of other foreign coins is as follows:

Ruble	Russia	= \$.515
Yen	Japan	= .498
Franc	France (Belgium)	= .193
Mark	Germany	= .238
Crown	Austria-Hungary	= .203
Lira	Italy	= .193
Peseta	Spain	= .193
Peso	Chile	= .365
Crown	Sweden	= .268

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12. 14,571. 13. 36,906. 14. 18,394. 15. 30,996. 16. 38,131.  
17. 28,984. 18. 10,137. 19. \$329.04. 20. \$379.21. 21. \$661.47.  
22. \$194.39. 23. \$190.39. 24. 24,323. 25. 14,489. 26. 300,884.  
27. 565,061. 28. 295,404. 29. 186,418. 30. 225,545. 31. 17,302.  
32. 11,025. 33. 3,472,273. 34. 2,698,987. 35. 3,125,092.

**Page 18.** — 2. \$98.67. 3. \$905.92. 4. \$4172.31. 5. \$558.21.  
6. \$247.50. 7. \$530.19. 8. \$255.06. 9. \$323.44. 10. \$1555.50.  
11. \$24.10. 12. \$1166.88. 13. \$2030.10. 14. \$124.41. 15. \$3657.20.  
16. \$2796.64.

**Page 19.** — 18. a. 1,574,125; b. 2,210,200; c. 3,623,700; d. 4,793,050;  
e. 5,165,700; f. 6,174,425; g. 5,583,325; h. 5,114,300; i. 6,219,400;  
j. 5,769,650. 19. a. 250,880; b. 352,256; c. 577,530; d. 763,904;  
e. 823,296; f. 984,064; g. 889,856; h. 815,104; i. 991,232; j. 919,552.  
20. a. 2,136,400; b. 2,999,680; c. 4,918,080; d. 6,505,120; e. 7,010,880;  
f. 8,379,920; g. 7,577,680; h. 6,941,120; i. 8,440,960; j. 7,830,560.  
21. a. 2,364,740; b. 3,320,288; c. 5,443,728; d. 7,200,392; e. 7,760,208;  
f. 9,275,572; g. 8,387,588; h. 7,682,992; i. 9,343,136; j. 8,667,496.  
22. a. 2,122,925; b. 2,980,760; c. 4,887,060; d. 6,464,090; e. 6,966,660;  
f. 8,327,065; g. 7,529,885; h. 6,897,340; i. 8,387,720; j. 7,781,170.  
23. a. 1,934,030; b. 2,715,536; c. 4,452,216; d. 5,888,924; e. 6,346,776;  
f. 7,586,134; g. 6,859,886; h. 6,283,624; i. 7,641,392; j. 7,088,812.  
24. a. 2,073,925; b. 2,911,960; c. 4,774,260; d. 6,314,890; e. 6,805,860;  
f. 8,134,865; g. 7,356,085; h. 6,738,140; i. 8,194,120; j. 7,601,570.  
25. a. 1,885,275; b. 2,647,080; c. 4,339,980; d. 5,740,470; e. 6,186,780;  
f. 7,394,895; g. 6,686,955; h. 6,125,220; i. 7,448,760; j. 6,910,110.



26. a. 2,064,125; b. 2,898,200; c. 4,751,700; d. 6,285,050; e. 6,773,700;  
f. 8,096,425; g. 7,321,325; h. 6,706,300; i. 8,155,400; j. 7,565,650.  
27. a. 2,321,620; b. 3,259,744; c. 5,344,464; d. 7,069,096; e. 7,618,704;  
f. 9,106,436; g. 8,234,644; h. 7,542,896; i. 9,172,768; j. 8,509,448.

Page 21. — 24. a. 24,120,111; b. 18,260,111; c. 9562,111; d. 8528,111;  
e. 7473,111; f. 6669,111; g. 8193,111; h. 7206,111; i. 12,700,111; j. 7263,111;  
k. 807,111; l. 817,111; m. 926,111; n. 863,111. 25. a. 29,278,111;  
b. 22,166,111; c. 11,607,111; d. 10,351,111; e. 9071,111; f. 7974,111;  
g. 9045,111; h. 8747,111; i. 15,416,111; j. 8816,111; k. 979,111;  
l. 992,111; m. 1124,111; n. 1048,111. 26. a. 21,882,111; b. 16,566,111;  
c. 8675,111; d. 7736,111; e. 6779,111; f. 5959,111; g. 7432,111;  
h. 6537,111; i. 11,521,111; j. 6589,111; k. 732,111; l. 741,111;  
m. 840,111; n. 783,111. 27. a. 32,258,111; b. 24,421,111; c. 12,788,111;  
d. 11,406,111; e. 9994,111; f. 8785,111; g. 10,957,111; h. 9638,111;  
i. 16,984,111; j. 9713,111; k. 1079,111; l. 1093,111; m. 1238,111;  
n. 1154,111. 28. a. 35,913,111; b. 27,188,111; c. 14,237,111;  
d. 12,697,111; e. 11,127,111; f. 9781,111; g. 12,198,111; h. 10,730,111;  
i. 18,909,111; j. 10,814,111; k. 1201,111; l. 1217,111; m. 1378,111;  
n. 1285,111. 29. a. 29,045,111; b. 21,988,111; c. 11,614,111;  
d. 10,269,111; e. 8998,111; f. 7910,111; g. 9865,111; h. 8677,111;  
i. 15,292,111; j. 8746,111; k. 971,111; l. 984,111; m. 1115,111;  
n. 1089,111. 30. a. 25,411,111; b. 19,238,111; c. 10,074,111;  
d. 8984,111; e. 7873,111; f. 6921,111; g. 8631,111; h. 7592,111;  
i. 13,379,111; j. 7652,111; k. 850,111; l. 861,111; m. 975,111;  
n. 909,111. 31. a. 29,492,111; b. 22,327,111; c. 11,692,111;  
d. 10,427,111; e. 9137,111; f. 8082,111; g. 10,017,111; h. 8811,111;  
i. 15,528,111; j. 8880,111; k. 986,111; l. 999,111; m. 1132,111;  
n. 1055,111. 32. a. 18,160,111; b. 13,748,111; c. 7199,111; d. 6420,111;  
e. 5626,111; f. 4946,111; g. 6168,111; h. 5425,111; i. 9561,111;  
j. 5468,111; k. 607,111; l. 615,111; m. 697,111; n. 650,111.  
33. a. 12,069,111; b. 9137,111; c. 4784,111; d. 4267,111; e. 3739,111;  
f. 3287,111; g. 4099,111; h. 3605,111; i. 6354,111; j. 3634,111;  
k. 403,111; l. 409,111; m. 463,111; n. 432,111. 34. a. 196,813,111;  
b. 148,999,111; c. 78,026,111; d. 69,585,111; e. 60,977,111; f. 53,603,111;  
g. 66,861,111; h. 58,802,111; i. 103,626,111; j. 59,265,111; k. 6585,111;  
l. 6670,111; m. 7556,111; n. 7045,111. 35. a. 237,850,111;  
b. 180,067,111; c. 94,295,111; d. 84,094,111; e. 73,602,111; f. 64,780,111;  
g. 80,790,111; h. 71,063,111; i. 125,233,111; j. 71,622,111; k. 7959,111;  
l. 8061,111; m. 9132,111; n. 8515,111. 36. a. 195,425,111;  
b. 147,949,111; c. 77,476,111; d. 69,095,111; e. 60,548,111; f. 53,225,111;  
g. 66,380,111; h. 58,388,111; i. 102,896,111; j. 58,847,111; k. 6639,111;  
l. 6623,111; m. 7503,111; n. 6966,111. 37. a. 152,938,111;  
b. 115,784,111; c. 60,632,111; d. 54,073,111; e. 47,384,111; f. 41,654,111;  
g. 51,948,111; h. 45,894,111; i. 80,525,111; j. 46,053,111; k. 5117,111;  
l. 5183,111; m. 5872,111; n. 5475,111.

Page 22. — 2. \$199.92. 3. 40 hr. 4. 16 da. 5. \$289.50.  
6. \$63.58.

Page 23. — 8. \$270. 9. \$217.05. 10. 1710 T. 11. \$5.30.  
12. \$1660; \$.87 $\frac{1}{2}$ . 13. \$5247.45.

Page 27. — 2.  $\frac{1}{10}$ . 3.  $\frac{4}{10}$ . 4.  $\frac{3}{10}$ . 5.  $\frac{5}{10}$ . 6.  $\frac{4}{10}$ . 7.  $\frac{3}{10}$ .  
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Page 28. — 2.  $\frac{1}{10}$ . 3.  $\frac{1}{10}$ . 4.  $\frac{1}{10}$ . 5.  $\frac{1}{10}$ . 6.  $\frac{1}{10}$ . 7.  $\frac{1}{10}$ . 8.  $\frac{1}{10}$ .  
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Page 29. — 2.  $\frac{5}{10}$ . 3.  $\frac{2}{10}$ . 4.  $\frac{1}{10}$ . 5.  $\frac{3}{10}$ . 6.  $\frac{2}{10}$ . 7.  $\frac{2}{10}$ .  
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Page 31. — 2.  $\frac{1}{10}$ . 3.  $\frac{1}{10}$ . 4.  $\frac{1}{10}$ . 5.  $\frac{1}{10}$ . 6.  $\frac{1}{10}$ . 7.  $\frac{1}{10}$ . 8.  $\frac{1}{10}$ .  
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Page 34. — 4.  $\frac{2}{10}$ . 5.  $\frac{6}{10}$ . 6.  $\frac{7}{10}$ . 7.  $\frac{6}{10}$ . 8.  $\frac{5}{10}$ . 9.  $\frac{2}{10}$ .  
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24.  $\frac{2}{10}$ . 25.  $\frac{344}{10}$ . 26.  $\frac{530}{10}$ . 27.  $\frac{865}{10}$ . 28.  $\frac{1935}{10}$ . 29.  $\frac{5649}{10}$ .  
30.  $\frac{3633}{10}$ . 31.  $\frac{1342}{10}$ . 32.  $\frac{4950}{10}$ . 33.  $\frac{9373}{10}$ . 34.  $\frac{32202}{10}$ . 35.  $\frac{6391}{10}$ .  
36.  $\frac{15420}{10}$ . 37.  $\frac{4553}{10}$ . 38.  $\frac{12798}{10}$ . 39.  $\frac{5173}{10}$ . 40.  $\frac{13063}{10}$ .  
41.  $\frac{16072}{10}$ . 42.  $\frac{15440}{10}$ . 43.  $\frac{129808}{10}$ . 44.  $\frac{29743}{10}$ . 45.  $\frac{80726}{10}$ .  
46.  $\frac{24970}{10}$ . 47.  $\frac{10929}{10}$ . 48.  $\frac{33823}{10}$ . 49.  $\frac{31644}{10}$ . 50.  $\frac{31221}{10}$ .  
51.  $\frac{84458}{10}$ . 52.  $\frac{35502}{10}$ . 53.  $\frac{18836}{10}$ . 54.  $\frac{22820}{10}$ . 55.  $\frac{39201}{10}$ .  
56.  $\frac{361150}{10}$ . 57.  $\frac{416050}{10}$ . 58.  $\frac{300}{10}$ . 59.  $\frac{195}{10}$ . 60.  $\frac{276}{10}$ .  
61.  $\frac{456}{10}$ . 62.  $\frac{105}{10}$ . 63.  $\frac{533}{10}$ .

Page 35. — 2.  $\frac{1}{10}$ . 3.  $\frac{1}{10}$ . 4.  $\frac{1}{10}$ . 5.  $\frac{1}{10}$ . 6.  $\frac{1}{10}$ . 7.  $\frac{1}{10}$ .  
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Page 36. — 18.  $\frac{9}{10}$ . 19.  $\frac{5}{10}$ . 20.  $\frac{10}{10}$ . 21.  $\frac{12}{10}$ . 22.  $\frac{6}{10}$ . 23.  $\frac{3}{10}$ .  
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30.  $\frac{16}{10}$ . 31.  $\frac{118}{10}$ . 32.  $\frac{64}{10}$ . 33.  $\frac{108}{10}$ . 34.  $\frac{2125}{10}$ . 35.  $\frac{7450}{10}$ .  
36.  $\frac{262}{10}$ . 37.  $\frac{9289}{10}$ . 38.  $\frac{12222}{10}$ . 39.  $\frac{75}{10}$ . 40.  $\frac{86060}{10}$ .  
41.  $\frac{34301}{10}$ . 42.  $\frac{1485}{10}$ .

Page 37.—2.  $\frac{1}{16}$ . 3.  $\frac{7}{32}$ . 4.  $\frac{11}{16}$ . 5.  $\frac{12}{16}$ . 6.  $\frac{11}{16}$ . 7.  $\frac{30}{16}$ .  
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Page 38.—14.  $\frac{1}{16}$ . 15.  $\frac{1}{16}$ . 16.  $\frac{1}{16}$ . 17.  $\frac{1}{16}$ . 18.  $\frac{1}{16}$ .  
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49. 3668 $\frac{1}{2}$ . 50. 1043 $\frac{1}{2}$ . 51. 816 $\frac{1}{2}$ . 52. 1375 $\frac{1}{2}$ . 53. 3306 $\frac{1}{2}$ .  
54. 3040 $\frac{1}{2}$ . 55. 666 $\frac{1}{2}$ . 56. 369 $\frac{1}{2}$ . 57. 2733 $\frac{1}{2}$ . 58. 883 $\frac{1}{2}$ .  
59. 787 $\frac{1}{2}$ . 60. 1006 $\frac{1}{2}$ .

Page 39.—61. 21. 62. 15. 63. 28 $\frac{1}{2}$ . 64. 45 $\frac{1}{2}$ . 65. 57 $\frac{1}{2}$ .  
66. 51 $\frac{1}{2}$ . 67. 193 $\frac{1}{2}$ . 68. 293 $\frac{1}{2}$ . 69. 1050. 70. 2058. 71. 1242.  
72. 2624. 73. 1 $\frac{1}{2}$ . 74.  $\frac{1}{2}$ . 75. 1 $\frac{1}{2}$ . 76. 1 $\frac{1}{2}$ . 77. 1 $\frac{1}{2}$ . 78.  $\frac{1}{2}$ .  
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100.  $\frac{1}{2}$ . 101. 1 $\frac{1}{2}$ . 102. 8 $\frac{1}{2}$ . 103. 8 $\frac{1}{2}$ . 104.  $\frac{1}{2}$ . 105. 50.  
106. 341 $\frac{1}{2}$ . 107. 65. 108. 16. 109.  $\frac{1}{2}$ . 110. 1 $\frac{1}{2}$ . 111. 3 $\frac{1}{2}$ .  
112.  $\frac{1}{2}$ . 113.  $\frac{1}{2}$ . 114.  $\frac{1}{2}$ . 115. 33 $\frac{1}{2}$ .

Page 40.—3.  $\frac{1}{16}$ . 4.  $\frac{1}{16}$ . 5.  $\frac{1}{16}$ . 6.  $\frac{1}{16}$ . 7.  $\frac{1}{16}$ . 8.  $\frac{1}{16}$ .  
9.  $\frac{1}{16}$ . 10.  $\frac{1}{16}$ . 11.  $\frac{1}{16}$ . 12.  $\frac{1}{16}$ . 13. 2. 14.  $\frac{1}{16}$ .

Page 41.—3. 96. 4. 539. 5. 4 $\frac{1}{2}$ . 6. 13 $\frac{1}{2}$ . 7. 1 $\frac{1}{2}$ . 8. 1 $\frac{1}{2}$ .  
9.  $\frac{1}{2}$ . 10. 7. 11. \$1300. 12. \$14,400. 13. 10,824 books.  
14. 64,725 men. 15. \$4500.

Page 42.—4. 10 $\frac{1}{2}$  lb.

Page 43.—7. 14 $\frac{1}{2}$  T. 8. \$.75. 9. 9 $\frac{1}{2}$ . 10. 510 lots. 11. 60 sheep.  
12. \$100. 13. \$1.66. 14. \$.780. 15. \$1.86. 16. \$350. 17. 5 $\frac{1}{2}$  lb.

Page 44.—18. \$2.17. 20. \$2.35. 21. \$70.50. 22. \$11. 23.  $\frac{1}{2}$ ;  
 $\frac{1}{2}$ ;  $\frac{1}{2}$ . 25. 85 pupils. 26. \$450. 27. \$50.

Page 45.—28. 6 da. 29. \$330.67. 30. \$600. 31. 2000 bu.  
32. \$3000. 33. 100 da. 34. 34 head. 35. \$1.60. 36. \$81.  
37. \$2.20. 38. \$214.50. 39. 120 lemons. 40. 896 ft.

Page 46.—41. \$40. 42. \$16,000; \$42,000. 43. \$67,500. 44. 60 A.;  
\$1000 gain. 45. 120 A. 46. \$145.50. 47. \$60. 48. 262 $\frac{1}{2}$  lb.  
49. 76 $\frac{1}{2}$  ft.; 60 $\frac{1}{2}$  ft.; 59 $\frac{1}{2}$  ft.; 67 $\frac{1}{2}$  ft.; 109 ft. 50. 55 $\frac{1}{2}$ . 51. \$7.66.

Page 49.—3. 91.983. 4. 164.97. 5. 36.502. 6. 134.7.  
7. 170.206. 8. 18.588. 9. 239.361. 10. 428.155. 11. 1.268.  
12. .6243. 13. 310.8. 14. 26.3. 15. 338.9923. 16. 25.823.  
17. 45.3407. 18. 35.755. 19. .6773. 20. 68.722. 21. .0467.  
22. \$23.59. 23. 4191. 24. 128.061. 25. 10.992. 26. 171.125.

Page 51.—2. .0805. 3. .1392. 4. .00112. 5. 5.98. 6. 20.16.  
7. 81.969. 8. 5.34. 9. 1.406. 10. .042105. 11. 1619.2.  
12. 3474. 13. 5943.2. 14. 854.34. 15. .001371. 16. .18046.  
17. 249.781. 18. 189.6832. 19. .242724. 20. 2.5144. 21. 48.608.  
22. 2217.75436. 23. 3.008. 24. 1.196. 25. 27.5. 26. 25.016.

# ANSWERS

V

27. 35.358. 28. 450.74. 29. 323.5. 30. 5.346. 31. 41.736.  
32. 36.4719. 33. 70.1337. 34. 2256.24. 35. 21.035. 36. .3495824.  
37. .28. 38. 71.442. 39. .000475. 40. 6.174784. 41. 4.97175.  
42. 1150.252. 43. .21056. 44. 1.1355. 45. 407.98485. 46. .52628.  
47. 12.5552. 48. .04311. 49. .00354. 50. 4.016016. 51. .00025.  
52. 39.13875.

Page 54.—3. 15. 4. 1.8. 5. 2.21. 6. 2.05. 7. 1200. 8. .012.  
9. 560. 10. 1.24. 11. 12.5. 12. 2.07. 13. 4. 14. 745. 15. 9.7.  
16. 248. 17. .018. 18. .0055. 19. 1,000,000. 20. 25. 21. 2.44.  
22. 115.8. 23. 16.4. 24. .0027. 25. 17,500. 26. .33. 27. .000011.  
28. 100,000. 29. 4. 30. .0002. 31. 210. 32. .000685. 33. .00023.  
34. 25. 35. 26.175. 36. 1700.82. 37. 14,096.4043.

Page 55.—1.  $\frac{1}{2}$ . 2.  $\frac{1}{3}$ . 3.  $\frac{1}{4}$ . 4.  $\frac{1}{5}$ . 5.  $\frac{1}{6}$ . 6.  $\frac{1}{7}$ . 7.  $\frac{1}{8}$ .  
8.  $\frac{1}{9}$ . 9.  $\frac{1}{10}$ . 10.  $\frac{1}{11}$ . 11.  $\frac{1}{12}$ . 12.  $\frac{1}{13}$ . 13.  $\frac{1}{14}$ . 14.  $\frac{1}{15}$ . 15.  $\frac{1}{16}$ .  
16.  $\frac{1}{17}$ .

Page 56.—1. .625. 2. .6. 3. .15. 4. .3125. 5. .875. 6. .44.  
7. .5625. 8. .59375. 9. .4375. 10. .68. 11. .63637. 12. .29037.  
13. .61644. 14. .54714. 15. .6296. 16. .47364. 17. 1.57144.  
18. 1.47054. 19. 1.29624. 20. .7551. 21. .5384. 22. .4545.  
23. .01664. 24. .4615. 25. 1.5882. 26. .72.

Page 57.—2. \$20. 3. \$6.25. 4. \$60. 5. \$6.25. 6. \$45.  
7. \$200.

Page 58.—2. 2560 lb. 3. 3000 lb. 4. 60 lb. 5. 200 yd.  
6. 260 yd. 7. 150 yd. 8. 12,800 lb. 9. 800 lb. 10. 32 collars.  
11. 60 bananas.  
1. \$14.27. 2. 50 bu. 3. \$1000. 4. 3.74624625. 5. 413.  
6. \$5000;  $\frac{1}{2}$ .

Page 59.—7. \$8.50. 8. 136 books. 9. \$60,000. 10. \$2018.75.  
11. \$350. 12. \$24,000. 13. \$9.10. 14. \$6000.

Page 60.—15. 2111.101. 16. 849.404. 17. 6270.835. 18. 101,278.089.  
19. 1034.25478. 20. \$6.56. 21. 60 apples. 22. \$750. 23. \$26.74.

Page 63.—3. 59.04. 4. \$23.94. 5. 55.23. 6. \$29.45. 7. 236.16.  
8. 5.4. 9. 15. 10.  $\frac{1}{2}$ . 11. 147.

Page 64.—12. \$30. 13. \$9. 14. \$50. 15. \$75. 16. \$30.  
17. \$17.50. 18. \$2.50; \$5; \$7.50. 19. \$1.20. 20. \$22. 21. \$.20.  
22. \$20. 23. \$20. 24. \$2.43. 25. \$1.76; \$1.87; \$2.04; \$2.09;  
\$2.20; \$2.64.

Page 65.—26. 1380 bu. 27. \$3000. 28. 240 A. wheat; 108 A. corn;  
168 A. oats and grass; 84 A. uncultivated. 29. \$375.75. 30. \$1080.  
31. \$100 per acre. 32. \$15. 33. \$32. 34. \$17.25. 35. \$43.13.  
36. \$74.76. 37. \$37.10. 38. \$24.28. 39. \$110.10. 40. 180 girls;  
220 boys. 41. \$7.50. 42. 25.2 T. 43. \$2422.50.

Page 66.—44. \$16.50. 45. The same. 46. \$73.95. 47. \$1750.  
48. \$292,702.50. 49. \$2.60; \$1.73. 50. \$220; \$198; \$132.

**Page 67.**—2. 40%      3.  $37\frac{1}{2}\%$       4. 60%      5. 75%      6. 25%  
 7.  $62\frac{1}{2}\%$       8. 20%      9.  $33\frac{1}{2}\%$       10. 15%      11. 60%      12.  $6\frac{1}{2}\%$   
 13.  $14\frac{1}{2}\%$       14. 50%      15.  $27\frac{1}{2}\%$       16. 30%      17.  $55\frac{1}{2}\%$       18. 6%  
 19. 25%      20.  $12\frac{1}{2}\%$       21.  $12\frac{1}{2}\%$       22.  $11\frac{1}{2}\%$       23. 20%      24. 25%  
 25.  $33\frac{1}{2}\%$       26.  $80\frac{1}{2}\%$       27.  $83\frac{1}{2}\%$       28.  $16\frac{1}{2}\%$       29. Mary, 60%; Henry, 72%.

**Page 68.**—30.  $66\frac{1}{2}\%$ .

**Page 69.**—34. 25%      35.  $33\frac{1}{2}\%$  gain.      36. Cost \$2.80;  $14\frac{1}{2}\%$  loss.  
 37. Gain \$2; 50%      38. Cost \$3.50;  $42\frac{1}{2}\%$       39. Gain \$.30; 25%  
 40. Cost \$100; 20% loss.      41. Gain \$2.50; 20%      42. Loss \$.15;  $14\frac{1}{2}\%$   
 43. Loss \$3; 20%      44. Loss \$.30;  $33\frac{1}{2}\%$       45. Gain \$2.50;  $33\frac{1}{2}\%$   
 46. Gain \$.50;  $28\frac{1}{2}\%$       47. 6%      48. 4%      49.  $33\frac{1}{2}\%$       50. 20%.

**Page 70.**—51.  $22\frac{3}{8}\%$       52. 15.4% gain.  
 1.  $2.868\frac{1}{2}\%$       2.  $.697\frac{1}{2}\%$       3.  $5\frac{1}{4}\%$       4.  $17\frac{1}{2}\%$ ;  $22\frac{1}{2}\%$       5.  $9\frac{1}{2}\%$   
 6.  $22\frac{1}{2}\%$       7. 25%.

**Page 71.**—8.  $9\frac{1}{2}\%$       9. 10%      10. \$500.      11. \$10;  $12\frac{1}{2}\%$   
 12. \$854.40.      13. 50%; 15%      14. 300%;  $33\frac{1}{2}\%$       15. 50%  
 16.  $21\frac{1}{2}\%$       17. \$1732.50.      18. 85%      19. \$1.20.

**Page 72.**—20. 20%      21.  $16\frac{1}{2}\%$       22.  $32\frac{1}{2}\%$       23. 25%  
 24. 25%      25.  $16\frac{1}{2}\%$       26. 20%      27. 40%      28.  $29\frac{1}{2}\%$       29.  $26\frac{1}{2}\%$   
 30.  $23\frac{1}{5}\%$       31.  $21\frac{1}{2}\%$       32. 25%      33.  $18\frac{1}{2}\%$       34. 20%      35. 30%  
 36. 20%      37.  $33\frac{1}{2}\%$       38.  $19\frac{1}{2}\%$       39. 25%.

**Page 75.**—1. \$564.35.

**Page 76.**—2. \$257.53.      3. \$365.80.      4. \$386.12.      5. \$12,677.94.  
 6. \$6.20.      7. \$119.      8. \$61.20.

**Page 78.**—1. \$159.88.      2. \$107.21.      3. \$17.57.

**Page 80.**—1. \$7.87.      2. \$97.78.

**Page 81.**—1. \$3; trade.      2. Cash.      3. Trade; 10%.      4. \$125;  
 because it is a reduction made for immediate payment in cash of goods sold on time.

**Page 82.**—5. 30%; trade.      6. Trade.      7. \$3.75.      8. Trade.      9. \$15.  
 10. \$30.      12. \$10.      13. \$1350.      14. \$20, \$10, \$4.

**Page 83.**—4. \$56.70.      5. \$72.      6. \$144.      7. \$26.60.      8. \$8.  
 9. \$238.03.      10. \$83.79.      11. \$223.13.      12. \$273.60.      13. \$3249.

**Page 84.**—14. \$31.92.      15. \$231.60.      16. 24.7%.      17. \$38.48.  
 18. 10% discount; \$2.60 better.      19. None.      20. (1) \$640; (2) \$567;  
 (3) \$482; (3) is best; \$208 better than (1), \$135 better than (2).  
 21. \$70.13.      22. \$1525.95.      23. 56.8%.

**Page 85.**—2. \$547.20.

**Page 86.**—3. \$564.48.      4. \$39.33.      5. \$407.80.      6. \$1289.25.  
 7. \$3182.50.      8. \$1508.64.

# ANSWERS

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**Page 87.** — 1. \$13.50. 2. \$68.75. 3. \$78.18. 4. \$365.75.  
5. \$4037.50.

**Page 88.** — 6. \$14. 7. \$378.50. 8. \$72. 9. \$96. 10. \$53.38.  
11. \$103.13. 12. \$59.57. 13. \$14. 14. \$18.75. 16. \$5484.37.

**Page 89.** — 17. 2%. 18. \$1851.35. 19. \$920 loss. 20. \$75.90.  
21. \$14,412.50. 22. \$6085.25. 23. \$306. 24. \$3940.

**Page 90.** — 25. \$210.75. 26. \$69,040.80. 27. \$1220.

**Page 93.** — 2. \$41. 3. \$240. 4. .003½. 5. .018; \$225. 6. \$22.15.  
7. .006; \$74.76. 8. .002½; \$18.25. 9. \$65.60.

**Page 94.** — 10. \$19.75. 11. .008½. 12. \$95.80. 13. (2) \$38.95;  
(3) \$228; (4) \$315.88; (5) \$213.75; (6) \$21.05; (7) \$71.03; (8) \$17.34;  
(9) \$62.32; (10) \$18.77; (12) \$91.01. 14. \$104.76. 15. \$176.  
16. \$225.

**Page 97.** — 3. 66 qt. 4. 65 qt. 5. 122 pt. 6. 211 qt. 7. 192½ pk.  
8. 832½ qt. 9. 1½ pk. 10. ½ bu. 11. 2 bu. 1 pk. 3 qt. 12. 8 bu. 3 pk.  
13. 1 bu. 7 qt. 14. 300,960 ft. 15. 15 yd. 1 ft. 16. 14 rd. 9 ft.  
17. 2 mi. 120 rd. 18. 4200 in. 19. 1440 in. 20. 4 ft. 9 in. 21. 1413 in.  
22. 170 ft. 23. 2436⅔ rd. 24. 123 sq. ft. 25. 14½ sq. ft.  
26. 2360 sq. rd. 27. 441 cu. ft. 28. 49,500 lb. 29. 450 lb. 30. 305 lb.  
31. 11,760 lb. 32. 30 lb. 2 oz. 33. 6770 lb. 34. 850 lb. 35. 1750 lb.  
36. 14,200 lb. 37. 1 L. T. 2060 lb. 38. 2 L. T. 920 lb.  
39. 33 L. T. 1080 lb. 40. 37 T. 10 cwt. 41. 20 T. 600 lb. 42. 187½ bu.  
43. 83½ bu. 44. 116½ bu. 45. 66½ bu. 46. 80 bu. 47. 178½ bu.  
48. 268 oz. 49. 300 oz. 50. 31,360 oz. 51. 23,600 oz. 52. 1347 oz.  
53. 108 min. 54. 165 min. 55. 940 sec. 56. 1425 sec. 57. 100 da.  
58. 30 wk. 5 da. 59. 2044 da. 60. 2 yr. 295 da. 61. 20,450 sec.  
62. 12,605 min. 63. 5 hr. 16 min. 64. 14 da. 6 hr. 65. 50 wk.  
66. 1 yr. 355 da. 67. 4 leap years. 68. 333 things. 69. 93½ doz.  
70. 1 gross 3 doz. 71. 302 doz. 72. 12 score. 73. 1100 things.  
74. 2 reams 2 quires. 75. 815 quires. 76. 19 quires 4 sheets.  
77. 1346 sheets. 78. 699 doz.

**Page 98.** — 4. 19¢. 5. 45¢. 6. 78¢. 7. \$1.14. 8. \$1.78.  
9. \$1.69. 10. \$1.40. 11. \$1.64. 12. 1 lb. 8 oz. 13. 3 lb. 1 oz.  
14. 18¢. 15. \$2.66. 16. 20¢. 17. 8¢. 18. \$1.07. 19. 3 lb. 4 oz.

**Page 99.** — 21. 57 yr. 2 mo. 8 da. 22. 84 yr. 9 mo. 10 da. 23. 42 yr.  
11 mo. 12 da. 24. 56 yr. 2 mo. 3 da. 25. 12 yr. 3 mo. 26 da.  
26. 71 yr. 3 mo. 6 da. 27. 76 yr. 7 mo. 25 da. 28. 73 yr. 2 mo. 7 da.

**Page 100.** — 1. ⅔¢; 18¢. 2. 38 L. T. 11½ long cwt.; 39 T. 18 cwt.  
3. 10,500 lb.; \$44.10. 4. 384 packages. 5. 13,671 lb.; 15,533 lb.  
6. 9500 lb.; \$5.47. 7. \$158.65. 8. 37 gal. 1 qt. 9. James,  
4 bu. 1 qt.; May, 4 bu. 6 qt.; Martha, 4 bu. 19 qt.; Ben, 5 bu. 29 qt.;  
\$2.58; \$2.68; \$2.94; \$3.78.

**Page 101.** — 10. 225 hr. 11. 7½ bu. 12. 80 posts. 13. 40 curbstones.  
14. 1 mi. 214 rd. farther; 2 mi. 6 rd. 15. 30 days' work. 16. \$30.10.  
17. 4½ in. 18. \$3.44. 19. 146 badges.

Page 102.—20. 2178 steps. 21. 2356 steps. 22. 5654 ft.  
23. 14.1872 ft.; 11.5192 ft.; 2.618 ft. 24. 1080 turns. 25. 240 sq. ft.  
26. \$12.60. 27. \$.95.

Page 103.—28. \$3.50. 29. \$40.32. 30. 111 boxes. 31. 113,796 ft.  
32. \$263.01. 33. 191 bottles. 34. 9500 packages. 35. \$1.65. 36.  $2\frac{1}{2}$  mi.  
37. 429,117 lb. 38. 13 mi. 252 rd. 2 ft.

Page 104.—39. \$4. 40. 48 bu. 136 baskets. 41. 8 yd. 25 $\frac{1}{2}$  in.  
42. 10,800 ft. 43. 42 gal. 2 qt. 1 pt. 44. \$8.40. 45. \$13.89.  
46. \$11.25. 47. 240 ft. 11 in.; 8 lb. 10 oz.; 27 bu. 2 pk. 5 qt. 48. \$115.08.  
49. \$74.31.

Page 105.—1. 78¢. 2. 79¢. 3. Any weight greater than 26 oz. and not more than 28 oz. 4. 21¢. 5. 4¢. 6. 15¢. 7. \$1.10. 8. 4¢.

Page 107.—1. \$.04. 2. \$.06. 3. \$.13. 4. \$.52. 5. \$.06.  
6. \$.14. 7. \$.71. 8. \$.08. 9. \$.08. 10. \$.23. 11. \$.05.  
12. \$.22. 13. \$.45. 14. \$.71. 15. \$.12. 16. \$.11. 17. \$.28.  
18. \$1.13. 19. \$.14. 20. \$.05.

Page 109.—13. 960 tiles.

Page 110.—2. 10 in. wasted. 3. 36 blotters. 4. 60 patterns.  
5. 10 book covers. 6. 50 rectangles. 7. 40 designs. 8. 6 designs.  
9. 12 holders; 12 holders.

Page 113.—1. 4450 ears. 2. \$18.75. 3. 33 doz. 4. \$4.80.

Page 114.—5. \$6.75. 6. \$8. 7. \$8. 8. \$26.86. 9. \$1.50.  
10. \$1.60. 11. \$19.20. 12. \$10.70. 13. \$69.76.

Page 115.—1. 7 waists; 38¢. 2. 8 $\frac{1}{2}$  in. 3. \$1.37. 4. \$4.80.  
5. \$78. 6. \$.55 $\frac{1}{2}$ .

Page 116.—7. \$.16 $\frac{1}{2}$ . 8. 31 $\frac{1}{2}$ %. 9. \$.25. 10. \$1.88. 11. 2 $\frac{1}{2}$  in.  
12. 8 $\frac{1}{2}$  in. 13. 48 tucks.

Page 117.—14. 3 yd. 31 in.

Page 119.—2. \$10.80. 3. \$152. 4. \$27. 5. \$118.40. 6. \$9.  
7. \$15.36. 8. \$19.25. 9. \$33.75. 10. \$28.50. 11. \$58.  
12. \$42.75. 13. \$180.95. 14. \$29.25. 15. \$15.75.

Page 120.—2. \$1675.80. 3. \$1226.55. 4. \$448. 5. \$573.80.  
6. \$16.25; \$516.25. 7. \$10.83; \$260.83. 8. \$9.60; \$369.60.  
9. \$7.13; \$482.13. 10. \$30.75; \$930.75. 11. \$2.90; \$127.90.  
12. \$11; \$336. 13. \$.51; \$26.01. 14. \$3.08; \$178.58. 15. \$7.05;  
\$157.05. 16. \$13.24; \$400.74. 17. \$6.71; \$132.21. 18. \$11.23;  
\$356.73. 19. \$80.03; \$835.03. 20. \$171.81; \$715.81. 21. \$9.90;  
\$90.70.

Page 121.—2. \$115.48. 3. \$91. 4. \$65.33. 5. \$101.28.  
6. \$4.58. 7. \$9. 8. \$19.85. 9. \$114.49. 10. \$73.02. 11. \$77.25  
12. \$169.30. 13. \$337.40. 14. \$219.75. 15. \$247.50. 16. \$2.40.  
17. \$4.77. 18. \$6.18. 19. \$10.50. 20. \$47.60. 21. \$60.50.  
22. \$38.25. 23. \$145.13. 24. \$10.66. 25. \$19.99.

**Page 122.**—2. \$2.50. 3. \$3.80. 4. \$13.02. 5. \$2.50.  
6. \$60.50. 7. \$57.60. 8. \$13.05. 9. \$57.96. 10. \$12.64.  
11. \$44.96. 12. \$13.40. 13. \$9.99. 14. \$30. 15. \$34.88.  
16. \$16.25. 17. \$10.83. 18. \$9.60. 19. \$7.13. 20. \$30.  
21. \$2.92. 22. \$10.83. 23. \$.47. 24. \$10.06. 25. \$5.31.  
26. \$2.50. 27. \$2.10. 28. \$31.10. 29. \$59.50. 30. \$31.36.  
31. \$25.34. 32. \$33.80. 33. \$8.85. 34. \$2.25. 35. \$13.  
36. \$2.67. 37. \$3.01. 38. \$3.07. 39. \$7. 40. \$11.57. 41. \$6.28.  
42. \$10.37. 43. \$79.28. 44. \$171.36. 45. \$9.90.

**Page 123.**—46. \$7.15. 47. \$15.98. 48. \$12.33. 49. \$22.25.  
50. \$77.19. 51. \$9.61. 52. \$31.93. 53. \$36.66. 54. \$779.69.  
55. \$966.60. 56. \$151.59. 57. \$895.13. 58. \$415.28. 59. \$38.48.  
60. \$207.81. 61. \$2246.44. 62. \$1423.15. 63. \$765.53.  
64. \$4044. 65. \$134. 66. \$7150. 67. \$1143.

**Page 126.**—2. \$11.38. 3. \$25.85. 4. \$4.04. 5. \$428.31.  
6. \$104.04. 7. \$157.14. 8. \$788.04. 9. \$61.20.

**Pages 127, 128.**—10. \$206.04. 11. \$154.52. 12. \$885.93.  
13. \$1206. 15. \$1.52. 16. \$157.99. 17. \$80.52. 18. \$1056.07.

**Pages 130, 131.**—7. \$6.25. 8. \$2.50; \$12.50. 9. July 1, 1917;  
\$25.50. 10. 31 certificates. 11. \$8.16. 13. \$193.80.

**Pages 135, 137.**—7. \$254.25. 18. \$255.50. 19. \$530.42.  
20. \$367.62. 21. \$129.23. 22. \$1219.50.

**Page 138.**—2. \$553.48. 3. \$588.19.

**Page 144.**—1. Aug. 1. 2. Aug. 22. 3. Nov. 13. 4. Oct. 10.  
5. Apr. 2. 6. May 17. 7. June 10. 8. Sept. 5.

**Page 145.**—9. Apr. 30; 29 da. 10. July 10; 25 da. 11. Nov. 10;  
41 da. 12. June 23; 22 da. 13. Oct. 14; 43 da. 14. Sept. 1; 31 da.  
15. July 23; 51 da. 16. Dec. 7; 36 da. 17. Sept. 3; 55 da. 18. Oct.  
24; 23 da. 20. \$6.10, \$293.90. 21. Nov. 8, 1917; \$2.25; \$147.75.  
22. Aug. 12, 1917; \$5.24; \$509.76. 23. Oct. 2, 1917; \$17.89; \$982.11.  
24. May 8, 1917; \$14.58; \$610.42. 25. May 4, 1917; \$5.70; \$564.30.  
26. July 7, 1917; \$4.25; \$420.75. 27. Oct. 1, 1917; \$8; \$1192. 28. Dec. 21,  
1917; \$1.33; \$398.67. 29. June 3, 1917; \$19.69; \$1105.31. 30. \$172.55.

**Page 146.**—31. \$148.67. 32. \$123.31. 33. \$204.

**Pages 148, 149.**—1.  $8\frac{1}{2}\%$ . 3.  $2\frac{2}{10}\%$ . 4. 17.9%. 5. 21%.  
6. \$360 a year. 7. To keep it is \$7 better a year. 8.  $8\frac{1}{2}\%$ . 9. \$124.75  
loss. 10. \$1820 better. 11. \$1180. 12. \$535. 13. \$74 profit.

**Page 150.**—14. (c) is the best. 15. 10%. 16.  $6\frac{1}{2}\%$ .  
17. \$27.92 a mo. 18. \$565. 19. The mortgage yields \$134.00 larger  
income.

**Page 151.**—20. \$1853. 21.  $7\frac{1}{10}\%$ . 22.  $8\frac{1}{11}\%$ . 23. 45%.  
24. 6%. 25. 136%.

**Page 152.**—2. \$47.93. 3. \$319.20. 4. \$430. 5. \$1.25.



**Page 154.** — 2. 246 sq. ft.      3. 144.9 sq. ft.      4. 2916 sq. yd.  
 5.  $24\frac{1}{4}$  sq. yd.      6.  $57\frac{1}{2}$  sq. yd.      7. 105 sq. yd.      8. 3 sq. yd.      9. 54  
 sq. yd.      10. 180 sq. yd.      11. 2400 sq. yd.  
 11. \$12.50.      12. 320 boxes.

**Page 155.** — 13. \$170.      14. \$46.69.      1. 11 hooks.

**Page 156.** — 2. 20 sq. in.; 60 sq. in.      3. 51 sq. in.      4. 16 in.  
 5. 96 sq. in.      6. 61 sq. in.

**Page 157.** — 8. 51 in.      9.  $17\frac{1}{2}$  sq. ft.      10. 192 books; 144 books.  
 11. 69 in.

**Page 158.** — 12.  $24\frac{1}{2}$  sq. ft.      13.  $\frac{1}{2}$  in.      14. 45 sq. ft.; 15 sq. ft.  
 15. 24 blocks.      16. 27 blocks.      17. 1368 tiles.      18. 200 red tiles;  
 1168 white tiles.

**Page 159.** — 19. 4 ft.      20. 5954 sq. ft.      21. 50 in.      22.  $12\frac{5}{8}$  sq. ft.

**Page 160.** — 23. 4000 cu. ft.      24. 480 sheets.      25. 550 sq. ft.  
 26. 684 ft.      27. 21,575 sq. ft.

**Page 168.** — 2. Illinois, 14%; Wisconsin, 6.6%; Minnesota, 6.9%; Iowa,  
 15.2%; Nebraska, 6.5%.

**Page 169.** — 3. Minnesota, 4.8%; North Dakota, 6.4%; Nebraska,  
 10.9%; Kansas, 14.8%; Washington, 5.8%.      4. Iowa, 13.9%; Illinois,  
 12.5%; Indiana, 6.7%; Missouri, 5.6%; Nebraska, 7.7%.      5. 125%  
 6. Texas, 32.6%; Georgia, 15.9%; Arkansas, 9.9%; South Carolina, 8.1%;  
 Oklahoma, 7.2%.

**Page 170.** — 7. Georgia, 48.9%; Arkansas, 30.4%; South Carolina, 25%;  
 Oklahoma, 22.1%.      8. \$3456.      9. \$691.20 more.      10.  $42\frac{1}{2}$ %.      11. 40%  
 12.  $42\frac{1}{2}$ %.

**Page 171.** — 13.  $66\frac{2}{3}$ %      14.  $31\frac{1}{2}$ %      15.  $20\frac{1}{2}$ %      16. 100%  
 17.  $106\frac{2}{3}$ %      18.  $14\frac{1}{2}$ %.

**Page 172.** — 1.  $33\frac{1}{3}$ %      2. \$133.33.      3. \$1204.      4. \$23,520  
 5. \$691.20.      6.  $14\frac{1}{2}$ %.

**Page 173.** — 7. \$4440.      1. \$960.      2. \$1242.

**Page 174.** — 3. \$2528.40.      4. \$2343.75.      5. 180%.      6. \$52.50.  
 7. 1000%.

**Page 175.** — 8. \$43.33.      9. 16.53%.      10. 400%.      11. \$288,000;  
 \$261,680.      12. \$1300.

**Page 177.** — 11. \$5.48.      12. \$1.79.      13. \$81.75.

**Page 178.** — 14. \$27.55.      15. \$78.05.      16. \$12.69.      17. \$2.12.  
 18. \$3.78.      19. \$2.48; \$3.15; \$2.70; \$10.80; \$9.76; \$7.02.

**Page 179.** — 20. \$7.01.      21. \$10.44.      22. \$3.13.      23. \$81.34.  
 24. \$144; \$49; \$42 19.      25. \$3.50.

**Page 180.**—26. 1100 cu. ft.; \$.88. 27. \$169.05. 28. \$11.75.  
 29. \$158.40. 30. \$8; \$19.49; \$18.88; \$4.80; \$8; \$10.88; \$6.72;  
 \$13.76; \$12.80. 31. a. \$17.64; b. \$26.29; c. \$90.90. 33. \$3.42;  
 \$28.50; \$49.

**Page 181.**—34. 20% inc.;  $9\frac{1}{2}\%$  inc.; 8% inc.;  $12\frac{1}{2}\%$  inc.;  
 10% inc.;  $6\frac{1}{2}\%$  dec.;  $16\frac{1}{2}\%$  dec.;  $14\frac{1}{2}\%$  dec.; 20% dec.; 20% inc.  
 35. \$34.30. 36. \$5.03. 37.  $33\frac{1}{2}\%$ ; 25%;  $23\frac{1}{2}\%$ . 38. \$19.24.  
 39. \$110.12.

**Page 182.**—40. \$23.14. 41. \$3.48. 42. \$42.56. 43. \$100.78.  
 44. \$75; \$37.50. 45. \$120. 46. 6%.

**Page 184.**—1. 236. 2. 191. 3. 407. 4. 1707. 5. \$195.94.  
 6. \$113.22. 7. \$595.61. 8. 516. 9. 432. 10. 429. 11. 3427.  
 12. \$127.83. 13. \$103.28. 14. \$926.88. 15. 312. 16. 300.  
 17. 348. 18. 3513. 19. \$147.84. 20. \$133.24. 21. \$505.87.

**Page 185.**—22. \$12,266.53. 23. \$11,599.21. 24. \$11,207.79.  
 25. \$11,048.76. 26. \$3028.51. 27. \$6736.50. 28. 63.622.  
 29. 89,348. 30. \$6272.12. 31. \$10,298.39. 32. 98.419. 33. 174.115.

**Page 186.**—35. 2872. 36. 4562. 37. 5587. 38. 6224. 39. 5377.  
 40. 86,501. 41. 79,824. 42. 90,930. 43. 75,115. 44. 87,829.  
 45. 88,173.

**Page 188.**—1. a. 101,988; b. 201,849; c. 65,971; d. \$226.13;  
 e. 38,764.995. 2. a. 183,216; b. 101,646; c. 1116; d. \$26.64;  
 e. 34,328.96. 3. a. 5; b. 52,414; c. 2859; d. \$1468.87;  
 e. 8077.62. 4. a. 174,386; b. 142,378; c. 201,757; d. \$38.45;  
 e. 331.945. 5. a. 124,689; b. 54,424; c. 6367; d. \$1124.38;  
 e. 2387.72. 6. a. 57,322; b. 107,633; c. 197,783; d. \$1711.67;  
 e. 622.989. 7. a. 70,206; b. 62,675; c. 203,176; d. \$547.19;  
 e. 3875.023. 8. a. 22,965; b. 83,108; c. 90,982; d. \$13.46;  
 e. 805.26. 9. a. 175,373; b. 317,823; c. 23,313; d. \$539.61;  
 e. 1369.166. 10. a. 71,089; b. 4909; c. 61,068; d. \$1220.67;  
 e. 223.49.

**Page 189.**—1. \$540.75. 2. \$367.04. 3. \$237,600. 4. \$151,500.  
 5. \$25,500. 6. \$7800. 7. \$270,000. 8. \$21,800. 9. \$1306.50.  
 10. \$637.50. 11. \$44,000. 12. \$14,000. 13. \$4001.50. 14. \$1050.  
 15. \$57,568.50. 16. \$100. 17. \$200. 18. \$656.50. 19. \$490.  
 20. \$7515. 21. \$1875. 22. \$341.25. 23. \$31,500. 24. \$11,009.  
 25. \$151.25. 26. \$4.375. 27. \$22.50. 28. \$8.50. 29. \$31.625.  
 30. \$700. 31. \$5280. 32. \$8700. 33. \$1.701. 34. \$1113.75.  
 35. \$33,552.75. 36. \$100.05. 37. 434.01. 38. \$9000. 39. 60.375.  
 40. 1.5625. 41. 30. 42.  $2.77\frac{1}{2}$ . 43. 44. 44.  $9\frac{1}{2}$ . 45. 15.27.

**Page 190.**—46. \$80,000. 47. \$105,000. 48. \$3600. 49. \$7474.50.  
 50. \$50,050. 51. \$300. 52. \$20. 53. \$1600. 54. \$7552.50.  
 55. \$32,224.50. 56. \$9375. 57. \$30. 58. \$350. 59. \$4433.  
 60. \$90. 61. \$69,0625. 62. \$602.25. 63. \$45,000. 64. \$53,500.  
 65. \$6120. 66. \$800. 67. 7500. 68. 150. 69. 1500. 70. 900.  
 71. 110. 72. 1100. 73. \$654.15. 74. 90,250. 75. 930. 76. \$25.55.  
 77. \$91.50.

Page 191. — 78. \$25.15. 79. \$16.65. 80. \$42.57. 81. \$7.19.

Page 192. — 82. \$8.41. 83. \$6.80.

1. 115 $\frac{1}{2}$ . 2. 106 $\frac{1}{2}$ . 3. 179 $\frac{1}{2}$ . 4. 135 $\frac{1}{2}$ . 5. 312 $\frac{3}{4}$ . 6. 103 $\frac{1}{2}$ .  
7. 61 $\frac{1}{2}$ . 8. 41 $\frac{1}{2}$ . 9. 84 $\frac{1}{2}$ .

Page 193. — 10. 77 $\frac{1}{2}$ . 11. 64 $\frac{1}{2}$ . 12. 59 $\frac{1}{2}$ . 13. 152 $\frac{1}{2}$ .  
14. 153 $\frac{1}{2}$ . 15. 181 $\frac{1}{2}$ . 16. 110 $\frac{1}{2}$ . 17. 208 $\frac{1}{2}$ . 18. 125 $\frac{1}{2}$ .  
19. 164 $\frac{1}{2}$ . 20. 75 $\frac{1}{2}$ . 21. 95 $\frac{1}{2}$ . 22. 363 $\frac{1}{2}$ . 23. 105 $\frac{1}{2}$ .  
24. 99 $\frac{1}{2}$ . 25. 79 $\frac{1}{2}$ . 26. 130 $\frac{1}{2}$ . 27. 23 $\frac{1}{2}$ . 28. 15 $\frac{1}{2}$ . 29. 54 $\frac{1}{2}$ .  
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36. 38 $\frac{1}{2}$ . 37. 37 $\frac{1}{2}$ . 38. 88 $\frac{1}{2}$ . 39. 90 $\frac{1}{2}$ . 40. a. 2747 $\frac{1}{2}$ .  
b. 1828 $\frac{1}{2}$ ; c. 1234 $\frac{1}{2}$ ; d. 1013 $\frac{1}{2}$ ; e. 1272 $\frac{1}{2}$ ; f. 959 $\frac{1}{2}$ ; g. 469 $\frac{1}{2}$ .  
h. 1169 $\frac{1}{2}$ ; i. 2221 $\frac{1}{2}$ ; j. 2443 $\frac{1}{2}$ . 41. a. 5010 $\frac{1}{2}$ ; b. 8333 $\frac{1}{2}$ .  
c. 2252 $\frac{1}{2}$ ; d. 1848 $\frac{1}{2}$ ; e. 2319 $\frac{1}{2}$ ; f. 1749 $\frac{1}{2}$ ; g. 856 $\frac{1}{2}$ ; h. 2132 $\frac{1}{2}$ .  
i. 4051 $\frac{1}{2}$ ; j. 4456 $\frac{1}{2}$ . 42. a. 5115 $\frac{1}{2}$ ; b. 3403 $\frac{1}{2}$ ; c. 2299 $\frac{1}{2}$ .  
d. 1887 $\frac{1}{2}$ ; e. 2368 $\frac{1}{2}$ ; f. 1786 $\frac{1}{2}$ ; g. 874 $\frac{1}{2}$ ; h. 2177 $\frac{1}{2}$ ; i. 4136 $\frac{1}{2}$ .  
j. 4560 $\frac{1}{2}$ . 43. a. 3818 $\frac{1}{2}$ ; b. 2540 $\frac{1}{2}$ ; c. 1716 $\frac{1}{2}$ ; d. 1409 $\frac{1}{2}$ .  
e. 1768 $\frac{1}{2}$ ; f. 1333 $\frac{1}{2}$ ; g. 652 $\frac{1}{2}$ ; h. 1625 $\frac{1}{2}$ ; i. 3087 $\frac{1}{2}$ ; j. 3396 $\frac{1}{2}$ .  
44. a. 3697 $\frac{1}{2}$ ; b. 2460 $\frac{1}{2}$ ; c. 1661 $\frac{1}{2}$ ; d. 1364 $\frac{1}{2}$ ; e. 1711 $\frac{1}{2}$ .  
f. 1291 $\frac{1}{2}$ ; g. 631 $\frac{1}{2}$ ; h. 1573 $\frac{1}{2}$ ; i. 2989 $\frac{1}{2}$ ; j. 3288 $\frac{1}{2}$ . 45. a. 4639 $\frac{1}{2}$ .  
b. 3087 $\frac{1}{2}$ ; c. 2085 $\frac{1}{2}$ ; d. 1712 $\frac{1}{2}$ ; e. 2148 $\frac{1}{2}$ ; f. 1620 $\frac{1}{2}$ ; g. 792 $\frac{1}{2}$ .  
h. 1976 $\frac{1}{2}$ ; i. 3751 $\frac{1}{2}$ ; j. 4126 $\frac{1}{2}$ . 46. a. 4795 $\frac{1}{2}$ ; b. 3190 $\frac{1}{2}$ ; c. 2155 $\frac{1}{2}$ .  
d. 1769 $\frac{1}{2}$ ; e. 2220 $\frac{1}{2}$ ; f. 1674 $\frac{1}{2}$ ; g. 819 $\frac{1}{2}$ ; h. 2041 $\frac{1}{2}$ ; i. 3877 $\frac{1}{2}$ .  
j. 4265 $\frac{1}{2}$ . 47. a. 5349 $\frac{1}{2}$ ; b. 3559 $\frac{1}{2}$ ; c. 2404 $\frac{1}{2}$ ; d. 1973 $\frac{1}{2}$ .  
e. 2476 $\frac{1}{2}$ ; f. 1867 $\frac{1}{2}$ ; g. 914 $\frac{1}{2}$ ; h. 2277 $\frac{1}{2}$ ; i. 4325 $\frac{1}{2}$ ; j. 4758 $\frac{1}{2}$ .  
48. a. 5130 $\frac{1}{2}$ ; b. 3413 $\frac{1}{2}$ ; c. 2306 $\frac{1}{2}$ ; d. 1893 $\frac{1}{2}$ ; e. 2375 $\frac{1}{2}$ .  
f. 1791 $\frac{1}{2}$ ; g. 876 $\frac{1}{2}$ ; h. 2183 $\frac{1}{2}$ ; i. 4148 $\frac{1}{2}$ ; j. 4563 $\frac{1}{2}$ . 49. a. 1387 $\frac{1}{2}$ .  
b. 923 $\frac{1}{2}$ ; c. 623 $\frac{1}{2}$ ; d. 512 $\frac{1}{2}$ ; e. 642 $\frac{1}{2}$ ; f. 484 $\frac{1}{2}$ ; g. 237 $\frac{1}{2}$ .  
h. 590 $\frac{1}{2}$ ; i. 1122 $\frac{1}{2}$ ; j. 1234 $\frac{1}{2}$ . 50. 100. 51. 100. 52. 10.  
53. 1000. 54. .05. 55. 100. 56. 300,000. 57. .1. 58. .00001.  
59. .1. 60. .003. 61. 500. 62. 60,000. 63. .003. 64. .19.  
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71. 4000. 72. 400. 73. 40.

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83. .2. 84. 1000. 85. 20. 86. .2. 87. 30. 88. 3000.  
89. .3. 90. 300. 91. 30,000. 92. .03. 93. 300. 94. 3000.  
95. 6000. 96. .06. 97. .3. 98. 3000. 99. .003. 100. 3.  
101. .03. 102. .00003. 103. .0003. 104. .003. 105. 3.  
106. .03. 107. 10. 108. .01. 109. .5. 110. 5. 111. 10.  
112. 1. 113. .1. 114. .06. 115. 100. 116. 100. 117. 1000.  
118. 400. 119. .2. 120. .02. 121. .6. 122. 100. 123. .0025.  
124. .0008. 125. 80. 126. 10,000. 127. 16. 128. 4000.  
129. .4. 130. .001.  
1. 8 $\frac{1}{2}$ . 2. 2 $\frac{1}{2}$ . 3. 2 $\frac{3}{5}$ . 4. 3 $\frac{1}{2}$ . 5. 3 $\frac{3}{5}$ . 6. 4 $\frac{1}{2}$ .

Page 195. — 7. 1 $\frac{1}{2}$ . 8. 1 $\frac{1}{2}$ . 9. 1 $\frac{1}{2}$ . 10. 1 $\frac{2}{5}$ . 11. 1 $\frac{1}{2}$ .  
12.  $\frac{1}{6}$ . 13. 1 $\frac{1}{2}$ . 14. 1 $\frac{1}{2}$ . 15. 1 $\frac{1}{2}$ . 16. 1 $\frac{1}{2}$ . 17. 1 $\frac{1}{2}$ .  
18. 1 $\frac{1}{2}$ .

# ANSWERS

xiii

43. \$.36.	44. \$1.39.	45. \$1.07.	46. \$2.60.	47. \$3.09.
48. \$.44.	49. \$1.95.	50. \$1.29.	51. 4964.	52. 4188 $\frac{1}{2}$ .
53. 403.	54. 2340 $\frac{1}{2}$ .	55. 3863.31.	56. 6116 $\frac{1}{2}$ .	57. 281 $\frac{1}{2}$ .
58. 1504 $\frac{1}{2}$ .	59. 680 $\frac{1}{2}$ .	60. 863 $\frac{7}{8}$ .	61. 708 $\frac{1}{2}$ .	62. 381 $\frac{1}{2}$ .
63. 118 $\frac{1}{2}$ .	64. 64 $\frac{1}{2}$ .	65. 108 $\frac{1}{2}$ .	66. 212 $\frac{1}{2}$ .	67. 7450.
68. 262 $\frac{1}{2}$ .	69. 9260 $\frac{1}{2}$ .	70. 12,222.	71. 75 $\frac{5}{8}$ .	72. 86,060.
73. 3430 $\frac{1}{2}$ .	74. 1485 $\frac{1}{2}$ .			

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80. 9.	81. 32.	82. 10.	83. 13 $\frac{1}{2}$ .	84. 12.
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90. 1.	91. 1.	92. 1.	93. 1.	94. 1.
95. 2 $\frac{1}{2}$ .	96. 1.	97. 1.	98. 1.	99. 1.
100. 1.	101. 1.	102. 1.	103. 1.	104. 1.
105. 1.	106. 1.	107. 1.	108. 1.	109. 1.
110. 1.	111. 1.	112. 1.	113. 1.	114. 1.
115. 1.	116. 1.	117. 1.	118. 1.	119. 1.
120. 1.	121. 1.	122. 1.	123. 1.	124. 1.
125. 1.	126. 1.	127. 1.	128. 1.	129. 1.
130. 1.	131. 1.	132. 1.	133. 1.	134. 1.
135. 1.	136. 1.	137. 1.	138. 1.	139. 1.
140. 1.	141. 1.	142. 1.	143. 1.	144. 1.
145. 1.	146. 1.	147. 1.	148. 1.	149. 1.
150. 1.	151. 1.	152. 1.	153. 1.	154. 1.
155. 1.	156. 1.	157. 1.	158. 1.	159. 1.
160. 1.	161. 1.	162. 1.	163. 1.	164. 1.

Page 197. — 162. 625.	163. 6.	164. 416 $\frac{1}{2}$ .	165. 3125.
166. 875.	167. 44.	168. 5625.	169. 59375.
170. 4375.	171. 68.	172. 6368 $\frac{1}{2}$ .	173. 2903 $\frac{1}{2}$ .
174. 6164 $\frac{1}{2}$ .	175. 5471 $\frac{1}{2}$ .	176. 6298 $\frac{1}{2}$ .	177. 4736 $\frac{1}{2}$ .
178. 15714 $\frac{1}{2}$ .	179. 14705 $\frac{1}{2}$ .	180. 12962 $\frac{1}{2}$ .	181. 7551 $\frac{1}{2}$ .
182. 5884 $\frac{1}{2}$ .	183. 4545 $\frac{1}{2}$ .	184. 0166 $\frac{1}{2}$ .	185. 4615 $\frac{1}{2}$ .
186. 15882 $\frac{1}{2}$ .	187. 5416 $\frac{1}{2}$ .	188. 1.	189. 1.
190. 1.	191. 1.	192. 1.	193. 1.
194. 1.	195. 1.	196. 1.	197. 1.
198. 1.	199. 1.	200. 1.	201. 1.
202. 1.	203. 1.	204. 1.	205. 1.
206. 1.	207. 1.	208. 1.	209. 1.
210. 1.	211. 1.	212. 1.	213. 1.
214. 1.	215. 1.	216. 1.	217. 1.
218. 1.	219. 1.	220. 1.	221. 1.
222. 1.	223. 1.	224. 1.	225. 1.
226. 1.	227. 1.	228. 1.	229. 1.
230. 1.	231. 1.	232. 1.	233. 1.
234. 1.	235. 1.	236. 1.	237. 1.
238. 1.	239. 1.	240. 1.	241. 1.
242. 1.	243. 1.	244. 1.	245. 1.

Page 198. — 1. \$.50. 2. 11 $\frac{1}{2}$  board feet. 3. 12 $\frac{1}{2}$  ft. 4. 8 in. by 12 in.; 96 sq. in.

Page 199. — 5. 11 in. 6. 6 $\frac{1}{2}$  in. 7. 37 in. 8. 38 board feet.  
 9. \$.06 $\frac{1}{2}$ . 10. 1 $\frac{1}{2}$  qt. of onions; 3 green peppers;  $\frac{1}{2}$  cup of salt;  $\frac{1}{2}$  cup of sugar;  $\frac{1}{2}$  cup of cinnamon; 1 $\frac{1}{2}$  pt. of vinegar. 11. 5 cups of flour; 10 tablespoonfuls of baking powder; 2 $\frac{1}{2}$  teaspoonfuls of salt; 2 $\frac{1}{2}$  tablespoonfuls of lard; 1 $\frac{1}{2}$  cups of milk; 2 $\frac{1}{2}$  tablespoonfuls of butter. 12.  $\frac{1}{2}$  of a bag.

13. 34¢. 14.  $3\frac{1}{2}$  qt. of milk;  $3\frac{1}{2}$  cups of sugar;  $3\frac{1}{2}$  cups of cream.  
15. 8 dresses; 18¢.

Page 200. — 16. 8 towels. 17.  $17\frac{1}{2}$  yd. 18. 4 yd. 19. \$1.96;  
\$ .49. 20. 474 plants. 21. \$23.70. 22.  $44\frac{1}{2}\%$ . 23. \$13.70;  $34\frac{1}{2}\%$ .  
24. \$2150.78. 25. \$1243.28.

Page 201. — 1. 234 lb.;  $61\frac{1}{2}$  lb.;  $52\frac{1}{2}$  lb.; 184 lb.

Page 202. — 2. 30 T. 3. 235 T. 4. \$12.25. 5. \$23.16. 6. \$24.33.  
7. 300 lb. of each. 8. \$2.60. 9. \$498.

Page 203. — 10. \$1140. 11. \$131.80. 12. \$1.00. 13. \$1.17.  
14. \$37.85.

Page 204. — 15. \$1.34. 16. \$1926. 17.  $44\frac{1}{2}\%$ . 18. \$75.  
19. \$3225. 20. \$328. 21. Loss \$135. 22. \$379.30.

Page 205. — 23.  $48\frac{1}{2}\%$ . 24. \$239.42. 25. \$86.40. 26. \$24.16.  
27. 8,435,084,131 lb. 28. 2,383,414,080 lb. 29.  $34.7\%$ . 30.  $78.6\%$ ;  
 $76.8\%$ . 31.  $25.3\%$ ;  $23.9\%$ .

Page 206. — 1. 7,036,337 wage earners; \$4,079,332,000; \$579.75.  
2. 268,491 establishments in 1909; 275,791 establishments in 1914; 7300.

Page 207. — 3.  $2.7\%$ . 4. \$4,362,710,000. 5.  $23.7\%$ . 6. \$20,672,052,-  
000; \$24,246,435,000. 7.  $17.3\%$ . 8.  $9.6\%$ . 9.  $12.8\%$ . 10.  $25.5\%$ .  
11.  $12.7\%$ . 12.  $21.9\%$ . 13.  $11.1\%$ . 14.  $28.3\%$ . 15.  $20.2\%$ . 16.  $26.6\%$ .

Page 208. — 19. 975 establishments in 1914; 1124 establishments in  
1909; \$464,249,813, value in 1914; \$507,166,710, value in 1909.  
20. \$42,916,897. 21. Worsteds goods,  $11.8\%$  dec.; woollen goods,  $3.1\%$   
dec.; carpets and rugs,  $2.9\%$  dec.; felt goods,  $15.5\%$  inc.; wool-felt hats,  
 $55.6\%$  dec.

2. \$664.68. 3. \$875.70.

Page 209. — 4., \$765.89. 5. \$42.53. 6.  $6\frac{1}{2}\%$ . 7. \$328.51.  
8. \$1.93. 9. \$39.24. 10. \$22.59. 11.  $6\frac{1}{11}\%$ . 12. \$114.29.  
13. \$10.04. 14. \$18.95.

Page 210. — 15. \$44.53. 16. \$424.25.

1. 75,000 gal.; \$7125. 2. 168 cars. 3. \$53,125. 4. \$8046.88. 5.  $47\frac{1}{2}\%$ .

Page 211. — 6. 73,767.1 lb.; \$118.03. 7. \$103.40. 8. \$44.27.  
9.  $428\frac{1}{2}$  lb.; 48¢. 10. \$304.29. 11. \$128,409.60. 12. \$66. 13. \$150.

Page 214. — 1. \$663.15.

Page 215. — 2. \$399.70.

Page 218. — 1. \$550.80. 2. \$1050; \$1051.75. 3. \$2252.25.  
4. \$1154.88. 5. \$430.42.

Page 219. — 7. \$549.56. 8. \$4551.80. 9. \$791.98. 10. \$311.85.

Page 220. — 1. 15¢. 2. \$1.25.

Page 221. — 3. \$2.55. 4. \$1.30.

Page 223. — 1. 58 da. 2. \$1183.47. 4. \$11,016.75. 5. \$591.80.  
6. \$1876.63.

Page 224. — 1. £2 7d., 51.25 francs, 41.25 marks; £10 2s. 11d., 256.25  
francs, 206.25 marks.

**Page 226.** — 1.  $45^\circ$ . 2.  $30^\circ$ . 3.  $30^\circ$ . 4.  $19^\circ$ . 5.  $80^\circ$ .  
 6.  $42^\circ 15'$ . 7.  $9^\circ 20'$ . 8.  $34^\circ 20'$ .  
 2. sq. in.; 30 sq. in. 3. sq. yd.; 48 sq. yd. 4. sq. ft.; 27 sq. ft.  
 5. sq. ft.; 35 sq. ft. 6. sq. ft.; 70 sq. ft. 7. sq. ft.; 60 sq. ft.

**Page 227.** — 3. 140 sq. ft. 4. 60 sq. ft.

**Page 228.** — 5. 150 sq. ft. 6. 450 sq. ft. 7. 320 sq. ft.; 90 sq. ft.;  
 150 sq. ft. 8. 696 sq. rd.; 2504 sq. rd. 9. 44 sq. yd.;  $97\frac{1}{2}$  sq. yd.  
 10. 180 sq. ft.; 660 sq. ft.

**Page 230.** — 2. 5 in. 3. 4 ft. 4. 13 ft. 5. 25 yd. 6. 8 in.  
 7. 15 ft. 8. 1300 sq. rd.

**Page 232.** — 1. 246 sq. ft. 2. 144.9 sq. ft. 3. 2916 sq. yd.  
 4.  $34\frac{1}{2}$  sq. yd. 5.  $517\frac{1}{2}$  sq. ft. 6. 945 sq. ft. 7. 27 sq. ft. 8. 486 sq. ft.  
 1.  $52\frac{1}{2}$  A. 2.  $28\frac{1}{2}$  A. 3. 75 A. 4. 25 rd. 5. 7 rd. 6. 80 rd.

**Page 233.** — 2. 1050 sq. in. 3. 10 rd.

**Page 234.** — 2. 440 sq. ft. 3. 308 sq. ft.

**Page 235.** — 1. 47.124 ft. 2. 78.54 ft. 3. 188.496 ft. 4. 33.5104 ft.  
 5. 39.27 ft. 6. 87.9048 yd. 7. 126.7112 ft. 8. 94.248 in.  
 9. 27.5762 $\frac{1}{2}$  yd. 10. 4 ft. 11. 40 yd.

**Page 236.** — 1. 78.54 sq. rd. 2. 314.16 sq. rd. 3. 254.4606 sq. ft.  
 4. 1017.8784 sq. ft. 5. 314.16 sq. in. 6. 1256.64 sq. in. 7. 5026.56  
 sq. rd. 8. 490.875 sq. yd. 9. 11,309.76 sq. ft. 10. .78 sq. ft.  
 11. 3.14 sq. rd. 12. 706.86 sq. in. 13. 962.11 sq. yd. 14. 314.16 sq. ft.  
 15. 78.54 sq. yd. 16. 49,087.5 sq. ft. 17. 11,309.76 sq. yd.  
 18. 1184.1176 sq. yd. 19. \$50.27.

**Page 237.** — 1. 432 sq. ft. 2. 1000 sq. in. 3. 118 sq. ft.  
 4. 412 sq. ft. 5. 170 sq. ft. 6. 334 sq. ft. 7. 766 sq. ft. 8. 96 sq. in.  
 9. 864 sq. in. 10. 24 sq. ft. 11. 937 $\frac{1}{2}$  sq. in. 12. 1176 sq. in.  
 13. 793 $\frac{1}{2}$  sq. ft. 14. 1261 $\frac{1}{2}$  sq. in.

**Page 238.** — 2. 250 cu. ft. 3. 684 cu. ft. 4. 800 cu. ft. 5. 9 cu. ft.  
 6.  $4\frac{1}{2}$  cu. ft. 7. 5 cu. yd. 8. 213 $\frac{1}{2}$  loads. 9. 1728 cakes.  
 10. 448 cu. ft. 11. 5 cords. 12. 22 $\frac{1}{2}$  cords. 13. 64 boxes.

**Page 239.** — 14. 22 $\frac{1}{2}$  cords; \$41.63. 15. 23 $\frac{1}{2}$  cords; \$42.79.  
 16. 28 $\frac{1}{2}$  cords; \$52.04. 17. 18 $\frac{1}{2}$  cords; \$34.69. 18. 106 $\frac{1}{2}$  loads.  
 19. 56 $\frac{1}{2}$  cords. 20. 27. 21. 1417 $\frac{1}{2}$  cu. in. 22. 111.475 cu. in. larger.  
 23. 126.6 cu. in. more. 24. 1621.5 cu. in.

**Page 240.** — 25. 10,936 $\frac{1}{2}$  cu. in. 26. 1595 $\frac{1}{2}$  cu. in. 27. Orange box  
 is 1440 cu. in. larger. 28. 124 $\frac{1}{2}$  cu. in.; 451 $\frac{1}{6}$  cu. in. 29. 268 $\frac{1}{2}$  loads.  
 30. 1168 $\frac{1}{2}$  bbl.

**Page 241.** — 31. 208 $\frac{1}{2}$  loads.

**Page 242.** — 1. 753 984 sq. in. 2. 1413.72 sq. in. 3. 62.832 sq. ft.  
 4. 20.944 sq. ft. 5. 8.3776 sq. ft. 6. 282.744 sq. ft. 7. 678.5856 sq. ft.  
 8. 155.5092 sq. ft. 10. 3180.87 cu. ft. 11. 14,726.25 cu. ft.  
 12. 4021.248 cu. ft. 13. 4712.4 cu. ft. 14. 100.5312 cu. ft.  
 15. 6031.872 cu. ft. 16. 12.5664 cu. ft. 17. 235.62 cu. ft.

**Page 243.** — 1. 803.56+ bu.      2. 520.70+ bu.      3. 1157.13+ bu.  
 4. 867.84+ bu.      5. 448.83 gal.      6. 4.04+ bbl.      7. 6.73+ bbl.  
 8. 109,714.28+ gal. ; 3482.99+ bbl.      9.  $12\frac{1}{2}$  bbl. ;  $1\frac{1}{2}$  hr.

**Page 244.** — 1. 7500 lb. ; 120 cu. ft.      2. 1 T. 12 cwt. 88 $\frac{1}{2}$  lb. ; 1 T.  
 7 cwt. 40 $\frac{3}{4}$  lb.      3. 29,687 $\frac{1}{2}$  lb.      4. 5714 $\frac{1}{2}$  bbl.      5. 672 bu.

**Page 245.** — 6.  $5\frac{1}{2}$  T.      7. 600 gal.      8. 108,000 lb.      9. 1958 $\frac{1}{2}$  bu.  
 10.  $31\frac{1}{2}$  T.  
 1. 8000 cu. ft. ; 35 $\frac{1}{2}$  lb.

**Page 246.** — 2. 2940 gal. ; 3840 gal. ; 4860 gal.      3. 187 $\frac{1}{2}$  gal.  
 4. 162,000 gal.      5. 38 $\frac{1}{4}$  bbl.      6. 3645 gal.      7. 1536 cu. in.      8. 86,400 lb.  
 9. 10 $\frac{1}{2}$  bbl.      10. 20 $\frac{1}{4}$  T.      11. 28 bbl.

**Page 248.** — 1. 201.0624 cu. in.      2. 1440 cu. in.      3. 3456 cu. in.  
 4. 94.248 cu. ft.      5. 600 cu. ft.

**Page 249.** — 2. 15 bd. ft.      3. 24 bd. ft.      4. 32 bd. ft.      5. 180  
 bd. ft.      6. 96 bd. ft.      7. 36 bd. ft.      8. 150 bd. ft.      9. 144 bd. ft.  
 10. 432 bd. ft.      11. 360 bd. ft.      12. 2000 bd. ft.      13. 720 bd. ft.

**Page 250.** — 14. 1200 bd. ft.      15. 240 bd. ft.      16. 720 bd. ft.  
 1. \$419.06.      2. \$28.      3. \$47.25.      4. \$26.25.      5. \$39.38.  
 6. \$10.50.      7. 880 bd. ft.      8. 12,800 bd. ft.      9. 3500 bd. ft.      10. 4800  
 bd. ft.      11. 1600 bd. ft.

**Page 251.** — 12. 1500 bd. ft.      13. 8000 bd. ft.      14. 21,000 bd. ft.  
 15. 8750 bd. ft.      16. 1200 bd. ft.      17. 16,666 $\frac{2}{3}$  bd. ft.      18. 18,200  
 bd. ft.      19. 24,000 bd. ft.      20. 7000 bd. ft.      21. 3600 bd. ft.  
 22. 10,000 bd. ft.      23. \$345.60 ; \$491.40.      24. \$48 ; \$672 ; \$768.  
 25. \$22.88 ; \$93.60.

1. 50 rd.      2. 15 ft.      3. 3280 tiles.      4. 400 sq. in. ; 80 in.      5. 1944  
 sq. in. ; 5832 cu. in.

**Page 252.** — 6. 918 boxes.      7. 216 cakes.      8. 12-in. cube has 264  
 sq. in. more surface.      9. \$57,600.      10. \$800.      11. \$7168.      12.  $1\frac{1}{2}$  ft.  
 13. 600,000 lb.      14. 56 yd.      15. \$10,640 ; \$52,800.

**Page 253.** — 16. 3320 sq. rd. ; 371.12 sq. rd. ; 4308.88 sq. rd.      17. 100  
 revolutions.      18. \$35.47.      19. 345 sq. ft.      20. \$100.45.      21. \$424.96.  
 22. 15 ft.      23. \$80.      24. \$666.67.

**Page 254.** — 25. \$468.      26. 2714.3424 cu. ft.      27. 960 blocks ;  
 110,400 lb.      28. 18 $\frac{1}{2}$  bbl.      29. 6000 lb.      30. \$72.50.      31. \$663.51.  
 32. \$67.50.      33. \$322.23.      34. \$167.68.

**Page 255.** — 35. \$191.67 ; \$94.58.      36. 4 ft.      37. \$712.80.      38. 28.05  
 bbl.      39. 1000.      40. 18,334.6 ft.      41. 9257 $\frac{1}{2}$  bu.      42. 4.1888 sq. ft.  
 43. Outflow through second pipe  $2\frac{1}{2}$  times that through first pipe.

**Page 256.** — 1. \$5013.14.      2. \$17,098.03.      3. \$24,735.13.  
 4. \$28,200.18.      5. \$41,683.52.

**Page 257.** — 8. 4,137,998. 7. 3,068,660. 8. 3,852,574.  
 9. \$19,556.88. 10. \$55,906.51. 11. \$21,781.41. 12. 2,697,503.  
 13. 2,057,997. 14. 380,906. 15. 19,486,296. 16. 45,641,176.  
 17. 76,849,156. 18. 55,642,792. 19. 31,005,450. 20. 24,825,878.  
 21. 85,502,932. 22. 28,958,208. 23. 57,513,488. 24. \$3530.748.  
 25. \$2219.7402. 26. \$2152.5765. 27. 50,070. 28. 3428. 29. 17,357.  
 30. 15,294 $\frac{1}{2}$ . 31. 70,508. 32. \$68.20 $\frac{1}{4}$ . 33. \$124.70 $\frac{1}{4}$ .  
 34. \$388.81 $\frac{1}{5}$ . 35. \$546.41 $\frac{1}{15}$ . 36. \$906.72 $\frac{1}{15}$ . 37.  $\frac{1}{15}$ . 38.  $\frac{1}{15}$ .  
 39.  $\frac{1}{5}$ . 40.  $\frac{1}{5}$ .

**Page 258.** — 41.  $\frac{1}{10}$ . 42.  $1\frac{1}{10}$ . 43.  $1\frac{1}{10}$ . 44.  $16\frac{1}{10}$ . 45. 167.609.  
 46. 20.577. 47. 182.932. 48. 100.482. 49. 36.206. 50. 53.361.  
 51. 15.751. 52. 50.131. 53. 19.75. 54. 1.969. 55. 180.741.  
 56. 28.219. 57. 2.077. 58. 3.24. 59. 9.966. 60. 1.927.  
 61. .011055. 62. .760125. 63. 7.762536. 64. 8.016008. 65. 11.0505.  
 66. 120.54. 67. .30335. 68. 30.04812. 69. 2.26 $\frac{1}{4}$ . 70. 565.  
 71. .022. 72. 750. 73. 33 $\frac{1}{4}$ . 74. .021. 75. .004. 76. 8.6.

**Page 259.** — 1. 426 $\frac{1}{2}$  ft. 2. 22,275 min. 3. 4620 ft. 4. 6 cwt. 25 lb.  
 5. 9768 ft. 6. \$5.04. 7. 324 min. 8. 90 bu. 9. 5 T. 3 cwt. 7 lb.  
 10. 15 lb. 12 oz. 11. 10 yr. 7 mo. 29 da. 12. 43 yr. 1 mo. 27 da.  
 13. 694 lb. 12 oz. 14. 5571 ft.

**Page 260.** — 15. 6 bu. 1 qt. 16. 15 lb. 9 $\frac{1}{2}$  oz. 17. 238 ft. 5 $\frac{1}{2}$  in.  
 18. 27 bu. 2 pk. 5 qt. 19. 28 yd. 9 $\frac{1}{2}$  in. 20. 1 A. 106 sq. rd. 20 sq. yd.  
 1 sq. ft. 72 sq. in. 21. 50 $\frac{1}{2}$  times. 22. 4455 times. 23. 21 lots.  
 24. 4 times. 25. \$7407. 26. \$3.14. 27. 62 $\frac{1}{2}$ %. 28. Gain, \$82.50.  
 29. 12%. 30. 18 $\frac{1}{3}$ %.

**Page 261.** — 31. 176 $\frac{1}{2}$  sq. ft. 32. 4 A. 12 sq. rd. 33. 11 $\frac{1}{2}$ %.  
 34. 5 $\frac{1}{2}$ %. 35. \$1200.40. 36. \$51.25. 37. 4%. 38. \$972.  
 39. \$376.10. 40. \$81. 41. 1 $\frac{1}{2}$ %.

**Page 262.** — 42. 87 $\frac{1}{2}$ %. 43. 1152 gal. 44. 640 bu. 45. \$73.55.  
 46. 13 $\frac{1}{2}$  cu. yd. 47. \$13.20. 48. \$54.98. 49. \$60.48. 50. \$635.04.  
 51. \$520.52. 52. \$547.40.

**Page 264.** — 5. 6 times. 6. No difference. 7. \$4000.

**Page 266.** — 1. 48 ft. 2. 81 ft. at widest; 76 ft. at narrowest.  
 3. \$8.10. 4. \$16.05. 5. \$17.71.

1. 16"  $\times$  12". 2. 8"  $\times$  12". 3. 8"  $\times$  14". 4. 8"  $\times$  12". 5. \$7.92;  
 \$.39 $\frac{1}{2}$ . 6. 28 $\phi$ . 7. 67 $\frac{1}{2}$  $\phi$ .

**Page 272.** — 1. 72. 2. 8400. 3. 240. 4. 448. 5. 30. 6. 48.  
 7. 32. 8. 30. 9. 70. 10. 50. 11. 76. 12. 138. 13. 200.  
 14. 180. 15. 144. 16. 2800. 17. 240. 18. 256. 19. 46.  
 20. 164. 21. 80. 22. 96. 23. 3500. 24. 152. 25. 55. 26. 36.

**Page 275.** — 3. 4. 4. 8. 5. 12. 6. 7. 7. 12. 8. 13. 9. 30.  
 10. 40. 11. 115. 12. 7. 13. 8. 14. 8. 15. 13. 16. 30.  
 17. 14. 18. 18. 19. 25. 20. 30. 21. 25.

**Page 276.** — 3. 5. 4. 8. 5. 55. 6. 6. 7. 9. 8. 6. 9. 12.  
 10. 9. 11. 10. 12. 5.



**Page 278.**—1. 10. 2. 48. 3. 15. 4. 7. 5. 36. 6. 6. 7. 100.  
8. 30. 9. 60. 10. 6. 11. 5. 12. 3. 13. 5. 14. 6. 15. 12.  
16. 12. 17. 27. 18. 33. 19. 13. 20. 23. 21. 17. 22. 5.  
23. 12. 24. 10. 25. 15. 26. 11. 27. 8. 28. 5. 29. 3. 30. 6.  
31. 7. 32. 3.

**Page 279.**—1. 60. 2. 52. 3. 90. 4. 5. 5. 60. 6. 9. 7. 10.  
8. 4. 9. 3.5.

**Page 280.**—12. 12. 13. 36. 14. 20. 15. 30. 16. 12. 17. 54.  
18. 96. 19. 12. 20. 42. 21. 12. 22. 16. 23. 7. 24. 12.  
25. 16. 26. 12. 27. 6. 28. 12. 29. 6. 30. 12. 31. 20.  
32. 36. 33. 15. 34. 6.

**Page 281.**—35. 30. 36.  $\frac{1}{2}$ . 37. 8. 38. 18. 39. 40. 40. 32.  
2. 25. 3. \$121; \$368. 4. 75 A.; 150 A.; 225 A. 5. \$900;  
\$1800; \$3600. 6. 72; 216. 7. 1.25; 5. 8. 8 yr.; 40 yr.

**Page 282.**—9. 128; 32. 10. 42 yr.; 7 yr. 11. 28. 12. 12 yr.;  
18 yr. 14. 57; 38. 15. 44. 16. 27 girls; 18 boys. 17. \$1800.  
18. 49; 21. 19. 88; 33.

**Page 283.**—20. 49. 21. 12 yr. 22.  $5\frac{1}{2}$ ;  $1\frac{1}{2}$ . 23. \$50. 24. 85 $\frac{1}{2}$ ;  
34 $\frac{1}{2}$ . 25. \$240; \$160. 26. 3000 lb.; 1500 lb. 27. 30; 60; 180.  
28. 180 girls; 150 boys.

**Page 284.**—30. 15.81; 10.54. 31. 90 A.; 80 A.; 60 A. 32. 90 rd.  
33. 80 rd.; 160 rd. 34. First, \$150; second, \$300; third, \$100. 35. \$60.  
36. \$200. 37. \$1600. 38. Cow, \$56; horse, \$140. 39. 30 ft.  
40. 200 A. 41. 30 marbles; 25 marbles.

**Page 289.**—3. \$30,000. 4. \$27,000. 5. \$100 a share. 6. \$6000.  
7. \$10,000.

**Page 291.**—1. \$6408. 2. \$26,740. 3. \$448. 4. \$11,412.  
6. \$8899.

**Page 292.**—7. \$1976.63. 8. \$5428.13. 9. \$51,250. 10. \$824.  
11. \$1890. 12. \$22,110. 13. \$7443.75. 14. \$26,752. 15. \$23,520.

**Page 293.**—1. \$175. 2. \$784. 3. \$240. 4. \$500. 5. \$180.  
6. \$16,000; \$800. 7. \$610. 8. \$236.25. 9. \$275.50. 10. \$940.  
11. \$240 lost.

**Page 294.**—12. \$333. 13. Gained, \$222. 14. \$88.

**Page 296.**—2. \$2355. 3. \$21,936.25. 4. \$22,385.

**Page 297.**—5. \$4930. 6. \$950. 7. \$10,620. 8. \$8662.50.  
9. \$17,360. 10. \$21,761.25. 11. \$7043.75. 12. \$68.75. 13. \$2720.  
14. \$95. 15. 141 A. 16. \$200. 17. \$160. 18. \$500. 19. \$180.  
20. \$600. 21. \$900. 22. \$200. 23. \$180. 24. \$600.

**Page 298.**—25. \$200. 27.  $6\frac{2}{3}\%$ . 28.  $2\frac{1}{4}\%$ . 29.  $7\frac{1}{2}\%$ .

**Page 299.**—2. \$253.75. 3. \$138. 4. \$256.50. 5. \$140.  
6. \$708.75. 7. \$208.75. 8. \$72. 9. \$428.75. 10. \$35.55.  
11. \$402. 12. \$195. 13. \$78.30. 14.  $\frac{2}{3}\%$ ; \$24. 15. \$58.50.  
16.  $2\frac{1}{2}\%$ ; \$2.50 per \$100. 17.  $\frac{3}{4}\%$ .

**Page 300.** — 18.  $\frac{7}{10}\%$ . 19.  $1\frac{1}{2}\%$ . 20. \$53.25. 21. 1%. 22.  $1\frac{1}{2}\%$ .  
 23. \$42. 24. Farmer's loss, \$210; company's loss, \$3290. 25. Com-  
 pany's loss, \$17,415; owner's loss, \$9585. 26. \$18,000; \$21,600; \$32,400;  
 \$36,000.

**Page 301.** — 1. 30.9%. 2. 17.3%. 3. 17.2%. 4. 34.6%.  
 5. 54.4%; 45.6%.

**Page 302.** — 1. \$264. 2. \$1875. 3. \$1000.

**Page 303.** — 4. \$60. 5. \$1250. 6. \$330. 7. \$2687.50. 8. 12,000 lb.  
 9. \$1920. 10. \$3.97. 11. \$11,147.25. 12. \$2135.75. 13. \$2160.

**Page 304.** — 14. 23.9%; 19.6%. 15. 6.9%. 16. 153.6%.  
 17. \$139,729,800. 18. \$330,000. 19.  $6\frac{1}{2}\frac{1}{2}\%$ .

**Page 305.** — 20. \$186,300. 21.  $18\frac{3}{4}$  mills. 22. \$379.97. 23.  $\frac{4}{5}$  of  
 a mill. 24.  $\frac{2}{5}$  of a mill. 25. \$3520; \$10,120; \$14,520. 26.  $5\frac{1}{2}\%$ ;  
 $21\frac{1}{4}\%$ ;  $26\frac{1}{4}\%$ . 27. Local, \$67,165; state, \$65,835.

**Page 306.** — 28. 4.8 mills. 29.  $59\frac{5}{8}\%$ ;  $41\frac{1}{8}\%$ ;  $4\frac{6}{8}\%$ . 30.  $\frac{1}{2}$  of  
 a mill. 31. \$53,938.50. 32. \$12,204.85. 33. \$25,328.70. 34. \$212.50.

**Page 310.** — 2. 29°; 1 hr. 56 min.; 74° W. 3. 45°; 3 hr.; 120° W.  
 4. 60°; 4 hr.; 15° W. 5. 30°; 2 hr.; 30° E. 6. 150°; 10 hr.; 120° W.  
 7. 120°; 8 hr.; 90° W. 8. 105°; 7 hr.; 30° E. 9. 105°; 7 hr.; 45° W.

**Page 311.** — 10. 11 min. 32 sec. after 12 P.M. 11. 17 min.  $45\frac{1}{2}$  sec.  
 after 11 A.M. 12. 58 min.  $20\frac{1}{2}$  sec. after 8 A.M. 13. 17 min. 33 sec. after  
 5 P.M. 14. 58 min.  $4\frac{1}{2}$  sec. after 5 P.M. 15. 36 min.  $49\frac{1}{2}$  sec. after  
 6 A.M. 16. 1 min.  $46\frac{1}{2}$  sec. after 6 P.M. 17. 12 min.  $11\frac{1}{2}$  sec. after  
 12 P.M. 18. 9 min. 8 sec. after 12 P.M. 19. 5 hr. 32 min.  $18\frac{1}{2}$  sec.  
 20. N.Y. time 2 hr. 39 min.  $35\frac{1}{2}$  sec. earlier.

**Page 312.** — 22. 48 min.  $11\frac{1}{2}$  sec. after 10 P.M., Sept. 5, at Petrograd.  
 23. Honolulu, 28 min.  $37\frac{1}{2}$  sec. after 6 A.M., March 4, 1917; San Francisco,  
 50 min.  $17\frac{1}{2}$  sec. after 8 A.M., March 4, 1917; Manila, 3 min.  $52\frac{1}{2}$  sec. after  
 1 A.M., March 5, 1917. 1. 17 da. 2. 21 da.

**Page 313.** — 1. 3 hr. 2. 11 A.M., Philadelphia = 8 A.M. San Francisco;  
 time for transmission of message was 45 min.

**Page 316.** — 1. \$990. 2. 3.1416 sq. rd. 3. \$1905.75. 4. 2 A. 96 sq. rd.  
 5. 324 cu. ft.; every 10 min. 48 sec. 6. \$39.20. 7. \$482.50.

**Page 317.** — 8. \$55.72. 9. \$160 less. 10. \$110.40. 11. \$79.50.  
 12. 3 mills on a dollar. 13. 58 min. 48 sec. after 2 P.M. 14. \$268, or \$27.  
 15.  $68\frac{1}{2}\%$ . 16. \$50; \$140. 17. 150.7968 in. long. 18. 7 in.; 21 in.

**Page 318.** — 19.  $6\frac{1}{2}\%$ . 20. \$73.10. 21. 16,380 lb. 22. \$450.  
 23. \$1.29. 24. 8 ft.  $8\frac{1}{11}$  in. 25. \$651.42. 26. \$7000. 27. \$6000.  
 28. \$57.

**Page 319.** — 29. \$181.43. 30. \$56.40. 31. 20 A. 32. 320 A.;  
 480 A.; 600 A. 33. \$8575. 34. \$630.40; \$639.86. 35. \$1119.57.  
 36. \$4200. 37.  $85\frac{1}{2}$  ft.

**Page 320.**—38. \$48.72. 39. 648 A. 40. \$1170.19. 41. 38 mi. an hour. 42. 2085 $\frac{1}{2}$  cu. yd. 43. \$3600. 44. 288 cakes. 45. 46.98 lb. water; 13.02 lb. solid. 46. 1121.112. 47. \$3500. 48. 40 A.

**Page 321.**—49. \$710. 50. \$28.80. 51. 2 $\frac{1}{2}$  mi. 52. 27 $\frac{1}{11}$ %. 53. \$429.84. 54. \$6.75. 55. \$207.50. 56. \$144. 57. \$168.48. 58. 60 shares. 59. \$151.59. 60. \$396.

**Page 322.**—61. \$608.40. 62. \$37,500. 63. \$525. 64. 12.5 mills on a dollar. 65. \$57.60. 66. \$787.50. 67. 21 mi. 68. \$26.45. 69. \$26.60. 70. 83 $\frac{1}{2}$ %.

**Page 323.**—71. 6288.2 sq. ft. 72. \$8.85. 73. 13 $\frac{1}{2}$  a quart. 74. \$502.50. 75. 1000 cu. ft. 76. \$39.25. 77. \$887.50. 78. \$99.84. 79. \$631.03. 80. 2.283 $\frac{1}{2}$ .

**Page 324.**—81. 3 $\frac{1}{2}$ . 82. 20 $\frac{5}{8}$ % gain. 83. 25%; \$13.50. 84. \$6. 85.  $\frac{1}{2}$ . 86. \$2812.50. 87. 17.4 mills on a dollar. 88. \$27,984. 89. 1010.121. 90. 1890 cu. ft.; 7110 cu. ft.

**Page 325.**—91. 13 wk. 92. \$51,568. 93. \$166.40. 94. \$50 a month. 95. 15 mills on a dollar. 96. 135,000 gal. 97. 6800 A. 98. \$20.

**Page 326.**—99. \$101.23. 101. 4%. 102. \$345.60. 103. 40 rd.; 80 rd. 104. \$43.70. 105. \$3217.50. 106. Ninety-nine hundred thousandths. 107. 20 ft. 108. 2 A.; 4 A.; 6 A.

**Page 327.**—109. \$2842.50. 110. \$144. 111. \$601.50. 112. \$282. 113. 3 $\frac{1}{4}$ %. 114. 4%. 115. \$120. 116. \$11,500. 117. 5 gal. 1 qt. .3 pt. 118. \$320.25.

**Page 328.**—119. \$3555. 120. 9 times. 121. \$2842. 122. 8%. 123. \$2937. 124. \$1176. 125. 33 min. 57 sec. after 11 A.M. 126. \$384. 127. \$1550.

**Page 329.**—128. \$137.77. 129. \$16,500. 130. \$54.84. 131. 58 min. after 11 A.M. 132. \$86.40. 133. \$94.50. 134. 30%. 135. 1 $\frac{1}{2}$ %. 136. \$1300.

**Page 330.**—137. \$1032.82. 138. \$.342 a pound. 139. \$2090. 140. 71.4% inc. 141. 100% inc. 142. 166.7% inc. 143. 1354.5% inc. 144. 120% inc. 145. 140% inc. 146. 16.7% dec. 147. 37.5% dec. 148. 100% inc. 149. 50% inc. 150. 175% inc.

**Page 331.**—2. 21. 3. 18. 4. 12. 5. 4. 6. 3. 7. 14. 8. 22. 9. 7. 10. 17. 11. 144.

**Page 332.**—3. 144. 4. 1260. 5. 480. 6. 1400. 7. 420. 8. 96. 9. 630. 10. 576. 11. 720. 12. 255. 13. 5400. 14. 882.

**Page 333.**—2. \$53.34. 3. \$15.79. 4. 96 bundles.

**Page 334.**—5. \$72.20. 6. \$305.22. 7. \$231.23.

1. \$134.40. 2. 12,960 tiles.

**Page 335.**—3. 40 sq. in. 4. 360 slates. 5. 240 slates; 450 lb. 6. 5760 slates; \$88. 7. 157 bunches; \$157. 8. \$70. 9. \$105. 10. 30 squares.

**Page 336.**—2. 15 double rolls. 3. \$18.

**Page 337.**—5. Because the ingrain is wide;  $55\frac{1}{2}$  yd. ingrain;  $73\frac{1}{2}$  yd. of Brussels. 6. 5; 5; 6; 6; 6. 7. 7; 6; 7; 8; 8. 8. \$7.20. 9. \$45.84.

**Page 339.**—3. \$30. 4. \$30. 5. \$728. 6. \$72. 7. \$127.50. 8. \$1200. 9. \$11.25. 10. \$8. 11. \$56.80. 12. \$32.32. 13. \$4128. 14. \$56. 15. 920 men. 16. \$1800. 17. \$4700. 18. \$50 a month. 19. 600 pupils.

**Page 340.**—2. 300. 3. 200. 4. 400. 5. 200. 6. 46. 7. 30. 8. 900. 9. 300. 10. \$3. 11. \$2. 12. \$3500. 13. \$1000. 14. \$4700. 15. 10 sq. mi.

**Page 341.**—2. 100. 3. 300. 4. 72. 5. 45. 6. 400. 7. 400. 8. 500. 9. 5. 10. \$40. 11. \$60. 12. \$384. 13. \$2000; \$800; \$10,000. 14. 549 pupils. 15. \$20. 16.  $\frac{1}{2}$ . 17. 1000. 18. \$160, loss. 19. 16,000.

**Page 342.**—2. \$1000. 3. \$680. 4. \$1200. 5. \$1375. 2. 6%.

**Page 343.**—3. 5%. 4. 5%. 2. 3 yr. 4 mo. 3. 20 yr.; 16 yr. 8 mo.; 12 yr. 6 mo. 4. 40 yr.; 33 yr. 4 mo.; 37 yr. 6 mo.; 30 yr. 5. Oct. 13, 1915.

**Page 345.**—1. 5:1. 2. 1:4. 3. 3:1. 4. 1:12. 5. 8:1. 6. 2:5. 7. 3:16. 8. 1:8. 9. 100:1. 10. 3:1. 11. 6:1. 12. 320:1.

**Page 346.**—3.  $18\frac{1}{2}$ . 4. 15. 5. 10. 6. 5. 7. 5. 8. 10. 9.  $\frac{1}{2}$ . 10. .5. 11. 2.5. 12. 90. 14. 30 da. 15. \$18. 16. 50 da.

**Page 347.**—17. 8. 18. \$1921.88. 19. \$4.17. 20. 12 oz. 21.  $41\frac{1}{2}$  mi. 22. \$40. 2. \$81; \$54; \$27. 3. \$5200; \$10,400.

**Page 349.**—1. 36; 49; 64; 81; 100; 144; 225. 2. 27; 64; 125; 216; 343; 512; 729; 1000; 1728. 3. 900; 2500; 3600; 6400; 14,400. 4. 8000; 27,000; 64,000; 125,000; 1,000,000. 5. 36; 64; 81; 125; 216; 343. 6.  $\frac{2}{3}$ ;  $\frac{1}{15}$ ;  $\frac{1}{5}$ ;  $\frac{64}{125}$ ;  $\frac{2}{3}$ ;  $2\frac{1}{3}$ . 7. .09; .0016; .0025; .36; .0036; .064; .000064; .216. 8. 225. 9. 256. 10. 324. 11. 484. 12. 625. 13. 42.25. 14. .5025. 15.  $2\frac{1}{4}$ . 16. 272 $\frac{1}{2}$ . 17. 225 sq. in. 18. 625 sq. ft. 19. 256 sq. yd. 20. 72 $\frac{1}{2}$  sq. ft. 21. 25 sq. in. 22. 72 $\frac{1}{2}$  sq. in. 23. 100 sq. yd. 24. 32 $\frac{1}{2}$  sq. yd. 25. 36 sq. mi. 26. 512 cu. in. 27. 8 cu. ft. 28. 34 $\frac{1}{2}$  cu. ft. 29. 74,088 cu. in. 30. 2 cu. yd. 2 cu. ft. 568 cu. in. 31. 1876 cu. ft. 64 cu. in.

**Page 351.**—2. 15. 3. 24. 4. 21. 5. 14. 6. 40. 7. 28. 8. 36. 9. 16. 10. 48. 11. 35. 12. 42. 13. 24. 14. 56. 15. 18. 16. 20. 17. 72. 18. 25. 19. 32. 20. 64. 21. 27.

**Page 354.**—5. 22. 6. 24. 7. 26. 8. 31. 9. 33. 10. 43. 11. 51. 12. 63. 13.  $\frac{1}{2}$ . 14.  $\frac{1}{2}$ . 15.  $\frac{1}{2}$ . 16. .5. 17. .15. 18. 2.1. 19. 2.5. 20. .25. 21. 17.75. 22. 22.91. 23. 34. 24. 66. 25. 14.5. 26. 13.35. 27. 122. 28. 163. 29. 369. 30. 4.56. 31. 9.84. 32. 13.875. 33. 23.25. 34. 56.5. 35. 75.125. 36. .92. 37. .114. 38. .024. 39. .661. 40. 335.

Page 358.—5. 1.863 m.; 18.00025 m. 6. 1 Km. 988 m. 7. 184.575 ml.

Page 359.—8.  $37\frac{1}{4}$  m. 9. 70.41 mi. 10. 6 Hm. 5 Dm. 1 m.  
11. 6400 Km.

Page 361.—2. \$72.90. 3. \$7.50. 4. 24 sq. m. 5. 24 steres.

Page 363.—1. 937.5 l. 2. 192,000 l.; 192 T. 3. \$165.  
4. 3,000,000 l. 5. 7.2 Hl. 6. 9000 bottles.

Page 364.—7. \$8880. 8. \$.45. 9. 1000 l. 10. 89  $\frac{1}{4}$  Kg.  
11.  $1\frac{1}{4}$  cu. m. 12. 3941.635 Kg. 13. 169.16 m. 14. \$63.75.  
15. \$1100. 16. 600,000 tiles. 17. 140,400 T. 18. 4680 cu. m.  
19. \$1.29 a yard.



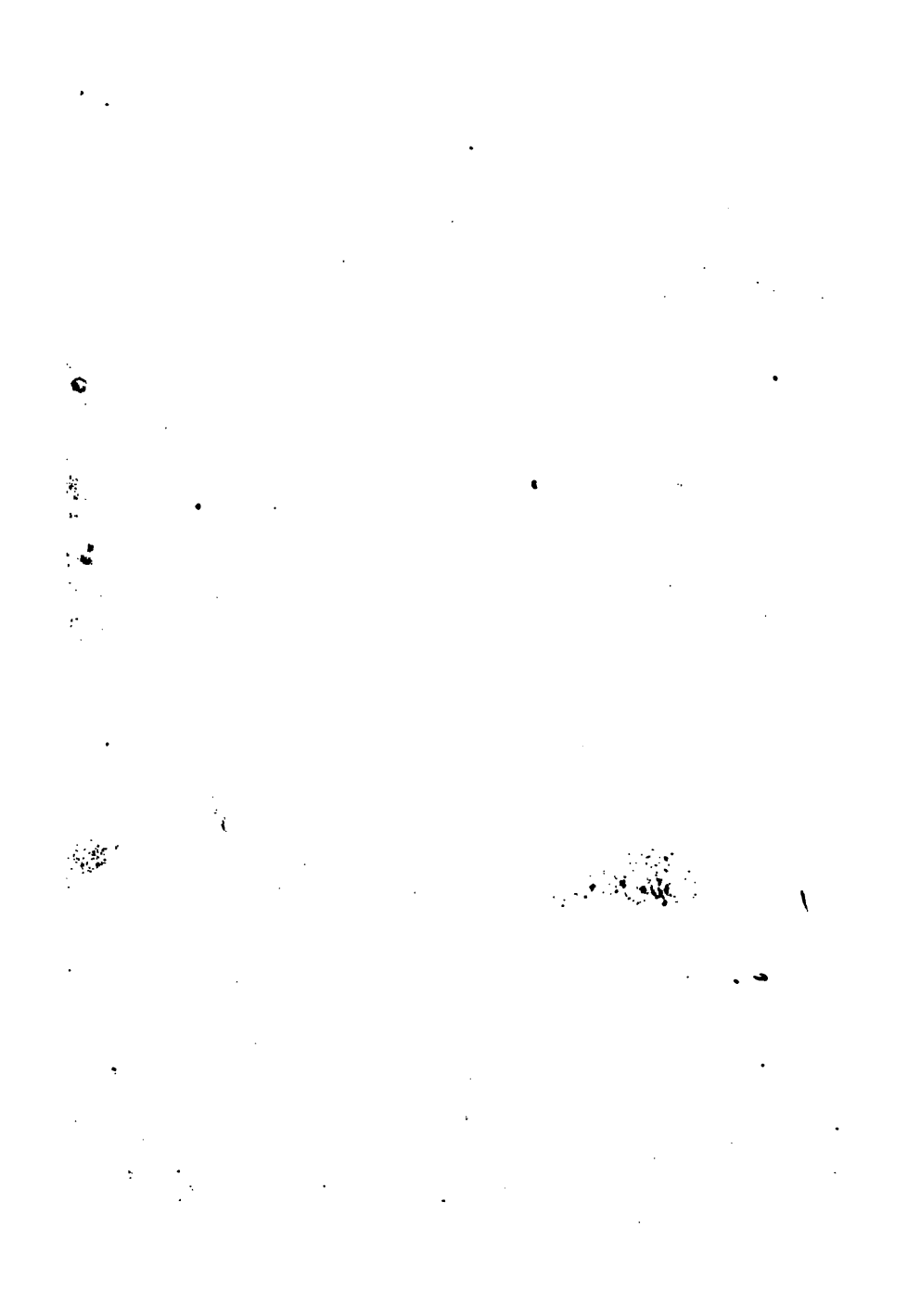
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1. The first part of the document is a list of names and titles, including "The Hon. Mr. Justice" and "The Hon. Mr. Justice".





Yellow & Purple



